Charged Up

Student Activity

Name ___________________

Class ___________________

Part 1 – Separable Differential Equations Introduced

1. A capacitor, like one used for a camera flash, is charged up. When it discharges rapidly, the rate of change of charge, *q*, with respect to time, *t*, is directly proportional to the charge. Write this as a differential equation.

The first step is to separate the variables, and then integrate and solve for *y*.

2. Find *y*(0), if $\frac{dy}{dx} = \sin(x)\cos^2(x)$ and $y\left(\frac{\pi}{2}\right)$ = 0. After integrating, use the initial condition *y* = 0 when $x = \frac{\pi}{2}$ to find the constant of integration. Then, substitute *x* = 0 to find *y*(0).

Let's return to the capacitor. Now that it is discharged, we need to get it charged up again. A 9V battery is connected to a 100kΩ resister, *R*, and 100μF capacitor, *C*.

The Law of Conservation of Energy gives us the differential equation $\frac{dq}{dt} \cdot R = V - \frac{q}{C} \rightarrow \frac{dq}{dt} \cdot R \cdot C = V \cdot C - q$.

After substituting the given information and simplifying, we get the differential equation $10\frac{dq}{dt} = 0.0009 - q$.

3. For the differential equation $10\frac{dq}{dt} = 0.0009 - q$, separate the variables and integrate.

4. Apply the initial condition when time = 0, the charge *q* = 0 and solve for *q*.

The syntax for **deSolve** is **deSolve(*y′*=f(*x*,*y*),*x*,*y*)** where *x* is the independent and *y* is the dependent variable. The **deSolve** command can be found in the HOME screen by pressing [F3] and selecting **C:deSolve(**.

5. On the HOME screen, type **deSolve(10*q′*=0.0009–*q* and *q*(0)=0,*t*,*q*)**. Write down this answer and reconcile it with your previous solution.

6. In the HOME screen enter **deSolve(*y′*=*y*/*x*,*x*,*y*)** to find the general solution of $\frac{dy}{dx} = \frac{y}{x}$. Write the answer. Show your work to solve this differential equation by hand and apply the initial condition $y(1) = 1$ to find the particular solution.

Part 2 – Homework/Extension – Practice with deSolve and Exploring DEs

Find the general solution for the following separable differential equations. Write the solution in an acceptable format, (for example, use *C* instead of @*7*). Show all the steps by hand if your teacher instructs you to do so.

1. $y' = k \cdot y$ **2.** $y' = \frac{x}{y}$ **3.** $y' = \frac{2x}{y^2}$ **4.** $y' = \frac{3x^2}{y}$

From the **HOME** screen, open the GDB labeled *diffq1*. On the **GRAPH** screen, observe the family of solutions to the differential equation from Question 4, $y' = \frac{3t^2}{y}$. Many particular solutions can come from a general solution. When you are finished viewing the family of functions, go to the **Y=** screen and delete the function.

5. Not all differential equations are separable. Use **deSolve** to find the solution to the non-separable differential equation $x \cdot y' = 3x^2 + 2 - y$. What does this graph look like if the integration constant is 0? Explain. Open the picture *diffq2* to view the graph.

Find the particular solution for the following equations. Show your work. Solve for *y*. Explore other differential equations on your own. Did you get any surprising results?

6. $y' = x \cdot y^2$ and $y(0) = 1$ **7.** $y' = 1 + y^2$ and $y(0) = 1$ **8.** $y' = 7y$ and $y(0) = \ln(e)$