EXPLORATIONS

Activity 14

Let Us Count the Ways!

Teacher Notes

Concepts

- ♦ Apply the multiplication counting principle
- ♦ Find the number of permutations in a data set
- ♦ Find the number of combinations in a data set

Calculator Skills

- ♦ Factorial: [PRB] !
- ♦ Permutations: [PRB] nPr
- ♦ Combinations: [PRB] nCr

Materials

- ♦ TI-30X IIS
- ♦ Student Activity pages (p. 141-142)

Objective

- ♦ In this activity, students will evaluate expressions using permutations and combinations of data elements on the calculator. They will solve problems using these counting principles.

Topics Covered

- ♦ Applying discrete mathematics
- ♦ Working with probability

Introduction

In a 1600-meter relay, a team's coach can arrange the four members of the relay team in any order. How many arrangements of the runners are possible?

Investigation

1. Show the students how to solve the problem above using the multiplication counting principle:

   [4 choices for the first runner] x [3 choices for the second runner] x [2 choices for the third runner] x [1 choice for the remaining runner]

   There are 4 x 3 x 2 x 1 = 24 possible orders in which the coach may arrange the runners. Explain that 4 x 3 x 2 x 1 = 4! (4 factorial).

2. Use the overhead calculator to demonstrate how to solve factorial problems on the TI-30X IIS.

Press:	The calculator shows:
[CLEAR] 4 [PRB] ▶ ▶	nPr nCr ! DEG
[ENTER] [ENTER]	4! 24 DEG

3. Explain that a particular arrangement of the elements of a set is called a permutation. In general, the number of permutations of a set of *n* objects is given by *n*! When all *n* objects are used.
4. Point out that, however, permutations do not always use all *n* objects. For example, suppose that there are nine runners who are qualified to run the 1600-meter relay and the coach must fill the four slots from this set of runners. How many different four-member arrangements are possible?

   Using the multiplication counting principle again:

   [The first runner can be any of the 9] x [The second runner can be any of the 8 remaining] x [The third runner can be any of the 7 remaining] x [The final slot could be filled from any of the 6 remaining]

   That is 9 x 8 x 7 x 6 = 3024. There are 3024 permutations of four runners chosen from nine.
5. Explain that the number of permutations of *r* objects taken from a set of *n* objects is given by *n*P*r* where

$$_nP_r = \frac{n!}{(n-r)!}.$$

   Solve the relay team permutation problem on the calculator, (n = 9 and r = 4)

Press:	The calculator shows:
[CLEAR] 9 [PRB]	nPr nCr ! DEG
[ENTER] 4 [ENTER]	9 nPr 4 3024 DEG

6. Ask the students how many permutations there are of the letters in MOM.

   This example illustrates that sometimes a data set will contain identical elements. When this occurs, some of the permutations will look the same.

   Consider these arrangements where the first M is denoted with bold-faced type:

MOM	O**M**M	**M**MO
MO**M**	OM**M**	M**M**O

   If you ignore the bold-faced type of the first M (**M**OM and MO**M** are both counted), you have 2! copies of each arrangement.

   Explain that if a set of *n* elements has one element repeated q_1 times, another element repeated q_2 times, and the last repeated q_K times, the number of distinguishable permutations of the *n* elements is given by:

   $$\frac{n!}{q_1!q_2!\ldots q_3!}$$

   Use the formula to find the number of permutations for the letters in MOM.

   $$\frac{3!}{2!1!.} = 3$$

   Show that when you eliminate duplicate arrangements, three arrangements remain.

7. Ask the students how many distinguishable permutations there are of the letters in the word BANDANA.

   Begin by having the students tell how many letters there are (n) and how many times each letter occurs (q_1, q_2,...q_k).

   There are 7 letters in all so n *= 7. With one B (q_1 = 1), three A's (q_2 = 3), two N's (q_3 = 2), and one D (q_4 = 1).*

   Write the problem and solve it without the calculator.

   $$\frac{7!}{1!3!2!1!} = \frac{7\times6\times5\times4\times3\times2\times1}{1\times(3\times2\times1)\times(2\times1)\times1} = 7\times6\times5\times2 = 420$$

   distinguishable permutations of the letters in BANDANA.

8. Use the TI-30X IIS to confirm the result given in the problem on the previous page.

Press:	The calculator shows:
[CLEAR] 7 [PRB] ▶ ▶ [ENTER]	7! DEG
[÷] [(] 1 [PRB] ▶ ▶ [ENTER] 3 [PRB] ▶ ▶ [ENTER] 2 [PRB] ▶ ▶ [ENTER] 1 [PRB] ▶ ▶ [ENTER] [)] [ENTER]	7!/(1!3!2!1!→ 420. DEG

9. Use the following example to explain combinations.

Suppose you have selected five CDs that you would like to purchase at the local music store, but you only have enough money to purchase three. In how many ways can you select three CDs?

You could start by trying out all the possibilities by stacking three CDs on the checkout counter. Since it doesn't matter how you stack the CDs you buy, all of the permutations of the three CDs are the same which gives ${}_3P_3$= 6 permutations of the three CDs. Eliminating the duplicate permutations gives

$$\frac{{}_5P_3}{{}_3P_3} = \frac{60}{6} = 10$$

Explain that selecting elements of a data set when the order does not matter is called a combination.

10. Show the formula for solving combinations. The number of combinations of *r* elements taken from a set of *n* elements is given by *n*C*r* where *n*C*r* = 11

$$\frac{{}_nP_r}{{}_rP_r} = \frac{n!}{(n-r)!r!}$$

Solve the CDs combination problem on the calculator (*n* + 5 and *r* = 3).

Press:	The calculator shows:
[CLEAR] 5 [PRB] ▶ [ENTER]	5 nCr DEG
3 [ENTER]	5 nCr 3 10. DEG

Wrap-up

Have the students to complete Student Activity Part 1 and Student Activity Part 2 individually or in groups of two.

Extension

Have students write problems that can be solved using combinations and permutations.

Solutions Part 1

Evaluate these expressions.

1.	$8!$	*40320*	4.	$(3!)(5!)$	*720*
2.	$9!$	*362880*	5.	$4! + 5!$	*144*
3.	$4!$	*24*	6.	$4(3!)$	*24*

Simplify these expressions.

7.	$\frac{9!}{8!}$	*9*	11.	${}_8P_1$	*8*
8.	$\frac{10!}{5!5!}$	*252*	12.	${}_8P_2$	*56*
9.	$\frac{9!}{4!3!2!}$	*1260*	13.	${}_{10}P_6$	*151200*
10.	${}_9P_5$	*15120*			

14. How many ways can a president, vice president, treasurer, recorder, and parliamentarian be selected from a club of 12 members?

    12 nPr 5 = 95,040

15. In the key of F, the first 13 notes of "I've Been Working on the Railroad" include five F's, two G's, two C's, two A's, and two B-flats. Write and evaluate an expression for the number of different melodies that can be formed from these 13 notes.

    $\frac{13!}{5!2!2!2!2!} = 3,243,240$

Solutions Part 2

Simplify each of these.

1.	${}_8C_2$	*28*	6.	${}_6C_6$	*1*
2.	${}_8C_4$	*70*	7.	${}_7C_1$	*7*
3.	${}_{16}C_4$	*1820*	8.	${}_7C_6$	*8*
4.	${}_8C_0$	*1*	9.	${}_5C_3 + {}_5C_2$	*20*
5.	${}_{18}C_{14}$	*3060*	10.	${}_8C_5 \times {}_8C_3$	*3136*

11. Charles grows seven different colors of roses in his garden. How many different combinations of three roses can be used to form a small bouquet?

    7 nCr 3 = 35

12. How many different combinations of five members of a basketball team can be selected from a team of 12 players?

    12 nCr 5 = 792

Student Activity 14

Name ______________________

Date ______________________

Let Us Count the Ways!

Objective: *In this activity, you will evaluate expressions using permutations and combinations of data elements on the calculator. You will solve problems using these counting principles.*

Part 1: Solving Factorials and Permutations

Evaluate these expressions.

1. 8!
2. 9!
3. 4!
4. (3!)(5!)
5. 4! + 5!
6. 4(3!)

Simplify these expressions.

7. $\frac{9!}{8!}$
8. $\frac{10!}{5!5!}$
9. $\frac{9!}{4!3!2!}$
10. ${}_9P_5$
11. ${}_8P_1$
12. ${}_8P_2$
13. ${}_{10}P_6$

14. How many ways can a president, vice president, treasurer, recorder, and parliamentarian be selected from a club of 12 members?

15. In the key of F, the first 13 notes of “I’ve Been Working on the Railroad” include five F’s, two G’s, two C’s, two A’s, and two B-flats. Write and evaluate an expression for the number of different melodies that can be formed from these 13 notes.

Part 2: Evaluating Combinations

Simplify each of these:

1. ${}_8C_2$
2. ${}_8C_4$
3. ${}_{16}C_4$
4. ${}_8C_0$
5. ${}_{18}C_{14}$
6. ${}_6C_6$
7. ${}_7C_1$
8. ${}_7C_6$
9. ${}_5C_3 + {}_5C_2$
10. ${}_8C_5 \times {}_8C_3$

11. Charles grows seven different colors of roses in his garden. How many different combinations of three roses can be used to form a small bouquet?

12. How many different combinations of five members of a basketball team can be selected from a team of 12 players?