Properties of an Ellipse

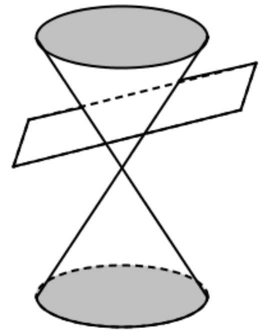
Student Activity

Name ________________

Class ________________

Introduction

All conic sections can be created by intersecting a plane and a right circular cone and simply changing the orientation of the plane. The diagram shown at right illustrates how to orient the plane to create a closed curve called an **ellipse**. How might you intersect a plane and a cone to create a circle, hyperbola, or parabola?



In this activity, you will focus on ellipses drawn on a coordinate plane, by answering the following two questions:

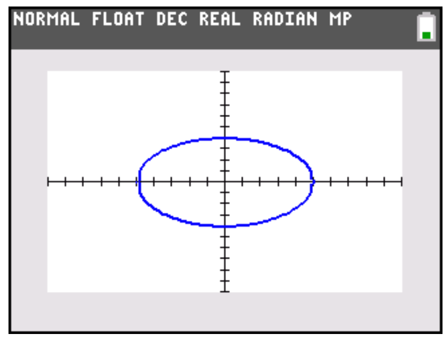
- How can we construct an ellipse on a coordinate plane?
- What interesting properties does an ellipse possess?

Change the display settings for the answers. Press [MODE], arrow down to **ANSWERS**, and press [ENTER] on **DEC**.

Problem 1 – Investigating the definition of an ellipse

An ellipse is shown at the right. It has foci at (3, 0) and (–3, 0). Use the following equation to graph the ellipse.

$$\mathbf{Y1} = 4\sqrt{1-\frac{x^2}{25}}, \quad \mathbf{Y2} = -4\sqrt{1-\frac{x^2}{25}}.$$



1. Trace the graph and record the coordinates of at least ten points in the table below. Find the distance from that point to both (3, 0) and (–3, 0). Then, find the total or sum of those distances.

Point	Distance from (3, 0)	Distance from (–3, 0)	Total Distance

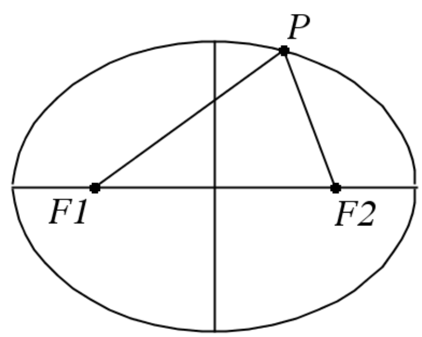
2. What do you notice about the distances and their sums?

3. Use your observations about these points and measurements to write a definition of an ellipse.

Problem 2 – An interesting property of an ellipse

Pick two of the points from the table on the previous page.

Using the given coordinates for *F1* and *F2*, find the distance between the points you picked and each point *F1* and *F2*.



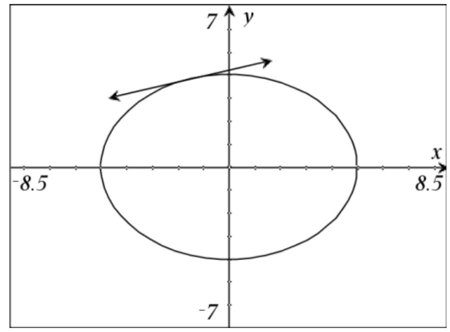
Point	*F1*	*F2*	Total Distance
	(–1, 0)	(1, 0)	
	(–1, 0)	(1, 0)	
	(–1, 0)	(2, 0)	
	(–1, 0)	(2, 0)	
	(–2, 0)	(4, 0)	
	(–2, 0)	(4, 0)	
	(–3, 0)	(4, 0)	
	(–3, 0)	(4, 0)	

4. What did you notice about the total distance?

5. What must be true about the foci?

6. Based on these observations, describe how to locate the foci of an ellipse.

On the graph at the right, place points at *F1*(3, 0), *F2*(–3, 0), and *P* the point of tangency. Draw a vector from *F1* to *P* and then another vector from *P* to *F2*.



7. What do you notice about direction of the vectors?

8. What is the relationship of these vectors?

Suppose these rays represent sound waves that originate *F1* and land at *F2*. Also consider that sound waves generally emanate in many directions, not just along a single ray.

9. Assume a person is standing at *F1* and whispers quietly. Where do you think a second person (not within earshot of the first person) would need to stand to hear this whisper clearly?

10. Summarize this property and explain how it relates to the foci and *any* point on the ellipse.

Problem 3 – Another interesting property of an ellipse

To determine the role that the location of the foci plays in determining the shape of an ellipse, graph the following equations of ellipses. The foci for each ellipse are given.

Foci	**Equations of Ellipses**
(–1, 0), (1, 0)	$\mathbf{Y1} = 2\sqrt{1-\frac{x^2}{5}}, \quad \mathbf{Y2} = -2\sqrt{1-\frac{x^2}{5}}$
(–5, 0), (5, 0)	$\mathbf{Y1} = 2\sqrt{1-\frac{x^2}{29}}, \quad \mathbf{Y2} = -2\sqrt{1-\frac{x^2}{29}}$
(–7, 0), (7, 0)	$\mathbf{Y1} = 2\sqrt{1-\frac{x^2}{53}}, \quad \mathbf{Y2} = -2\sqrt{1-\frac{x^2}{53}}$
(0, 0), (0, 0)	$\mathbf{Y1} = 2\sqrt{1-\frac{x^2}{4}}, \quad \mathbf{Y2} = -2\sqrt{1-\frac{x^2}{4}}$

11. What happens to the shape of the ellipse as the distance between the foci increases?

When you draw the foci closer, what happens?

When happens to the shape when the foci coincide? Does the definition of an ellipse still hold true?