We are excited that you will be using these interactive investigations to assist your students in exploring and learning about Transformational Geometry. They are designed so that students investigate and discover the mathematics in under 15 seconds on any one of these TI-Nspire™ learning platforms:

- TI-Nspire™ CX handheld
- TI-Nspire™ CX Teacher or Student Software
- TI-Nspire™ App for iPad®

To obtain the optimal results for your students, follow these suggestions:

- **Always do the "Tour" activity (Lesson 0 or Lesson 0 Vector) first**, before doing any of the other lessons. The Tour contains information that is needed to understand how to do the other lessons. Follow the recommended sequence: Tour first (Lesson 0), then Lesson 1, Lesson 2, etc.

- Transformational Geometry experts recommend investigations of transformations first without a grid and also without numerical properties. Then introduce numerical properties such as side lengths, angle measures, coordinates, as needed, with or without a grid.

- The best approach to using these activities in the classroom is to do part of each activity as a whole class, then have the students collaborate on other parts of the activity. **See the Suggested Lesson Plans beginning on page 4.**

- These activities are designed as a thorough first exposure to geometric transformations in middle grades and high school. Students should PLAY – INVESTIGATE – EXPLORE – DISCOVER using this technology. Also encourage students to make conjectures both verbally and handwritten. Use other paper and pencil activities, including compass and straightedge.

- Ensure that your students realize that observing something occur repeatedly does not "prove" it, we need to show that it works for **all** possibilities. To disprove requires only one counter example.

- The use of the **Think-Pair-Share** protocol works well with these activities. First students should do the exercise individually and make their own conjectures. They then share their ideas with a partner or group. Finally, the students share their conjectures with the class. The activities work best when students **read and follow the directions**.

- These lessons are created to investigate one or two concepts at a time. To investigate the concepts in a different order, create your own lessons utilizing this technology using the Options menu. **See the Teacher Tip on the bottom of page 3 of this document.**

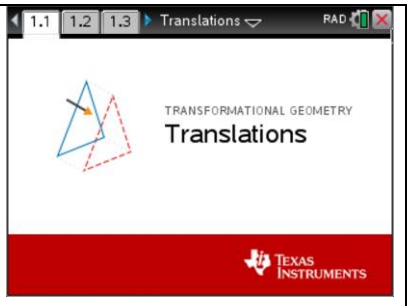
- To investigate the geometric transformations more deeply, use the other TI-Nspire activities found at the MathNspired.com website. These are located within the Geometry section and the Transformational Geometry subsection.

- The directions are written primarily for the TI-Nspire handheld. If using the computer software, use the mouse to select and move objects. If using the iPad app, tap on the appropriate icons and figures to move objects.

About the Translations "Lesson Bundle"

In the Translations "lesson bundle", students will explore translations and their properties. Throughout the lessons, students are encouraged to make observations related to their investigations, leading them to discovering the properties of translated figures. This lesson bundle includes the following lessons:

- **Lesson 0: Translations Tour.** Explore the defining properties of translations and learn how to use the translations files. ***This must be done first before doing the other lessons.*** Page 5.
- **Lesson 0 Vectors: Tour.** If you want the students to use a vector approach first, use this lesson as the tour. Otherwise students will study vectors in Lesson 5. Page 9.
- **Lesson 1: Angles & Sides.** Explore the relationship between measures of corresponding angles and lengths of corresponding sides of translated triangles. Page 13.
- **Lesson 2: Perimeters & Areas.** Explore the relationship between the perimeters and areas of translated triangles. Page 16.
- **Lesson 3: Grid & Coordinates**. Investigate the coordinates of vertices of triangles that have been translated and look for patterns. Represent the coordinates algebraically. Page 20.
- **Lesson 4: Translate by Hand**. Perform translations "by hand" on a grid. The *Translations_Lesson4.tns* file has four examples, with the first one being a tutorial. Page 24.
- **Lesson 5: Vectors.** Discover properties of triangles translated by vectors. Also explore the effects that moving the vector has on the translated figures. Page 28
- **Lesson 6: Corresponding Sides.** Investigate properties of corresponding sides of translated figures, other than their lengths and look for patterns. Page 32.
- **Lesson 7: Self-Assessment**. Summarize, review, explore and extend ideas about translations. Page 36.



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity
Lesson dependent

TI-Nspire document
Translations.tns
Translations_Lesson4.tns

Math Objectives

Students will:

- verify experimentally the properties of translations by direction or vector
- learn to identify and perform translations
- understand the effects that a translation has upon a triangle by easily manipulating the triangle
- explore the relationship between the corresponding parts of the pre-image and image triangles
- explain how underlying properties relate to translation
- discover how to translate figures by hand

Vocabulary

- Translation
- Image/Pre-Image
- Corresponding sides
- Corresponding angles
- Horizontal
- Vertical
- Slope (m)
- Notation: $\overline{AB}$ represents the segment, AB represents its length.
- Area
- Perimeter
- Parallel lines
- Perpendicular lines

TI-Nspire™ Navigator™

- Use Class Capture to monitor students' use of the TI-Nspire document.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Teacher Tip: These lessons are created to investigate one or two concepts at a time. To investigate other concepts, press the icon or the shortcut key [O] to open the Options menu. Use the [tab] key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar [␣] to select or un-select the options.
In this way, you can ***create your own investigations***.

Suggested Lesson Plans

Students should work in groups of four and be encouraged to discuss what they discover.

Lesson 0 Do page 1 with the class as they work in groups of four. Continue with page 2 with the class and encourage discussions. Lead the students as they do exercises 5 and 6 on page 3. Next have each group finish exercises 7 through 10. When students are finished, discuss the answers to exercises 8, 9, and 10 with the class.

Lesson 0 Vector Do exercises 1 through 5 with the class. Then have each group finish exercises 6 through 10. When students are finished, discuss the answers to exercises 8, 9, and 10 with the class.

Lesson 1 Do exercises 1 and 2 with the class. Then in each group of four, have each student pick a different part of exercise 3: i, ii, iii, or iv. Have the students discuss and compare their results. Next have each group finish exercises 4 through 6. When students are finished, discuss the answers to exercises 4, 5, and 6 with the class.

Lesson 2 Do exercises 1 and 2 a through d with the class. Then in each group of four, have each student pick a different part of exercise 2e: i, ii, iii, or iv. Have students complete exercises 2 f, g, h. Do exercise 3 a through d with the class. Then in each group of four, have each student pick a different part of exercise 3e: i, ii, iii, or iv. Have students complete exercises 3 f, g, h, and 4. Discuss answers as needed.

Lesson 3 Do exercises 1 and 2 with the class. Have the students do exercise 3 in groups, then discuss the answers. Next have students do exercises 4 and 5 in groups, then discuss the answers. Assign exercises 6 and 7 and discuss the answers.

Lesson 4 This exercises uses a different tns file: *Translations_Lesson4.tns*. Have students do exercise 1 in groups and use the tutorial as needed. Discuss the answers in class. Then have the students finish exercises 2, 3, and 4 and discuss the answers in their groups and with the class as needed.

Lesson 5 Do exercises 1 and 2 with the class. Have students do exercises 3, 4, and 5 in groups. Discuss the answers. We suggest that you at least do exercise 3 with the class and then decide how to proceed with exercises 4 and 5. Do exercise 6 with the class as needed. Then have students do exercise 7 in groups and discuss the answers.

Lesson 6 Do exercises 1 and 2 a through d with the class. Start exercise 2e with the class, calculating one or two of the slopes. Have the students finish 2e, f, g. Do exercise 3 with the class in groups. Have students finish exercises 4, 5, and 6, then discuss the answers with the class.

Lesson 7 Do exercises with the class as you think necessary. Assign any or all exercises and then discuss the answers and solutions with the class.

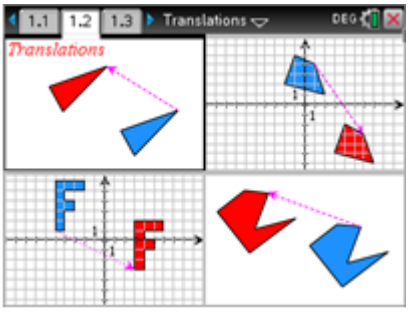
Lesson 0: Translations Tour

In this activity, you will investigate the defining properties of the transformation known as a translation by moving a triangle up, down, left, and right a number of units.
You will also learn how to easily and quickly maneuver within all the Translations activities using shortcut keys or the tab key.
Open the document: *Translations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.2. (ctrl ▶)

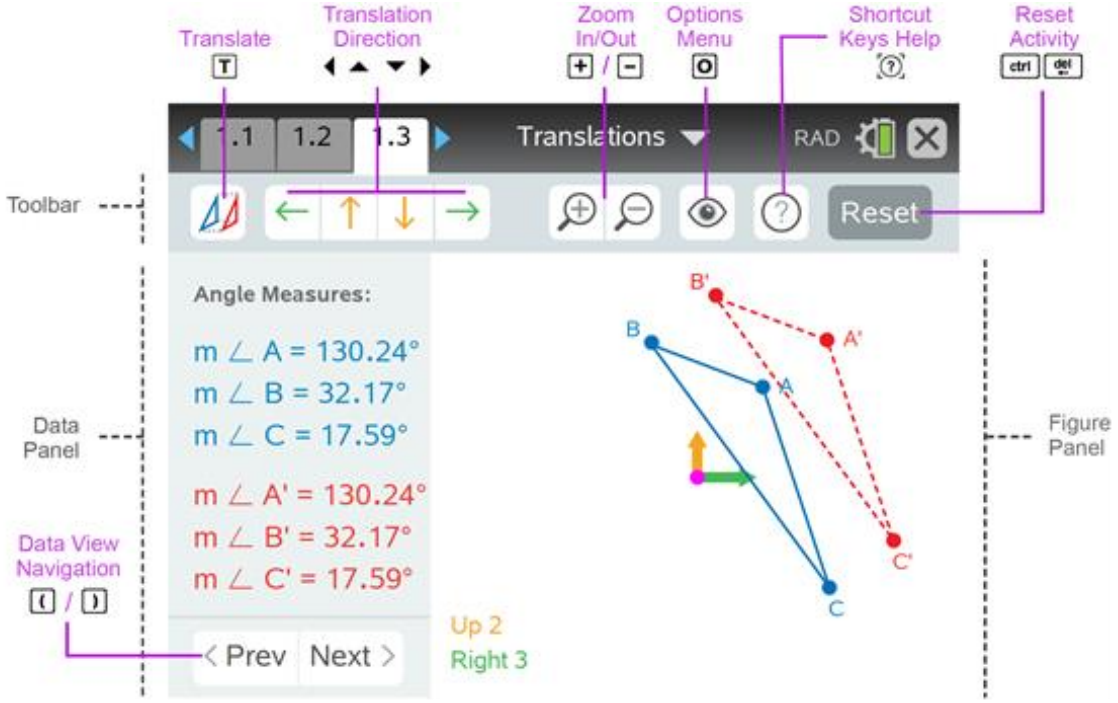
On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)



1. What do the 4 parts of the screen have in common?
   Make two conjectures.
   A ***conjecture*** is an opinion or conclusion based upon what is observed. Quickly discuss with your group.
   Sample Answer(s):

Figures are the same size, same shape (congruent) in each pane. The corresponding sides are congruent and corresponding angles are congruent. The pink dashed ray is drawn from the blue figure to the red one. One figure is red and the other is blue.

Move to page 1.3. (ctrl ▶) Look at the figure below of an overview of the main translations page and shortcut keys. ***Especially notice what the shortcut keys T, +, and – represent.***



Navigating to and Selecting Screen Options or Objects

Handheld Tech Tip:
To choose an option or object, use any of the following 3 methods:

- Use the touchpad to move the pointer over the option or object and press the center of the touchpad ( ) to select (**click**) it.
- Use [tab] to move to the next option or object on the screen and use [shift] [tab] to go to the previous option or object.
- Use a **shortcut key** (ex: [A] for vertex A, [T] to Translate, etc.). Letters [A], [B], [C] are located at the bottom of the handheld.

Use the method that works best for you: **click**, **tab** or **shortcut key**.

iPad Tech Tip:
To choose a command or object, tap the icon or the object.

2. On the handheld, press the tab key ([tab]) multiple times and notice each of the icons and points as they are highlighted. To go in the opposite direction, press [shift] [tab]. Investigate.

3. Shortcut keys provide a fast way to perform actions and/or select objects on the screen on the handheld. A list of all shortcuts can be found in the Shortcut Keys Help menu (click on or press [ctrl] [trig]). **Look at this list now.** Use as needed. Press [enter] or [esc] to close the Shortcut Keys Help menu.

4. To translate ΔABC, press the Translate key (click on or press [T]).
   Press the up arrow (▲) on the touchpad two times, then press the right arrow (▶) three times.
   Zoom in ([+]) or out ([−]) as needed. Observe what happens on the screen.

   Blue ΔABC is called the pre-image and red $\Delta A'B'C'$ is called the image.
   $\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."

5. To move and grab a vertex, press the letter key that corresponds to the vertex such as A ([A]), and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move vertex A. Grab and move point A to play, explore and discover ideas and investigate patterns.

Note: You can also use the **tab** key or **click** on the vertex that is needed.
(On the iPad®, tap the desired point and move it.)

What appears to be the relationship between ΔABC and $\Delta A'B'C'$? Discuss in your groups.

Answer: They appear to be the same shape and same size, i.e., congruent.

Grab and move vertex B (B). Grab and move vertex C (C). Observe.

Discuss with your partner or group: what appears to be true about the pre-image and its image? Write your conjecture below. A ***conjecture*** is an opinion or conclusion based upon what is observed.

Answer:
The red image $\Delta A'B'C'$ appears to be congruent to the blue pre-image ΔABC when it is translated.

6. Reset the page. Press Reset (ctrl del).
Press the down arrow (▼) on the touchpad four times, then press the right arrow (▶) five times.
Translate ΔABC (click on  or press (T). Zoom  in (+) or out (−) as needed.

   a. Discuss with your group as to what happens to every point on the pre-image ΔABC to obtain the image $\Delta A'B'C'$.

Proceed as follows for better visualization:

Open the Options menu (press  or (O). Press the down arrow (▼) 5 times and press the space bar (␣) to put a check mark on 'Resultant Vector'. Press enter or esc to close the menu.

Notice the three rays on the screen: the green ray is the horizontal component vector, the orange ray is the vertical component vector, while the pink ray is called the **resultant** vector.

Grab point W (W) and move it to coincide with vertex A. What point coincides with V? **Answer: A'**

Grab point W (W) and move it to coincide with vertex B. What point coincides with V? **Answer: B'**

Grab point W (W) and move it to coincide with vertex C. What point coincides with V? **Answer: C'**

Discuss in your group.

   b. Write a conjecture with respect to what happens to every point on the pre-image to obtain the image.

Answer: Every point on the pre-image is moved (slid) 4 units down and 5 units to the right.

7. Reset the page. Press Reset (ctrl del).
Press the down arrow (▼) on the touchpad three times, then press the left arrow (◀) five times.

Translate ΔABC (click on or press [T]).

Zoom in ([+]) or out ([−]) as needed. Observe what happens on the screen.
Proceed as follows for better visualization:

Open the Options menu (press or ([O]). Press the down arrow (▼) 5 times and press the space bar ([␣]) to put a check mark(select it) on 'Resultant Vector'. Then move to 'Connected Segments' and press the space bar ([␣]) to select it. Press [enter] or [esc] to close the menu.

Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, *and* $\overline{CC'}$.

Grab point W ([W]) and move it to coincide with vertex A. What point coincides with V? **Answer: A'**

Grab point W ([W]) and move it to coincide with vertex B. What point coincides with V? **Answer: B'**

Grab point W ([W]) and move it to coincide with vertex C. What point coincides with V? **Answer: C'**

Grab point V ([V]) and move it around the screen. Discuss what you observe with your group.

Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, *and* $\overline{CC'}$. Discuss with your group what you notice about those 3 segments and write your conjecture(s) below.

Sample Answer(s): The dashed segments, $\overline{AA'}$, $\overline{BB'}$, *and* $\overline{CC'}$, appear to be the same length, which is the same as the length of vector $\overrightarrow{WV}$. The three dashed segments and vector $\overrightarrow{WV}$ appear to be parallel to each other.

8. Many different triangles have been translated in several different direction.
Based upon your observations, write several conjectures about what seems to be true about a triangle and its translation.
A ***conjecture*** is an opinion or conclusion based upon what is observed.

Sample Answer(s): The pre-image triangle and its image triangle are congruent. The corresponding sides and corresponding angles are congruent. The segments that join corresponding vertices are congruent and parallel to each other and to the translation vector.

9. In a translation ΔABC is typically called the **pre-image** triangle and

$\Delta A'B'C'$ is typically called the **image** triangle.

How is $\Delta A'B'C'$ read? **Triangle A prime, B prime, C prime**

10. What is another word or words that you could use to describe what a translation does?
Answer: A translation 'slides' or 'moves' a figure.

Lesson 0: Translations Tour Vector

In this activity, you will investigate the defining properties of the transformation known as a translation from a vector standpoint. You will also learn how to easily and quickly maneuver within all the Translations activities using shortcut keys or the tab key.

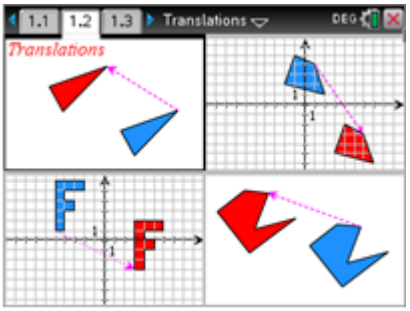
Open the document: *Translations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.2. (ctrl ▶)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

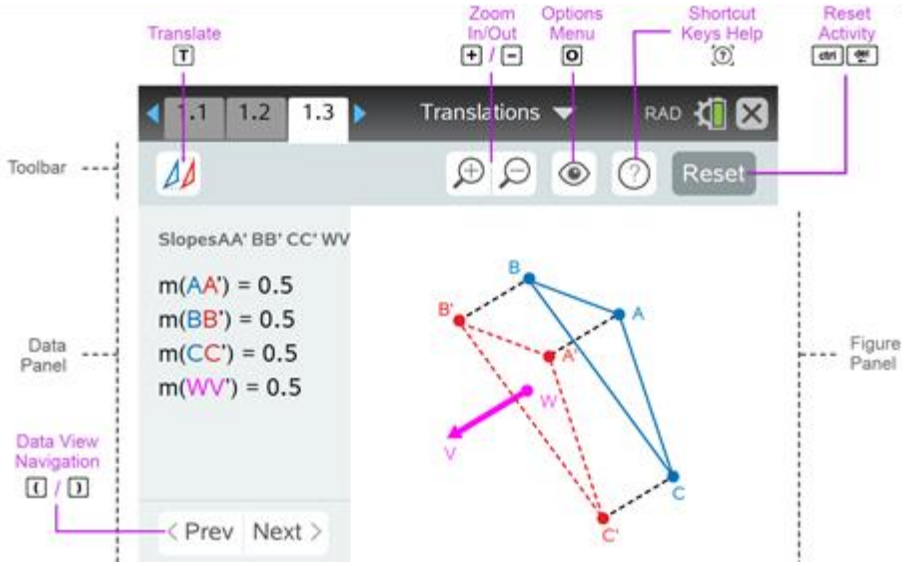
1. What do the 4 parts of the screen have in common? Make two conjectures.
   A ***conjecture*** is an opinion or conclusion based upon what is observed. Quickly discuss with your group.



Sample Answer(s):

Figures are the same size and same shape (congruent) in each pane. The corresponding sides are congruent and also corresponding angles are congruent. The pink dashed ray is drawn from the blue figure to the red one. One figure is red and the other is blue.

Move to page 1.3. (ctrl ▶) Look at the figure below of an overview of the main translations page and shortcut keys. ***Especially notice what the shortcut keys T, +, and – represent.***



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Use the method that works best for you: **click**, **tab** or **shortcut key**.

iPad Tech Tip:
To choose a command or object, tap the icon or the object.

2. Press [menu] to open the menu.
   (On the iPad®, tap on the wrench icon to open the menu.)
   Press [2] (2: Translation Mode), [2] (2: Vector).

3. On the handheld, press the tab key ([tab]) multiple times and notice each of the icons and points as they are highlighted. To go in the opposite direction, press [shift] [tab]. Investigate.

4. Shortcut keys provide a fast way to perform actions and/or select objects on the screen on the handheld. A list of all shortcuts can be found in the Shortcut Keys Help menu (click on or press [ctrl] [trig]). **Look at this list now.** Use as needed. Press [enter] or [esc] to close the Shortcut Keys Help menu.

5. To translate ΔABC about the **vector** $\overrightarrow{WV}$, press the Translate key (click on or press [T]).
   A **vector** is a directed line segment which has both length and direction.
   Zoom in ([+]) or out ([−]) as needed. Observe what happens on the screen.

   Blue ΔABC is called the pre-image and red $\Delta A'B'C'$ is called the image.
   $\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."

6. To move and grab a vertex, press the letter key that corresponds to the vertex such as A (A), and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move vertex A. Grab and move point A to play, explore and discover ideas and investigate patterns.

Note: You can also use the **tab** key or **click** on the vertex that is needed.
(On the iPad®, tap the desired point and move it.)

What appears to be the relationship between ΔABC and $\Delta A'B'C'$? Discuss with your groups.
Answer: They appear to be the same shape and same size, i.e., congruent.

Grab and move vertex B (B). Grab and move vertex C (C). Observe the triangles.

Discuss with your partner or group: what appears to be true about the pre-image and its image?
Write your conjecture about the two triangles below.
A ***conjecture*** is an opinion or conclusion based upon what is observed.

Answer: The red image $\Delta A'B'C'$ appears to be congruent to the blue pre-image ΔABC when it is translated.

7. Grab point W (W) and move it to coincide with vertex A. What point coincides with V? **Answer: A'**
Grab point W (W) and move it to coincide with vertex B. What point coincides with V? **Answer: B'**
Grab point W (W) and move it to coincide with vertex C. What point coincides with V? **Answer: C'**

With your group, discuss and describe how the vector affects the translation.

Answer: The blue pre-image triangle "slides" along the vector to get to the red image triangle.

8. Open the Options menu (press or (O). Use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move to 'Connected Segments' and press the space bar (␣) to put a check mark there. Press enter or esc to close the menu.
Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, *and* $\overline{CC'}$.

Grab and move the three vertices (A, B, C) and observe the three dashed segments.

To change the length and direction of vector $\overrightarrow{WV}$, grab point V (V) and move it around the screen.

Discuss in your groups what you observe. What do you observe?

Answer: The blue pre-image triangle does not move or change shape. The red image triangle changes position but does not change size or shape.

Grab point W (W) and move it to coincide with vertex A. What point coincides with V? **Answer: A'**
Grab point W (W) and move it to coincide with vertex B. What point coincides with V? **Answer: B'**
Grab point W (W) and move it to coincide with vertex C. What point coincides with V? **Answer: C'**

Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, *and* $\overline{CC'}$. Discuss in your group what you notice about those 3 segments and the vector $\overrightarrow{WV}$. Write conjecture(s) below.

Sample answers:

$\overline{AA'} \cong \overline{BB'} \cong \overline{CC'} \cong \overline{WV}$

$\overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'} \parallel \overline{WV}$

9. Many different triangles have been translated in several different directions about several different vectors.

   Based upon your observations, write several conjectures about what seems to be true about a triangle and its translation about a vector.
   A ***conjecture*** is an opinion or conclusion based upon what is observed.

   Sample Answer(s):
   The pre-image triangle and its image triangle are congruent. The corresponding sides and corresponding angles are congruent. The segments that join corresponding vertices are congruent and parallel to each other and to the translation vector.

10. In a translation ΔABC is typically called the **pre-image** triangle and

    $\Delta A'B'C'$ is typically called the **image** triangle.

    How is $\Delta A'B'C'$ read? **Triangle A prime, B prime, C prime**

11. What is another word or words that could be used to describe what a translation does?

    Sample Answer(s): A translation 'slides' or 'moves' a figure which can be determined by a vector.

Lesson 1: Angles & Sides

In this lesson, you will investigate the measures of angles and lengths of sides of triangles that have been translated in different ways. Open the document: *Translations.tns.*

It is important that one of the Translations Tours be done before any Translations lessons.

PLAY INVESTIGATE EXPLORE DISCOVER

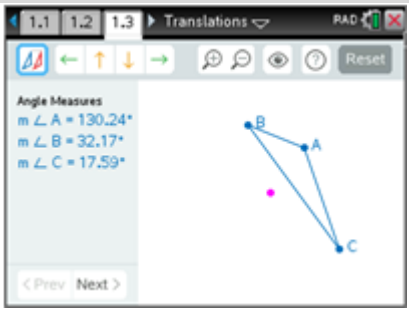
Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad®, tap on the wrench icon to open the menu.)

   Press 1 (1: Templates), 1 (1: Angles & Sides).



2. To translate ΔABC up 2 units and to the left 3 units, click on or press T, then press the up arrow (▲) twice and the left arrow (◀) three times.

   Zoom in (+) or out (−) as needed.

   a. Record the Original angle measures (first measures displayed) in the first row of the following table.

   b. Investigate and mentally make note of Angle Measures by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
   Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Up 2, Left 3	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Original	**130.24°**	**32.17°**	**17.59°**	**130.24°**	**32.17°**	**17.59°**
Figure 1	**90°**	**57.53°**	**32.47°**	**90°**	**57.53°**	**32.47°**

   c. Discuss observations in your group. Write a conjecture about the angle measures.

   Answer: The corresponding angles have the same measure (are congruent). Make sure that students don't incorrectly say 'all' angles are equal, but that 'corresponding' angles are.

d. Click on Next > or press [›] to see the lengths of the sides of the triangles.
Record the Original side lengths (first measures displayed) in the first row of the following table.

e. Investigate and mentally make note of Side Lengths by grabbing and moving each of the three vertices of ΔABC ([A], [B], [C]) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Up 2, Left 3	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{C'A'}$
Original	**7u**	**13.04u**	**11u**	**7u**	**13.04u**	**11u**
Figure 1	**8.54u**	**9.9u**	**10.05u**	**8.54u**	**9.9u**	**10.05u**

f. Discuss observations in your group. Write a conjecture about the lengths of the sides.

Answer: The corresponding sides have the same length (are congruent).

3. Reset the page. Press Reset ([ctrl] [del]).
Repeat what was done in exercise 2, but with each person in the group doing a different translation. Each person in the group should choose one from the following:

i) Translate ΔABC down 4 units and to the right 2 units.
ii) Translate ΔABC up 5 units.
iii) Translate ΔABC down 1 unit and to the left 4 units.
iv) Translate ΔABC up 6 units and to the left 3 units.

a. Record the Original angle measures (first measures displayed) in the first row of the table below.

b. Investigate and mentally make note of Angle Measures by grabbing and moving each of the three vertices of ΔABC ([A], [B], [C]) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Circle: i ii iii iv	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Original	**130.24°**	**32.17°**	**17.59°**	**130.24°**	**32.17°**	**17.59°**
Figure 1	**69.34°**	**66.12°**	**44.53°**	**69.34°**	**66.12°**	**44.53°**

c. Discuss observations in your group. Is your conjecture about the angle measures still true?

Answer: Yes.

d. Click on Next > or press) to see the lengths of the sides of the triangles.
Record the Original side lengths (first measures displayed) in the first row of the table below.

e. Investigate and mentally make note of Side Lengths by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Circle: i ii iii iv	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{C'A'}$
Original	**7.07u**	**9.43u**	**9.22u**	**7.07u**	**9.43u**	**9.22u**
Figure 1	**7.28u**	**12.21u**	**8u**	**7.28u**	**12.21u**	**8u**

f. Discuss observations in your group. Is your conjecture about the lengths of the sides still true?
Answer: Yes.

4. Many different triangles have been translated in several directions.
Generalize explorations and investigations by responding to the following:
a. If a triangle is translated, what appears to be true about the measures of the angles of the pre-image and image triangles?
Answer: The corresponding angles have the same measure (must use the word 'corresponding').
b. If a triangle is translated, what appears to be true about the lengths of the sides of the pre-image and image triangles?
Answer: The corresponding sides have the same length (must use the word 'corresponding').

5. Because the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent, the triangles are congruent.
Therefore, a translation is a **rigid motion**, or an **isometry**.
We also say that a translation is a **distance-preserving** and an **angle-preserving** transformation.

6. ΔDEF has been translated down 7 units and to the right 8 units. Answer the following.
a. If $m\angle D = 35^o$, $m\angle D' =$ **35^o**.

b. If EF = 8 cm, E'F' = **8cm** .

c. If $m\angle E = 120^o$, what other angle has a measure of 120^o? $\angle E'$

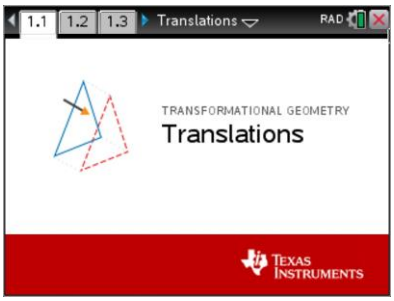
d. If DF = 3 in, what other segment has a length of 3 in? **D'F'**

Lesson 2: Perimeters & Areas

In this lesson, you will investigate the perimeters and areas of triangles that have been translated in different directions.

Open the document: *Translations.tns.*

It is important that one of the Translations Tour be done before any Translations lessons.



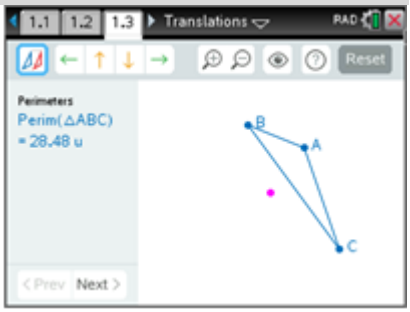
PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad®, tap on the wrench icon to open the menu.)
   Press 1 (1: Templates), 2 (2: Perimeter & Area).



2. Translate ΔABC down 4 and to the left 5.

   Press the down arrow (▼) four times and the left arrow (◀) five times then click on or press T. Zoom in (+) or out (−) as needed.

   a. Record the Original perimeters (first measures displayed) in the appropriate places of the **Down 4 Left 5** section in the table below.

   b. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Down 4 Left 5	Perimeter ΔABC	Perimeter $\Delta A'B'C'$		**Up 3 Right 6**	Perimeter ΔABC	Perimeter $\Delta A'B'C'$
Original	**28.48u**	**28.48u**		Original	**28.48u**	**28.48u**
Figure 1	**24u**	**24u**		Figure 1	**24.3u**	**24.3u**

c. Reset the page. Press Reset (ctrl del).
Translate ΔABC up 3 and to the right 6.
Press the up arrow (▲) three times and the right arrow (▶) six times then click on  or press T. Zoom  in (+) or out (−) as needed.
Record the Original perimeters in the appropriate places of the **Up 3 Right 6** section in the previous table.

d. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the previous table.

e. Reset the page. Press Reset (ctrl del).
Repeat what was done in parts a – d, but with each person in the group choosing a different translation. Each person in the group should choose one from the following:
i) Translate ΔABC down 4 units and to the right 2 units.
ii) Translate ΔABC up 5 units.
iii) Translate ΔABC down 1 unit and to the left 4 units.
iv) Translate ΔABC up 6 units and to the left 3 units.
Record the Original perimeters in the appropriate places in the following table.
Sample Answer(s):

Circle: i ii iii iv	Perimeter ΔABC	Perimeter $\Delta A'B'C'$
Original	**28.48u**	**28.48u**
Figure 1	**32.28u**	**32.28u**

f. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row" Figure 1" in the previous table.

g. Many different triangles were translated in several different directions.
Make a conjecture about the perimeters of translated triangles.
A ***conjecture*** is an opinion or conclusion based upon what is observed.
Answer: If a triangle is translated in any direction, the image triangle and the pre-image triangle have the same perimeter.

h. Based on explorations of translated triangles in previous lessons, explain why this conjecture is true.

Answer: In Lesson 1, we discovered that corresponding sides have the same length when triangles are translated. If you add equal numbers, the sums (perimeters) will be equal also.

3. Do a similar exploration about the areas of translated triangles.

a. Reset the page. Press Reset (ctrl del).
Translate ΔABC down 4 and to the left 5.
Press the down arrow (▼) four times and the left arrow (◀) five times then click on or press T. Zoom in (+) or out (−) as needed.
Click on Next or press) to explore the areas of the triangles.

Record the Original areas (first measures displayed) in the appropriate places of the **Down 4 Left 5** section in the table below.

Sample Answer(s):

Down 4 Left 5	Area ΔABC	Area $\Delta A'B'C'$		**Up 3 Right 6**	Area ΔABC	Area $\Delta A'B'C'$
Original	**19.5 sq u**	**19.5 sq u**		Original	**19.5 sq u**	**19.5 sq u**
Figure 1	**16 sq u**	**16 sq u**		Figure 1	**28 sq u**	**28 sq u**

b. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the previous table.

c. Reset the page. Press Reset (ctrl del).
Translate ΔABC up 3 and to the right 6. Click on Next or press) to explore the areas of the triangles
Record the Original areas in the appropriate places of the **Up 3 Right 6** section in the previous table.

d. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the previous table.

e. Reset the page. Press Reset (ctrl del).
Repeat what was done in parts a – d, but each person in the group choosing a different translation. Record the Original areas in the appropriate place in the following table.
Each person in the group should choose one from the following:
i) Translate ΔABC down 4 units and to the right 2 units.
ii) Translate ΔABC up 5 units.
iii) Translate ΔABC down 1 unit and to the left 4 units.
iv) Translate ΔABC up 6 units and to the left 3 units.
Click on Next or press) to explore the areas of the triangles
Record the Original areas in the appropriate place in the following table.
Sample Answer(s):

Circle: i ii iii iv	Area ΔABC	Area $\Delta A'B'C'$
Original	**19.5 sq. u**	**19.5 sq. u**
Figure 1	**32.5 sq. u**	**32.5 sq. u**

f. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the previous table.

g. Many different triangles were translated in several different directions.
Make a conjecture about the areas of translated triangles.
A **conjecture** is an opinion or conclusion based upon what is observed.
If a triangle is translated in any direction, the image triangle and the pre-image triangle have the same area.

h. Based on explorations of translated triangles in previous lessons, explain why this conjecture is true.
In translated triangles, corresponding bases are congruent, corresponding heights are congruent, so $A = \frac{1}{2} \cdot base \cdot height$ will be equal for the image and pre-image triangles.

4. ΔJKL is translated to the left 4 units. The perimeter of ΔJKL is 40 cm and its area is 60 sq. cm.

a. What is the perimeter of $\Delta J'K'L'$? **40 cm**

b. What is the area of $\Delta J'K'L'$? **60 sq cm**

Lesson 3: Grid & Coordinates

In this lesson, you will investigate the coordinates of vertices of translated triangles and look for patterns.

Open the document: *Translations.tns.*

It is important that one of the Translations Tours be done before any Translations lessons.

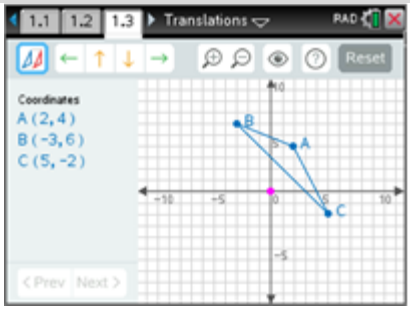
PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad, tap the wrench icon to open the menu.)
   Press 1 (1: Templates), 5 (5: Grid & Coordinates).



2. Translate ΔABC **to the right 5 units** by pressing the right arrow (▶) 5 times.

   Then click on or press T. Zoom in (+) or out (−) as needed.

   a. Record the Original coordinates (first coordinates displayed) in the first row of the table below. Look for patterns.

   b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
   Record a set of data observed in row "Figure 1" in the following table.
   Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
   Look for patterns among the coordinates of corresponding vertices.
   Which coordinates remain the same? Which coordinates change? How? Discuss.
   Sample Answer(s): The y-coordinates remain the same. The x-coordinates change, increasing by 5.

Translate Right 5	A	B	C	A'	B'	C'
Original	**(2, 4)**	**(– 3, 6)**	**(5, – 2)**	**(7, 4)**	**(2, 6)**	**(10, – 2)**
Figure 1	**(3, 3)**	**(– 4, 6)**	**(6, – 3)**	**(8, 3)**	**(1, 6)**	**(11, – 3)**
Figure 2	**(2, 5)**	**(– 3,3)**	**(3, – 4)**	**(7, 5)**	**(2, 3)**	**(8, – 4)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (1, 2), what are the coordinates of the corresponding point on the image triangle?
That is (1, 2) → **(6, 2)** '→' means "maps to"

Similarly, the point (– 3, 7) would be translated to? That is (– 3, 7) → **(2, 7)**

d. Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y), what are the coordinates of the corresponding point on the image triangle?

That is (x, y) → **(x + 5, y)** '→' means "maps to"

3. Reset the page. Press Reset (ctrl del).
Translate ΔABC **down 4 units** by pressing the down arrow (▼) 4 times.
Then click on or press (T). Zoom in (+) or out (–) as needed.

a. Record the Original coordinates (first coordinates displayed) in the first row of the following table. Look for patterns.

b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Look for patterns among the coordinates of corresponding vertices.
Which coordinates remain the same? Which coordinates change? How? Discuss.
Sample Answer(s): The x-coordinates remain the same. The y-coordinates change, decreasing by 4.

Translate Down 4	A	B	C	A'	B'	C'
Original	**(2, 4)**	**(– 3, 6)**	**(5, – 2)**	**(2, 0)**	**(– 3, 2)**	**(5, – 6)**
Figure 1	**(4, 6)**	**(– 3, 5)**	**(2, – 4)**	**(4, 2)**	**(– 3, 1)**	**(2, – 8)**
Figure 2	**(3, 8)**	**(– 4, 3)**	**(5, 0)**	**(3, 4)**	**(– 4,– 1)**	**(5, – 4)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (1, 2), what are the coordinates of its corresponding point on the image triangle?
That is (1, 2) → **(1, – 2)** '→' means "maps to"

Similarly, the point (– 3, 7) would be translated to? That is (– 3, 7) → **(– 3, 3)**

d. Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y), what are the coordinates of its corresponding point on the image triangle?

That is (x, y) → **(x, y – 4)** '→' means "maps to"

4. Reset the page. Press Reset (ctrl del).
Translate ΔABC **to the left 3 units and up 2 units**.
Then click on  or press [T]). Zoom  in ([+]) or out ([–]) as needed.

a. Record the Original coordinates (first coordinates displayed) in the first row of the following table. Look for patterns.

b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC ([A], [B], [C]) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Look for patterns among the coordinates of corresponding vertices.

Sample Answer(s):

Translate **Up 2 Left 3**	A	B	C	A'	B'	C'
Original	**(2, 4)**	**(– 3, 6)**	**(5, – 2)**	**(– 1, 6)**	**(– 6, 8)**	**(2, 0)**
Figure 1	**(0, 7)**	**(– 4, 3)**	**(6, 0)**	**(– 3, 9)**	**(– 7, 5)**	**(3, 2)**
Figure 2	**(0, 0)**	**(– 3, 6)**	**(5, 1)**	**(– 3, 2)**	**(– 6, 8)**	**(2, 3)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (1, 2), what are the coordinates of its corresponding point on the image triangle?
That is (1, 2) → **(– 2, 4)** '→' means "maps to"

Similarly, the point (– 3, 7) would be translated to? That is (– 3, 7) → **(– 6, 9)**

d. Generalize this: If a point on the pre-image triangle has coordinates (x, y), what are the coordinates of the corresponding point on the image triangle?

That is (x, y) → **(x – 3, y + 2)** '→' means "maps to"

5. Given: ΔDEF is translated to the right 4 units and down 2 units.

   a. If D has coordinates (5, 7), what are the coordinates for D'? **(9, 5)**

   b. If E has coordinate (– 3, – 7), what are the coordinates of E'? **(1, – 9)**

   c. If F' has coordinates (1, 6), what are the coordinates of F? **(– 3, 8)**

   d. If D has coordinates (x, y), what are the coordinates for D'? **(x + 4, y – 2)**

6. Given: ΔPQR is translated up 5 units and to the left 6 units.

   a. If P has coordinates (5, 7), what are the coordinates for P'? **(– 1, 12)**

   b. If Q has coordinate (– 3, – 7), what are the coordinates of Q'? **(– 9, – 2)**

   c. If R' has coordinates (1, 6), what are the coordinates of R? **(7, 1)**

   d. If P has coordinates (x, y), what are the coordinates for P'? **(x – 6, y + 5)**

7. Given: ΔXYZ is translated to the left 3 units and down 9 units.

   a. If X has coordinates (5, 7), what are the coordinates for X'? **(2, – 2)**

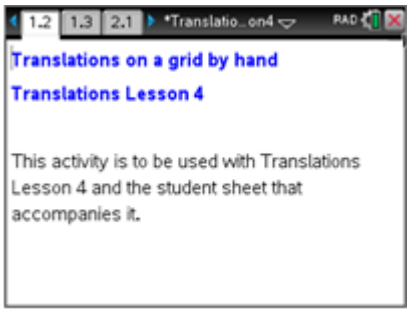
   b. If Y has coordinate (– 3, – 7), what are the coordinates of Y'? **(– 6, – 16)**

   c. If Z' has coordinates (1, 6), what are the coordinates of Z? **(4, 15)**

   d. If X has coordinates (x, y), what are the coordinates for X'? **(x – 3, y – 9)**

Lesson 4: Translate by Hand

In this lesson, you will translate a triangle on a grid without technology. Open the document: *Translations_Lesson4.tns.*



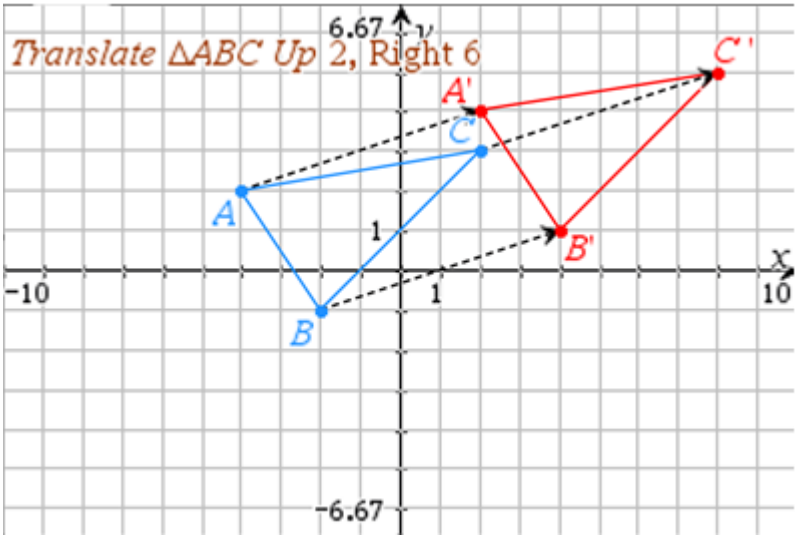
It is important that one of the Translations Tours be done before any Translations lessons.

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.2. (ctrl ▶) Read page 1.2.

On the handheld, press ctrl ▶ and ◀ to navigate through the pages of the lesson.
On the iPad®, select the page thumbnail in the page sorter panel.

1. **Move to page 1.3.** (ctrl ▶) **Translate ΔABC up 2 units, right 6 units using a straightedge.**
   Read and follow the directions using the figure below.
   To check your answer or to get help, press the right arrow (▶) on the touchpad to advance a step and press the left arrow (◀) to go back a step, as needed.
   Label the vertices and show the three dashed segments that connect corresponding vertices.



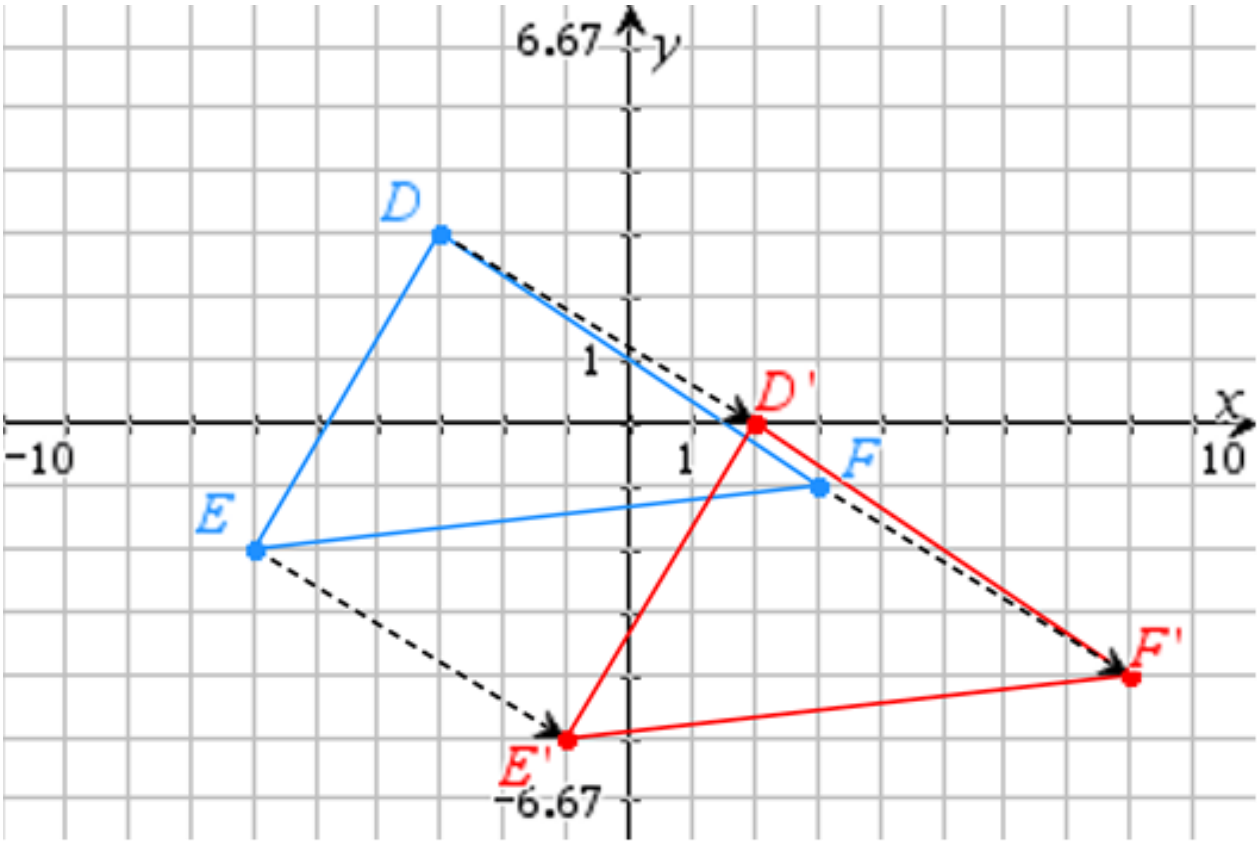
List the coordinates of each of the six vertices:

A: **(– 4, 2)** B: **(– 2, – 1)** C: **(2, 3)**

A': **(2, 4)** B': **(4, 1)** C': **(8, 5)**

If a point on ΔABC has coordinates (x, y), what will be the coordinates of its image on $\Delta A'B'C'$? **(x + 6, y + 2)**

2. **Translate ΔDEF down 3 units, right 5 units using a straightedge.**
Label the vertices and show the three dashed segments that connect corresponding vertices.



List the coordinates of each of the six vertices:

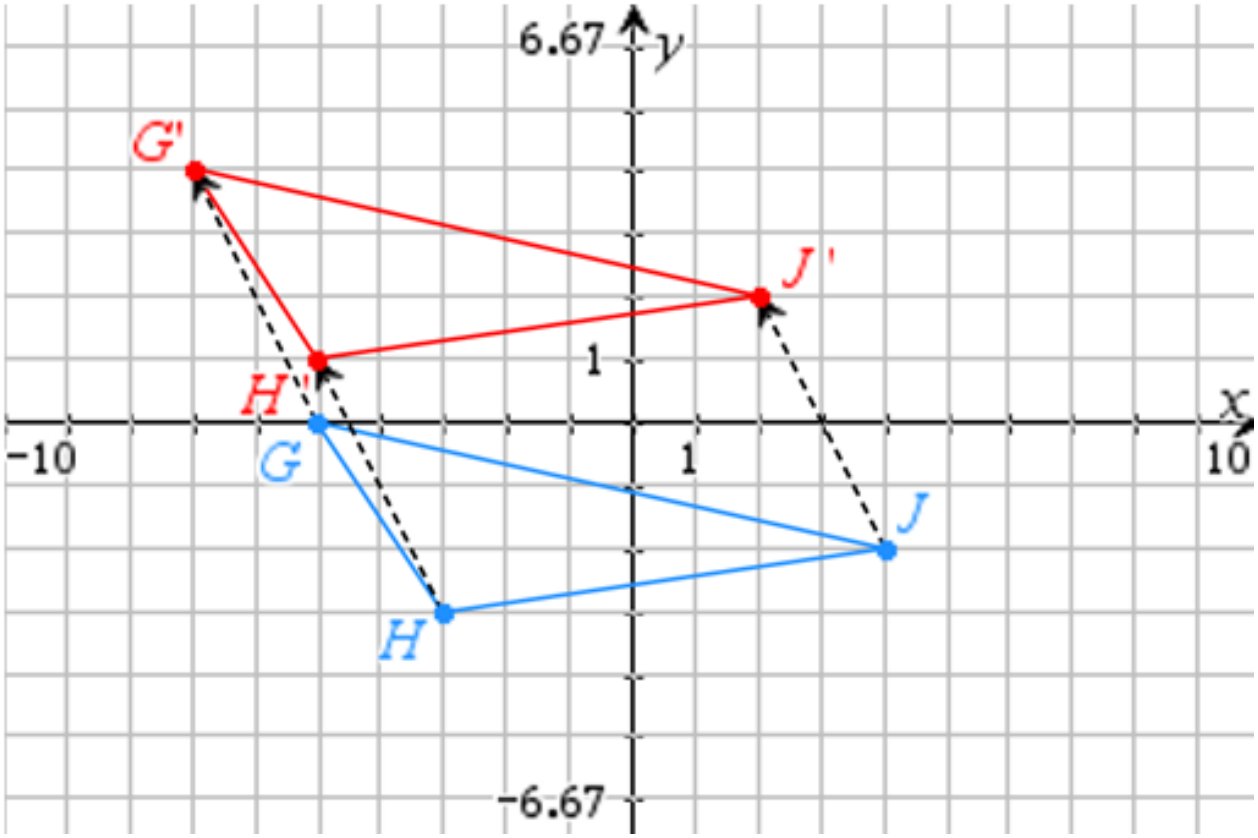
D: **(– 3, 3)** E: **(– 6, – 2)** F: **(3, – 1)**

D': **(2, 0)** E': **(– 1, – 5)** F': **(8, – 4)**

If a point on ΔDEF has coordinates (x, y), what will be the coordinates of its image on $\Delta D'E'F'$? **(x + 5, y – 3)**

Using the expressions listed in your last answer above, check your answers for the coordinates of the six vertices. Make corrections as needed.

3. **Translate ΔGHJ up 4 units, left 2 units using a straightedge.**
Label the vertices and show the three dashed segments that connect corresponding vertices.



List the coordinates of each of the six vertices:

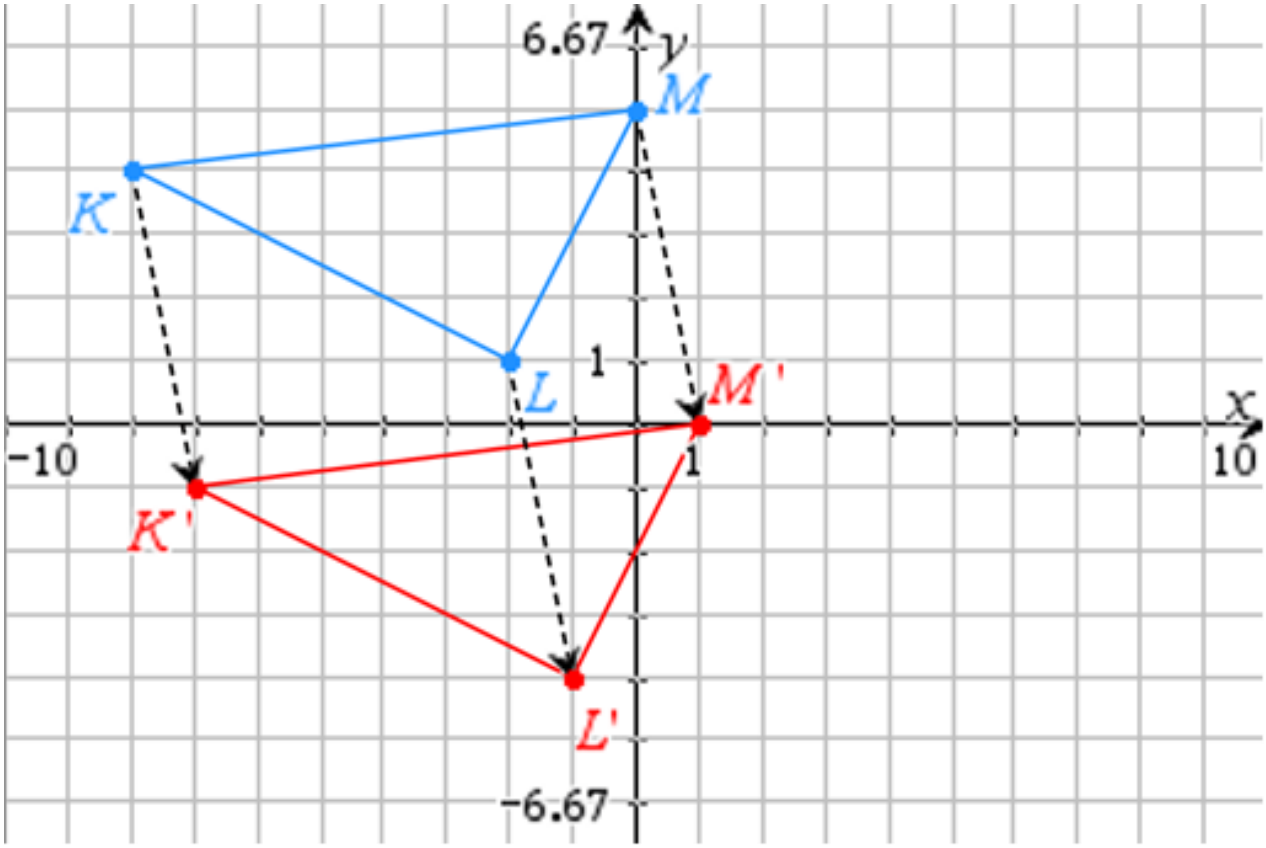
G: **(– 5, 0)** H: **(– 3, – 3)** J: **(4, – 2)**

G': **(– 7, 4)** H': **(– 5, 1)** J': **(2, 2)**

If a point on ΔGHJ has coordinates (x, y), what will be the coordinates of its image on $\Delta G'H'J'$? **(x – 2, y + 4)**

Using the expressions listed in your last answer above, check your answers for the coordinates of the six vertices. Make corrections as needed.

4. **Translate ΔKLM down 5 units, right 1 unit using a straightedge.**
Label the vertices and show the three dashed segments that connect corresponding vertices.



List the coordinates of each of the six vertices:

K: **(– 8, 4)** L: **(– 2, 1)** M: **(0, 5)**

K': **(– 7, – 1)** L': **(– 1, – 4)** M': **(1 , 0)**

If a point on ΔKLM has coordinates (x, y), what will be the coordinates of its image on $\Delta K'L'M'$? **(x + 1, y – 5)**

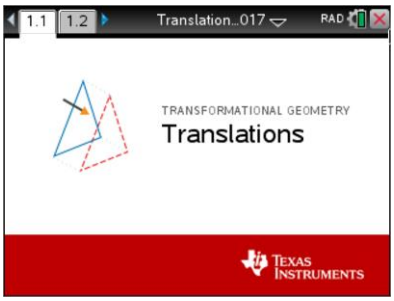
Using the expressions listed in your last answer above, check your answers for the coordinates of the six vertices. Make corrections as needed.

Lesson 5: Vectors

In this lesson, you will investigate how to translate a triangle using a vector and how this relates to previous work with translations.

Open the document: *Translations.tns.*

It is important that one of the Translations Tours be done before any Translations lessons.

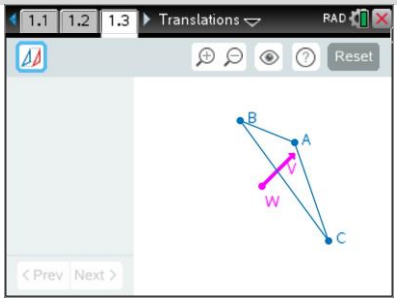


PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.
   (On the iPad, tap the wrench icon to open the menu.)
   Press 1 (1: Templates), 8 (8: Vector).



2. Translate ΔABC by vector $\overrightarrow{WV}$ by clicking on or pressing T.
   Zoom in (+) or out (−) as needed.
   a. Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the **vector** $\overrightarrow{WV}$.
      A **vector** is a directed line segment which has both length and direction.
      Discuss in your groups what seems to be true about vector $\overrightarrow{WV}$ and these three dashed segments.

   b. Grab the endpoint of the vector, W, by clicking on it or pressing W and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move point W so that it coincides with vertex A.
      What point coincides with V? **Answer: A'**
      Move point W so that it coincides with vertex B. What point coincides with V? **Answer: B'**
      Move point W so that it coincides with vertex C. What point coincides with V? **Answer: C'**

      Discuss in your groups what seems to be true about the dashed segments, $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the vector $\overrightarrow{WV}$. Write your conjecture(s) below.
      Answer: The lengths of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and $\overrightarrow{WV}$ are equal, that is, AA' = BB' = CC'.
      The segments: $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and $\overrightarrow{WV}$, are parallel.

c. Click on or press [T] to undo the translation. Grab and move point W ([W]) and redo the translation by clicking on or by pressing [T] again. What do you observe?
Grab and move point W ([W]) about the screen. What do you observe?
Does moving point W change the result of the translation?
Discuss in your groups and write a conjecture below.
Answer: No, the location of the endpoint of the translation vector does not affect the result of the translation.

3. Investigate what happens when point V on the vector is moved.

a. Grab point V by clicking on it or pressing [V] and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move V to several places on the screen. Look at the dashed segments $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the vector $\overrightarrow{WV}$. Discuss in your groups what you observe.

b. Grab and move each of the three vertices of ΔABC ([A], [B], [C]) to create different shaped triangles. Discuss in your groups what segment lengths appear to be equal. Write a conjecture about segment lengths.
Answer: **AA' = BB' = CC' = WV. When a triangle is translated by a vector, the segments formed by joining corresponding vertices from the pre-image to the image triangle are equal in length, and equal to the length of vector VW.**

c. To confirm or disprove your conjecture, open the Options menu (press or ([O]).
Select "AA' BB' CC' WV' " by putting a check mark in the box next to it using the space bar key (␣). Press [esc] .
Look at the lengths displayed.

d. Further investigate by grabbing and moving each of the three vertices ([A], [B], [C]) and look at the lengths displayed as you do this. Is your conjecture still true? Discuss with your group.

e. Investigate further by grabbing and moving point V ([V]) about the screen.
Is your conjecture still true? Discuss in your group.

f. Grab and move point W. How does that affect what is displayed? Discuss in your group.
Based on your investigations, what seems to be true about the dashed segments $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the vector $\overrightarrow{WV}$?
Answer: $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, **and the vector** $\overrightarrow{WV}$ **appear to be congruent and parallel.**

4. Open the Options menu (press or (O).
   a. Select "Slope AA' BB' CC' WV' " by putting a check mark in the box next to it using the space bar key (␣). Press esc. Click on Next or press) to see the next set of data.
   Look at the slopes displayed. Make a conjecture based upon those values. Discuss with your group.
   Answer: The slopes of these four segments are equal, which means that the four segments are parallel.
   b. Investigate further by grabbing and moving each of the three vertices (A, B, C) and look at the slopes displayed. Is your conjecture still true? Discuss in your group.

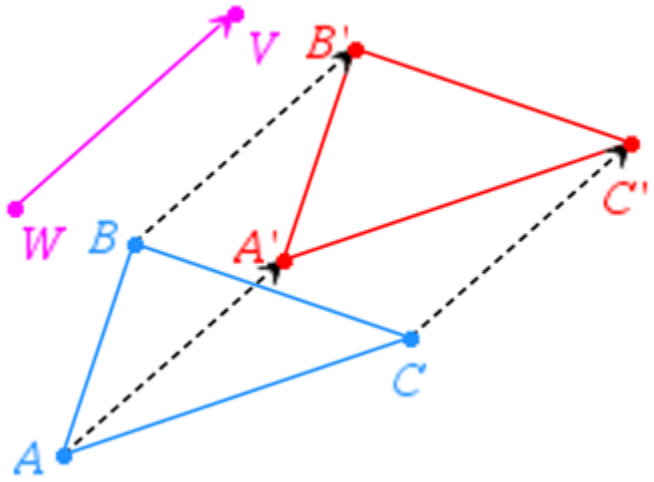
   c. Investigate further by grabbing and moving point V (V) about the screen.
   Is your conjecture still true? Discuss in your group.

   d. Grab and move point W. How does that affect what is displayed? Discuss in your group.
   Based on your investigations, what seems to be true about the dashed segments $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the vector $\overrightarrow{WV}$?
   Answer: **They are parallel.**
   e. Grab the endpoint of the vector, W, by clicking on it or pressing W and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move point W so that it coincides with vertex A. What point coincides with V? **Answer: A'**
   Move point W so that it coincides with vertex B. What point coincides with V? **Answer: B'**
   Move point W so that it coincides with vertex C. What point coincides with V? **Answer: C'**

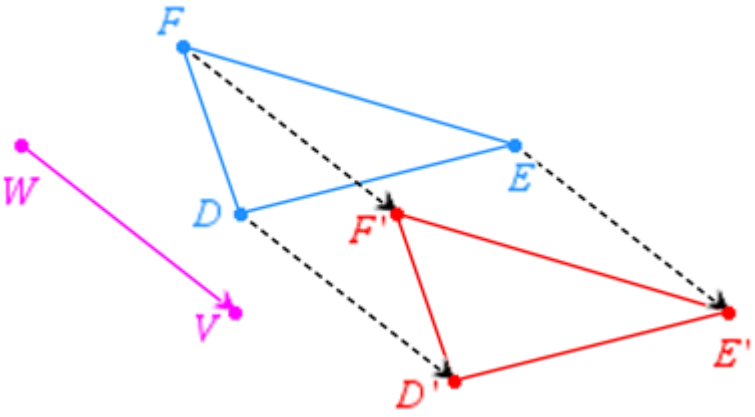
   Based on your investigations, what seems to be true about the dashed segments $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the vector $\overrightarrow{WV}$?
   Answer: They are parallel.
5. Many triangles have been translated by a given vector. Answer the following based on this activity.
   a. What is a vector?
   Answer: A vector is a directed line segment.

   b. A vector has both **length** and **direction**.
   c. A translation can also be called a **slide**.
   d. Lines that are parallel have the same **slope**.
   e. Based on your discoveries, write a definition for translating a triangle about a vector.
   If a triangle is translated by a vector, each point on the image is the same distance away from its corresponding point on the pre-image and in the same direction.

6. We have been using technology to translate the triangles. Now do this "by hand" using a straightedge.
Translate ΔABC by vector $\overrightarrow{WV}$. Also, draw the dashed segments, $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$.



7. Translate ΔDEF by vector $\overrightarrow{WV}$. Also, draw the dashed segments, $\overline{DD'}$, $\overline{EE'}$, $\overline{FF'}$.



Using the figure above, answer the following questions.

a. List 3 other segments that are parallel to $\overline{EE'}$:

Answer: $\overline{DD'}$, $\overline{FF'}$, $\overline{WV}$

b. If DD' = 4 cm, then what other segments have a length of 4 cm?

Answer: EE' = FF' = WV = 4 cm = DD'

Lesson 6: Corresponding Sides

In this lesson, you will investigate the corresponding sides (not their lengths) of translated triangles and look for patterns.

Open the document: *Translations.tns.*

It is important that one of the Translations Tours be done before any Translations lessons.

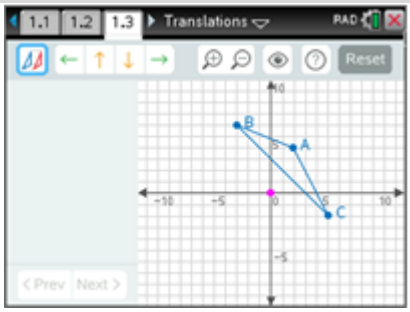
PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad, tap the wrench icon to open the menu.)
   Press 1 (1: Templates), 4 (4: Grid).



2. Translate ΔABC **to the right 5 units** by pressing the right arrow (▶) 5 times.

   Then click on or press T. Zoom in (+) or out (−) as needed.

   a. Look at corresponding sides, $\overline{AB}$ *and* $\overline{A'B'}$. We have already established that these two segments are congruent (have the same length).
   What else appears to be true about these two segments?
   Answer: They appear to be parallel.

   b. Look at corresponding sides, $\overline{BC}$ *and* $\overline{B'C'}$. We have already established that these two segments are congruent (have the same length).
   What else appears to be true about these two segments?
   Answer: They appear to be parallel.

   c. Look at corresponding sides, $\overline{CA}$ *and* $\overline{C'A'}$. We have already established that these two segments are congruent (have the same length).
   What else appears to be true about these two segments?
   Answer: They appear to be parallel.

d. If segments (lines) are to be parallel, what must be true about their slopes?
Answer: Their slopes must be the same (equal).

e. Calculate the slope of each pair of corresponding sides. Record your answers as fractions:

Slope of $\overline{AB} = -\frac{2}{5}$ Slope of $\overline{A'B'} = -\frac{2}{5}$

Slope of $\overline{BC} = -1$ Slope of $\overline{B'C'} = -1$

Slope of $\overline{CA} = -2$ Slope of $\overline{C'A'} = -2$

f. Based upon the results in part e above, is each pair of corresponding sides parallel?
Answer: Yes, each pair of corresponding sides have the same slope.

g. This is not enough evidence to prove this conjecture for all triangles. Let's investigate more examples.

3. Press menu to open the menu.

(On the iPad, tap the wrench icon to open the menu.)
Press 1 (1: Templates), 6 (6: Slopes Sides).
Translate ΔABC **up 3 units** by pressing the up arrow (▲) 3 times and to the **left 6 units** by pressing the left arrow (◀) 6 times.
Then click on or press T. Zoom in (+) or out (−) as needed.

a. Record the Original slopes (first slopes displayed) in the first row of the following table. Look for patterns.

Sample Answer(s):

Translate Up 3,Left 6	m($\overline{AB}$)	m($\overline{BC}$)	m($\overline{CA}$)	m($\overline{A'B'}$)	m($\overline{B'C'}$)	m($\overline{C'A'}$)
Original	**– 0.4**	**– 1**	**– 2**	**– 0.4**	**– 1**	**– 2**
Figure 1	**– 0.25**	**– 2**	**5**	**– 0.25**	**– 2**	**5**
Figure 2	**0.75**	**– 1.5**	**3**	**0.75**	**– 1.5**	**3**

b. Investigate and mentally make note of the slopes by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the previous table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" in the previous table.
Look for patterns among the slopes of corresponding sides.

c. Using the pattern observed in the previous table, state a conjecture.

Answer: If a triangle is translated, each side of the image triangle is parallel to its corresponding side of its pre-image triangle.

4. Reset the page. Press Reset (ctrl del).
Repeat what was done in exercise 3, but with each person in the group doing a different translation. Each person in the group should choose one from the following:
i) Translate ΔABC down 4 units and to the right 2 units.
ii) Translate ΔABC up 5 units.
iii) Translate ΔABC down 1 unit and to the left 4 units.
iv) Translate ΔABC up 6 units and to the left 3 units.

Then click on  or press T). Zoom  in (+) or out (–) as needed.

a. Record the Original slopes (first slopes displayed) in the first row of the following table.
Look for patterns.

Sample Answer(s):

Translate i ii iii iv	m($\overline{AB}$)	m($\overline{BC}$)	m(CA)	m($\overline{A'B'}$)	m($\overline{B'C'}$)	m($\overline{C'A'}$)
Original	**– 0.4**	**– 1**	**– 2**	**– 0.4**	**– 1**	**– 2**
Figure 1	**0.2**	**– 2.5**	**2**	**0.2**	**– 2.5**	**2**
Figure 2	**0**	**und**	**1**	**0**	**und**	**1**

b. Investigate and mentally make note of the slopes by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the previous table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" in the previous table.
Look for patterns among the slopes of corresponding sides.

c. Using the pattern observed in the previous table, is your conjecture still true?

Answer: Yes, each pair of corresponding sides have the same slope.

5. Many different triangles have been translated in several directions.
Generalize explorations and investigations by responding to the following:
If a triangle is translated, what appears to be true about the corresponding sides of the pre-image and image triangles?

Answer: If a triangle is translated, each side of the image triangle is parallel to its corresponding side of its pre-image triangle.

6. ΔDEF has been translated down 7 units and to the right 8 units. Answer the following.

a. State three pairs of segments that are parallel: $\overline{DE} \parallel \overline{D'E'}$ $\overline{EF} \parallel \overline{E'F'}$ $\overline{DF} \parallel \overline{D'F'}$

b. If $\overline{DE}$ has a slope of $-\frac{4}{7}$, what other segment has a slope of $-\frac{4}{7}$? $\overline{D'E'}$

c. If $\overline{EF}$ is horizontal, what other segment will be horizontal? $\overline{E'F'}$

What is its slope? **0 (zero)**

d. If $\overline{F'D'}$ has a slope that is undefined, what other segment will have a slope that is undefined?
$\overline{FD}$

What word can be used to describe $\overline{F'D'}$? **vertical**

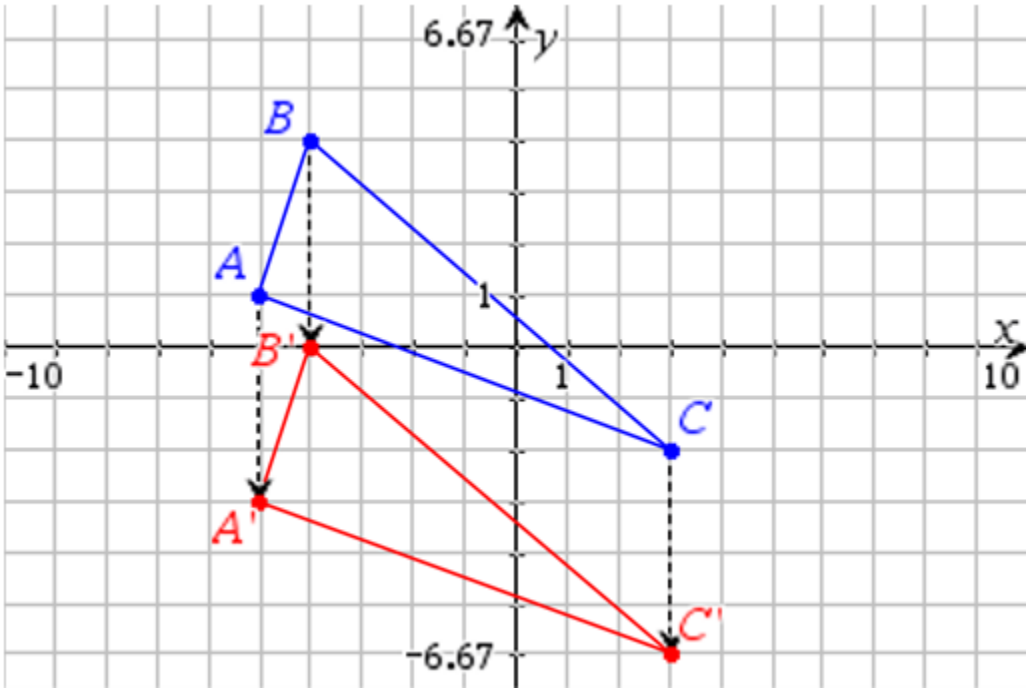
Lesson 7: Self-Assessment

In this lesson, you will be given the opportunity to summarize, review, explore and extend ideas about Translations.

It is important that one of the Translations Tours be done before any Translations lessons.

1 – 4. Make a sketch of each on the grid supplied.

1. Translate ΔABC down 4 units. Then fill in the blanks with appropriate responses.



a. If $m\angle C = 35^o$, then $m\angle C' = 35^o$

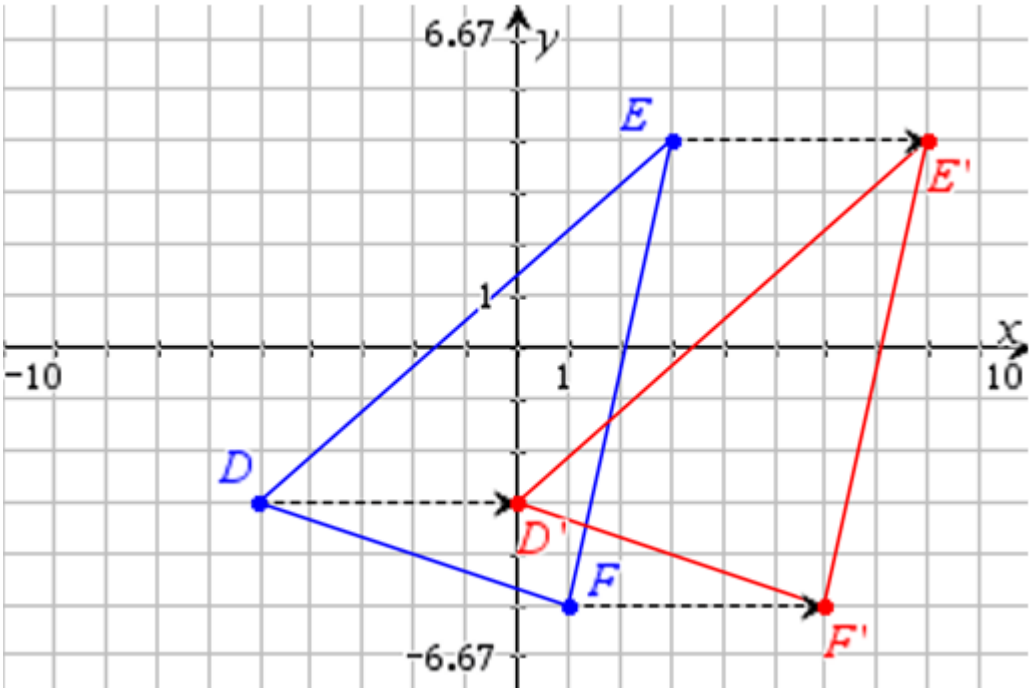
b. If BC = 8 cm, then $B'C' = 8cm$

c. If the slope of $\overline{BC} = -\frac{6}{7}$, then the slope of $\overline{B'C'} = -\frac{6}{7}$

d. If the perimeter of ΔABC = 17 in, then the perimeter of $\Delta A'B'C' = 17in$

e. If the coordinates of a point G on ΔABC are (x, y), then the coordinates of G' are **(x, y – 4)**

2. Translate ΔDEF 5 units to the right. Then fill in the blanks with appropriate responses.



a. If $m\angle F = 70^o$, then $m\angle F' = 70^o$

b. if the slope of $\overline{DE} = \frac{7}{8}$, then the slope of $\overline{D'E'} = \frac{7}{8}$

c. If the coordinates of E are (3, 4), then the coordinates of **E'** are **(8, 4)**

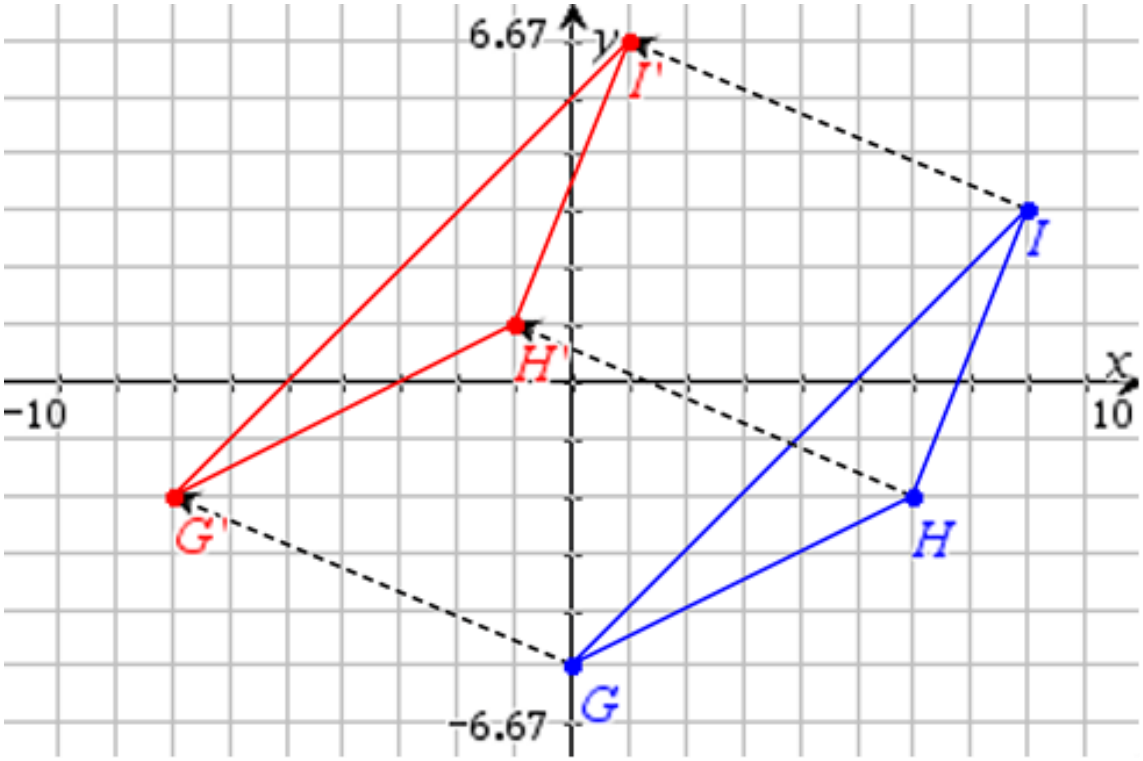
d. If the area of ΔDEF is 24 sq cm, then the area of $\Delta D'E'F'$ is **24 sq cm**

e. If the coordinates of a point H on ΔDEF are (x, y), then the coordinates of H' are **(x + 5, y)**

f. Name two segments that are parallel to $\overline{DD'}$ and state their slopes.

$\overline{DD'} \parallel \overline{EE'} \parallel \overline{FF'}$ $\quad m(\overline{DD'}) = m\left(\overline{EE'}\right) = m\left(\overline{FF'}\right) = 0$

3. Translate ΔGHI up 3 units and to the left 6 units. Then fill in the blanks with appropriate responses.



a. If GH = 9 in, then $\overline{G'H'} = 9in$

b. If the perimeter of ΔGHI is 36 cm, then the perimeter of $\Delta G'H'I'$ is $36cm$

c. If the slope of $\overline{HI} = \frac{5}{2}$, then the slope of $\overline{H'I'} = \frac{5}{2}$

d. If the coordinates of H are (6, – 2), then the coordinates of **H' are (0, 1)**

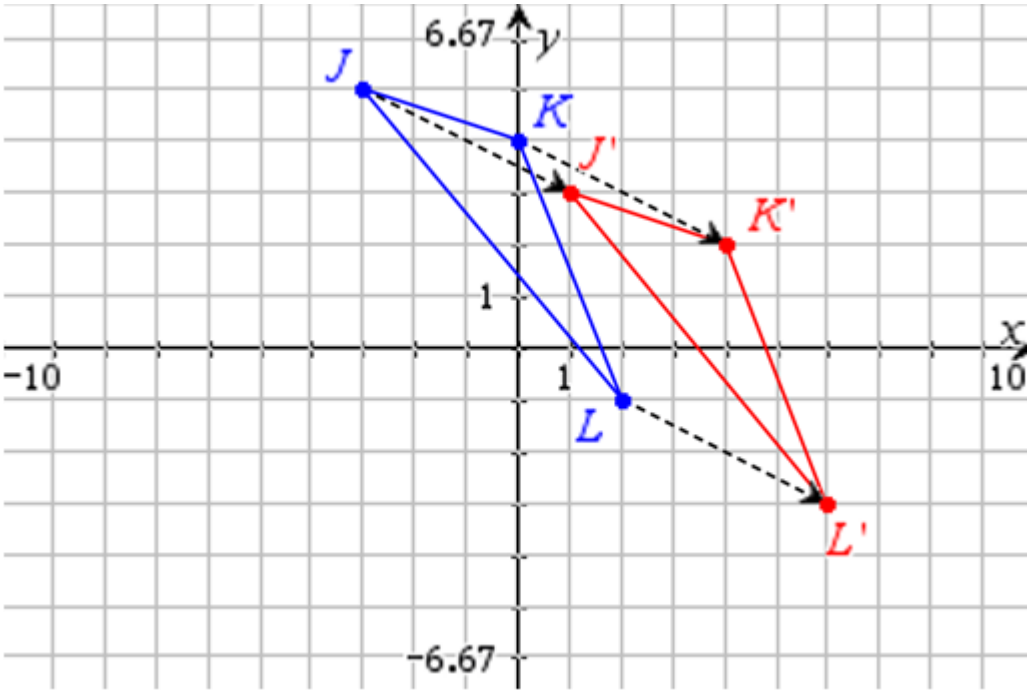
e. If the coordinates of a point P on ΔGHI are (x, y), then the coordinates of P' are **(x – 6, y + 3)**

f. Name three sets of parallel segments and list the slope of each:

$\overline{GG'} \parallel \overline{HH'} \parallel \overline{II'}$ $slope\ is$ $-\frac{1}{2}$ $\quad\quad$ $\overline{HI} \parallel \overline{H'I'}$ $slope\ is$ $\frac{5}{2}$

$\overline{GH} \parallel \overline{G'H'}$ $slope\ is$ $\frac{1}{2}$ $\quad\quad$ $\overline{GI} \parallel \overline{G'I'}$ $slope\ is$ 1

4. Translate ΔJKL to the right 4 units and down 2 units. Then fill in the blanks with appropriate responses.



a. If $m\angle K = 125^o$, then the $m\angle K' = 125^o$

b. If JL = 24 in, then **J'L' = 24 in**

c. If the area of $\Delta JKL = 40\,sq\,in$, then the area of $\Delta J'K'L' = 40\,sq\,in$

d. If the slope of $\overline{JK} = -\frac{1}{3}$, then the slope of $\overline{J'K'} = -\frac{1}{3}$

e. If the coordinates of L are (2, – 1), then the coordinates of **L' are (6, – 3)**

f. If the coordinates of a point Q on ΔJKL are (x, y), then the coordinates of Q' are **(x + 4, y – 2)**

g. Name three sets of parallel segments and list the slope of each:

$\overline{JJ'} \parallel \overline{KK'} \parallel \overline{LL'}$ *slope is* $-\frac{1}{2}$

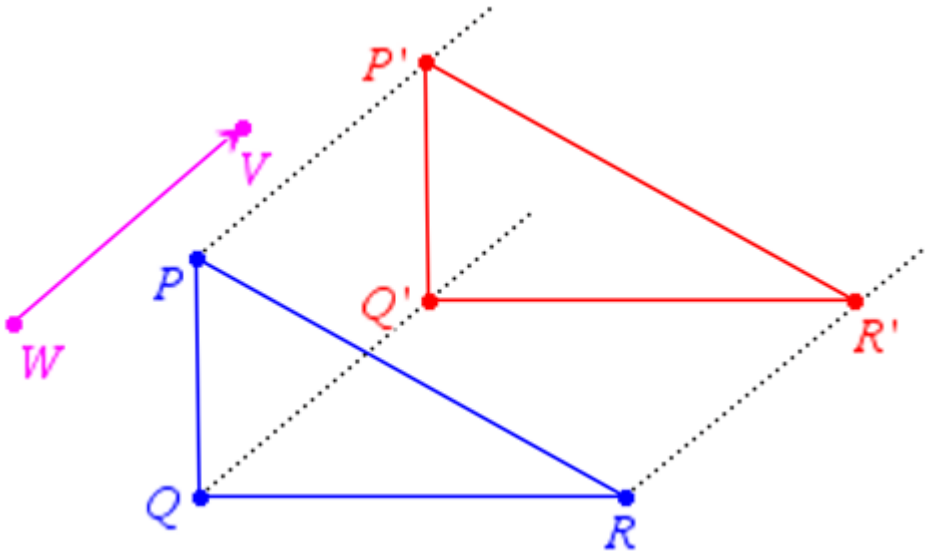
$\overline{KL} \parallel \overline{K'L'}$ *slope is* $-\frac{5}{2}$

$\overline{JK} \parallel \overline{J'K'}$ *slope is* $-\frac{1}{3}$

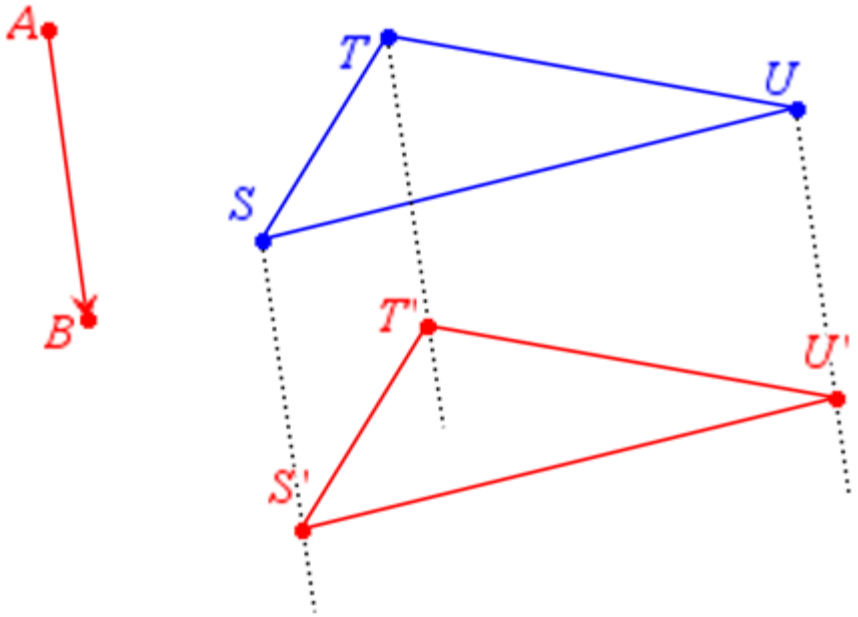
$\overline{JL} \parallel \overline{J'L'}$ *slope is* $-\frac{6}{5}$

5 – 8. Make a sketch of each in the space provided.

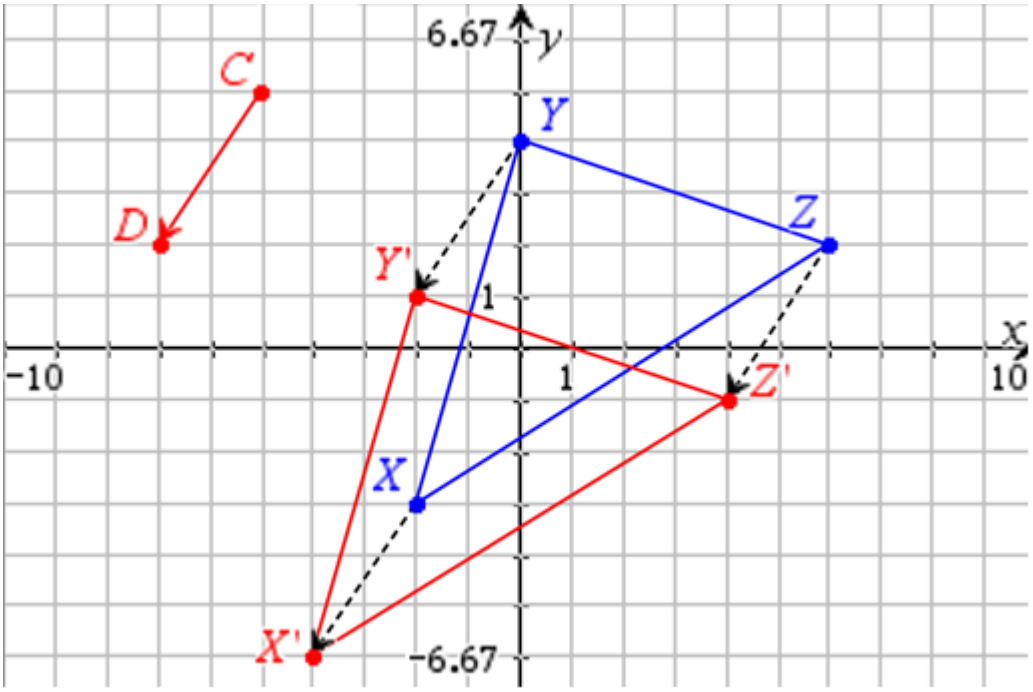
5. Translate ΔPQR about $\overrightarrow{WV}$. Use a straightedge.



6. Translate ΔSTU about $\overrightarrow{AB}$. Use a straightedge.



7. Translate ΔXYZ by vector $\overrightarrow{CD}$. Use a straightedge.



a. What segments are parallel to vector $\overrightarrow{CD}$? $\overline{YY'}, \overline{ZZ'}, \overline{XX'}$

What is the slope of each of those segments? $-\frac{3}{2}$

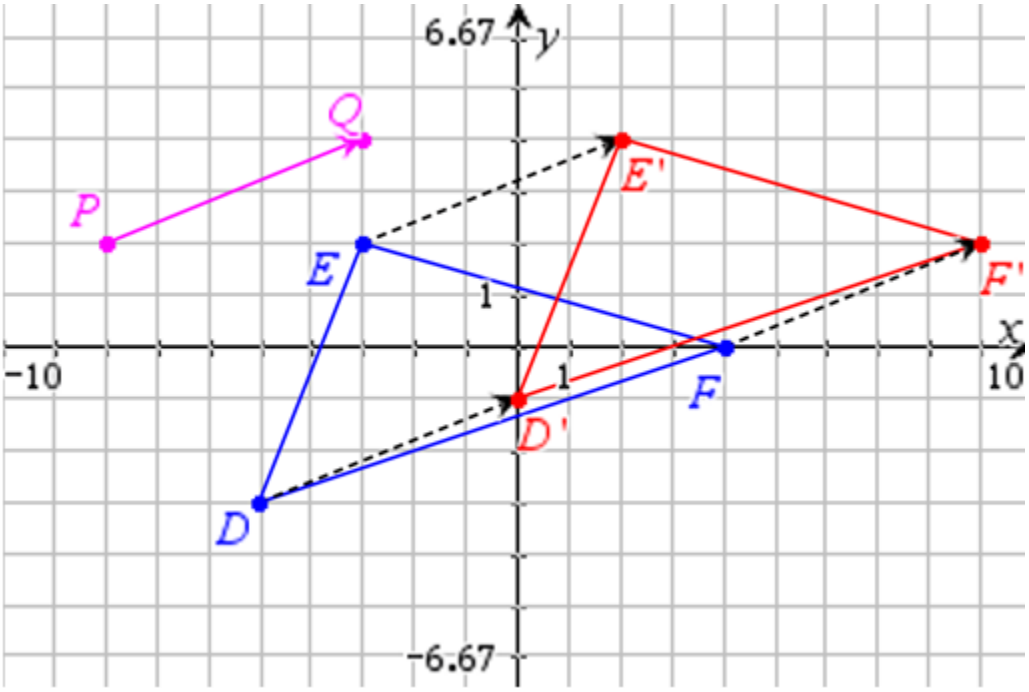
b. Name three other pairs of segments that are also parallel and state their slopes:

$\overline{YZ} \parallel \overline{Y'Z'}$ *slope is* $-\frac{1}{3}$

$\overline{XY} \parallel \overline{X'Y'}$ *slope is* $\frac{7}{2}$

$\overline{XZ} \parallel \overline{X'Z'}$ *slope is* $\frac{5}{8}$

8. Translate ΔDEF by vector $\overrightarrow{PQ}$.



a. What segments are parallel to vector $\overrightarrow{PQ}$? $\overline{EE'},\ \overline{DD'},\ \overline{FF'}$

What is the slope of each of those segments? $\frac{2}{5}$

b. Name three other pairs of segments that are also parallel and state their slopes:

$\overline{DE} \parallel \overline{D'E'}$ *slope is* $\frac{5}{2}$

$\overline{DF} \parallel \overline{D'F'}$ *slope is* $\frac{1}{3}$

$\overline{EF} \parallel \overline{E'F'}$ *slope is* $-\frac{2}{7}$

9. Given: ΔDEF is translated to the left 7 units and up 5 units.

a. If D has coordinates (5, 7), what are the coordinates for D'? **(– 2, 12)**

b. If E has coordinate (– 3, – 7), what are the coordinates of E'? **(– 10, – 2)**

c. If F' has coordinates (1, 6), what are the coordinates of F? **(8, 1)**

d. If D has coordinates (x, y), what are the coordinates for D'? **(x – 7, y + 5)**

e. If E' has coordinates (p, q), what are the coordinates for E? **(p + 7, q – 5)**

10. Given: ΔPQR is translated to the right 8 units and down 3 units.

a. If P has coordinates (5, 7), what are the coordinates for P'? **(13, 4)**

b. If Q has coordinate (– 3, – 7), what are the coordinates of Q'? **(5, – 10)**

c. If R' has coordinates (1, 6), what are the coordinates of R? **(– 7, 9)**

d. If P has coordinates (x, y), what are the coordinates for P'? **(x + 8, y – 3)**

e. If P' has coordinates (a, b), what are the coordinates for P? **(a – 8, b + 3)**

11. Given: ΔXYZ is translated down 7 units and to the left 10 units.

a. If X has coordinates (5, 7), what are the coordinates for X'? **(– 5, 0)**

b. If Y has coordinate (– 3, – 7), what are the coordinates of Y'? **(– 13, – 14)**

c. If Z' has coordinates (1, 6), what are the coordinates of Z? **(11, 13)**

d. If X has coordinates (a, b), what are the coordinates for X'? **(a – 10, b – 7)**

e. If E' has coordinates (c, d), what are the coordinates for E? **(c + 10, d + 7)**