EXPLORATIONS

CHAPTER 3

AC Circuit Analysis

This chapter discusses basic concepts in the analysis of AC circuits.

The Sine Wave

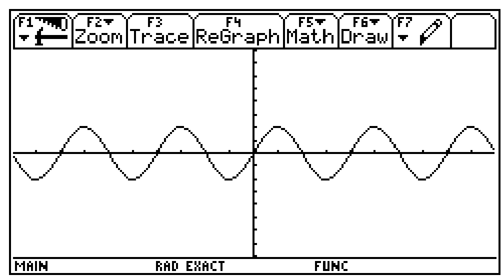
AC circuit analysis usually begins with the mathematical expression for a sine wave:

$$v(t) = Vp \sin(wt+\theta)$$

where

Vp = the peak voltage
w = the angular velocity of the generator
t = time
θ = the phase shift.

A sample sine wave is shown in Figure 1.



(Figure 1)

The angular velocity **w** is equal to $2\pi * $ frequency, or **w =2πf**.

Using this substitution, the equation can be re-written as:

$$v(t) = Vp \sin(2\pi ft+\theta)$$

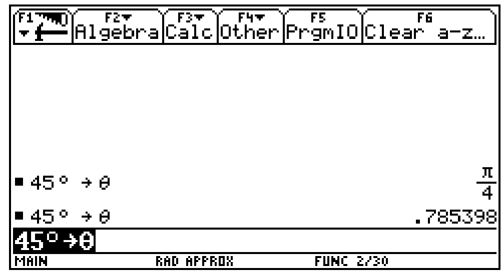
This equation allows you to calculate a voltage at an instance in time. The frequency **f** and phase shift **θ** are constants. Frequency determines how many peaks occur over a given period of time. Phase shift determines where the sine wave crosses the Y axis. In the sine wave shown in Figure 1 there is no phase shift.

Degrees and Radians in the Sine Equation

If you have ever tried to use the sine wave equation, you may have found it somewhat confusing. The **$2\pi * f * t$** part of the equation is in radians, while the phase shift θ is normally in degrees. Unfortunately you cannot mix radians and degrees in a sine function. However the TI-92 provides a convenient way to express an angle in degrees, even if it is to be used in radians.

*To specify degrees, simply press **2nd D** after the number. This means the number is entered in degrees, but is stored in the default system, as shown in Figure 2.*

1. Set Mode to **Radian** and **Exact** (*See Chapter 1 for instructions on setting the **Mode***)
2. Press **45**
3. Press [2nd] **D** [STO▸] [θ] [ENTER]
   *Do not type **STO>**. This key is located next to the space bar.*
4. Press [♦]
5. Press [ENTER]



(Figure 2)

You can display an approximate answer by pressing [♦] and [ENTER] as demonstrated in steps 7 and 8 in the previous example. See Chapter 1 for more information on these modes.

The actual angle stored is shown on the right of the screen.

Note that **45°** equals **π/4** radians in **Exact** mode and **0.785398** radians in **Approximate** mode.

Example:

Graphing a Sine Wave

Let's start by graphing the sine wave equation using the powerful graphics capability in the TI-92.

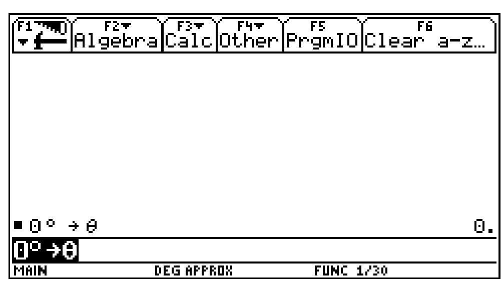
When you store a value, you store it as a named variable. You can then use the name instead of the value in equations. A variable name can be 1 to 8 characters and contain letters and numbers. See Chapter 1 for more information.

We need to begin by setting the **Mode**, angle θ and frequency **f**.

1. Set Mode to **Degree**

To set the variables:

2. Type **0**
3. Press [2nd] **D**
4. Press [STO▸]
5. Press [θ]
6. Press [ENTER]



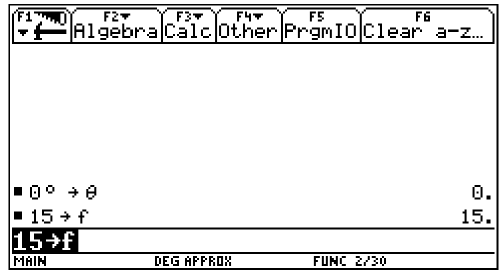
(Figure 3)

This stores **0º** in the variable θ, as shown in Figure 3.

Enter :

7. Type **15**
8. Press [STO▸] **f** [ENTER]

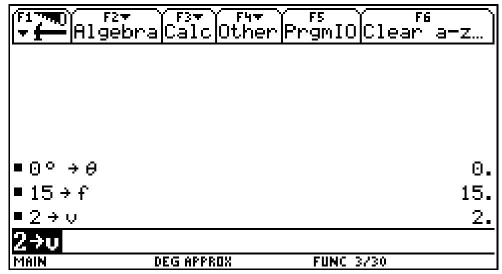
to store **15 Hz** as the frequency, as in Figure 4.



(Figure 4)

Enter:

9. Type **2** [STO▸] **v** [ENTER]
   See Figure 5.



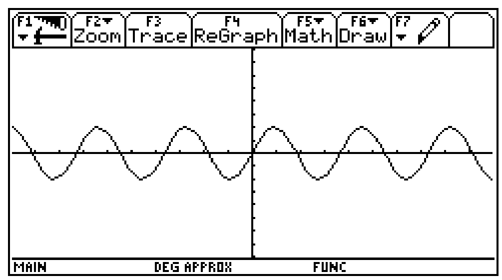
(Figure 5)

Next enter the equation:

10. Press [♦] [Y=]
11. Clear all equations from the **Y= Editor**
12. Press [ENTER] at the **y1=** prompt
13. Type **v** [×] [SIN] **2** [×] **π** [×] **f** [×] **x** [+] [θ] [)]

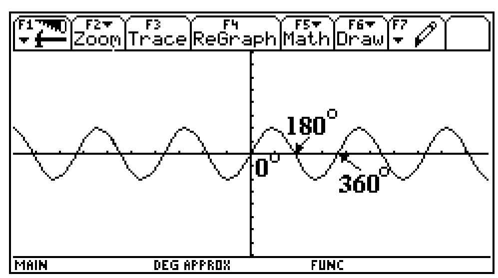
To graph the equation:

14. Press [♦] [GRAPH]
15. Press [F2] **Zoom**
16. Press **6: ZoomStd**
    See Figure 6.



(Figure 6)

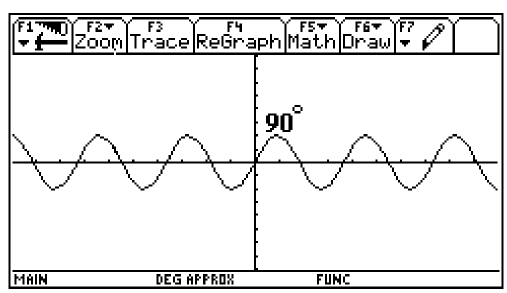
Even though you are in **Degree** mode, a sine wave appears on the screen, as shown. Note that the beginning of the sine wave occurs where it crosses the y axis. If you think of the sine wave being produced by the rotation of a generator, the x axis can be shown in degrees (a specific voltage exists for each degree of rotation). In this case the sine wave begins at **0°**, as shown in Figure 7.



(Figure 7)

A **90°** phase shift occurs at the first peak, as shown in Figure 8.

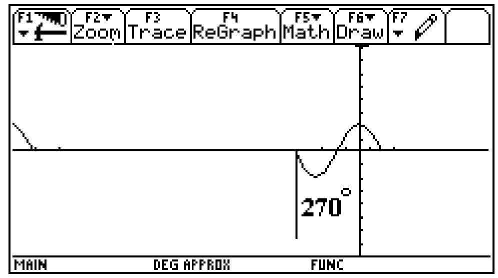
The equation for the sine wave assumes that the wave starts at **x = 0°.** If the waveform starts somewhere else, it has been **phase shifted** by a certain number of degrees.



(Figure 8)

Phase Shifts

A sine wave can be phase shifted to the right or left of the origin. Phase shift is usually expressed in degrees, and is an angular distance from the origin. When measuring phase shift, imagine how many degrees (positive or negative) the waveform must be shifted to move it to the origin. In Figure 9, the waveform must be moved **270°** in the positive direction in order for it to start at the origin. We thus say the waveform has a phase shift of **270°**.



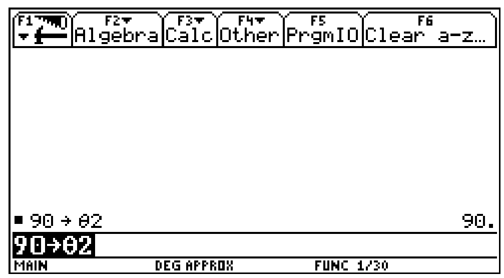
(Figure 9)

To help you better understand phase shift, superimpose two sine waves and observe the phase difference between them.

Let's start by introducing a **90°** phase shift to the sine wave equation.

To store **90°** in the variable **θ2**, enter the following:

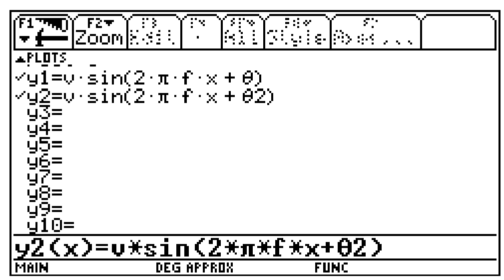
1. Press [◆] [HOME]
2. Type **90**
3. Press [2nd] **D**
4. Type [STO▶] **θ2**
5. Press [ENTER]
   See Figure 10.



(Figure 10)

Now, enter a new sine wave equation to graph:

6. Press [◆] [Y=]
7. Press [ENTER] at the **y2**= prompt
8. Type **v** [×] [SIN] **2** [×] [π] [×] **f** [×] **x** [+] [θ] **2** [)]
9. Press [ENTER]
   See Figure 11



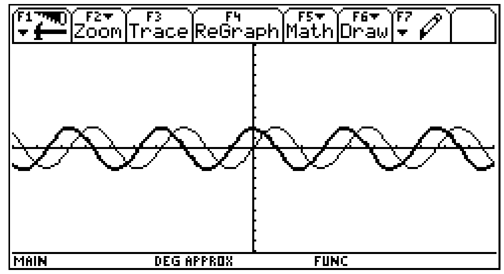
(Figure 11)

Now, make the second trace thicker than the first one so that we can distinguish between the two:

10. Press [▲] to highlight the equation in **y2**
11. Press [F6]
12. Press **4**

To graph the equations:
13. Press [◆] [GRAPH]

After the TI-92 graphs the equations:
14. Press [F2] **Zoom**
15. Press **6: ZoomStd**
See Figure 12.



(Figure 12)

Both sine waves appear on the display. The thicker trace shows a **90°** phase shift. It crosses the Y axis at a positive peak.

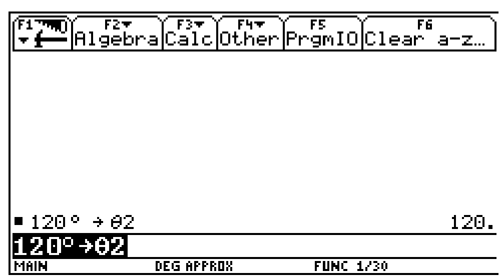
The figure above is similar to the display on a dual trace oscilloscope. On a scope the phase difference between two waveforms is calculated by first measuring the distance between peaks, then dividing by the period and multiplying by 360.

Three-Phase Waveforms

A three-phase generator has three sets of coils evenly spaced on the rotor. This means that each coil is **360°/3**, or **120°** from the others. Let's graphically calculate the phase shift between the two waveforms. Use the same method that is used on an oscilloscope to determine phase shift, or

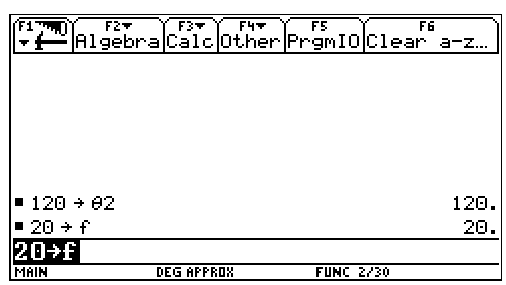
$$\text{Phase Shift} = \frac{\text{Time between peaks}}{\text{Period}} * 360$$

To measure the phase shift, first change the phase shift of **θ2** to **120°**:
1. Press [◆] [HOME]
2. Type **120**
3. Press [2nd] **D** [STO▸] [θ] **2** [ENTER]
See Figure 13.



(Figure 13)

Now change the frequency to **20 Hz**:
4. Type **20** [STO▸] **f** [ENTER]
See Figure 14.

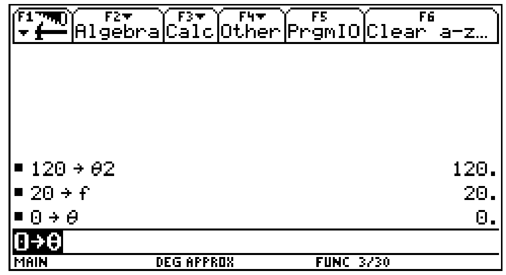


(Figure 14)

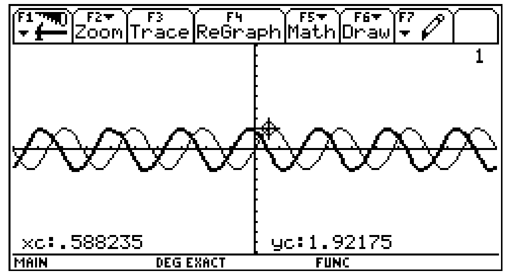
Change the phase shift of the first waveform to **0°**:
5. Type **0** [STO▸] [θ] [ENTER]
See Figure 15.

Now graph the waveforms and find the phase shift between them:

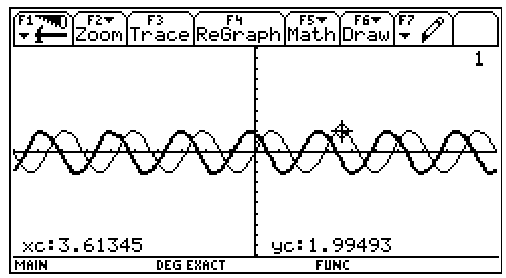
6. Press [◆] [GRAPH]
7. Press [F3] **Trace**
8. Use the ⊕ key and move the cursor to the top of the first peak past the Y axis. The time (or **xc** reading) should be about **.5882**.
   See Figure 16.
9. Press the ⊕ key and move the cursor to the next positive peak on the same waveform. It should read about **3.613**.
   See Figure 17.



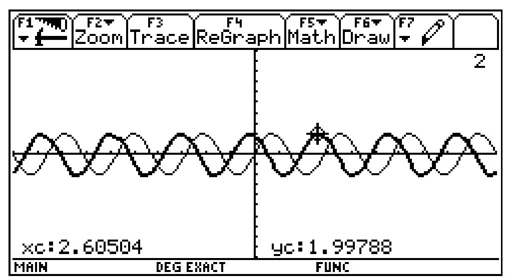
(Figure 15)



(Figure 16)



(Figure 17)



(Figure 18)

The difference between these two readings is the period:

3.613-.5882 = 3.02

10. Use the ⊕ key and move the cursor to the other waveform by pressing down.
11. Now move to the peak immediately to the left. It should read **2.605**.
    See Figure 18.

The difference between the peak of the other waveform at **3.613** and this value is the time between peaks, or:

3.613 - 2.605 = 1.008

The phase shift can then be calculated by:

(1.008/3.02)∗360 = 120°

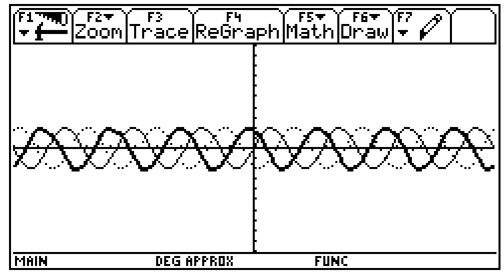
The third waveform of a three-phase network can be displayed by first storing **240°** in a new variable called **θ3**.

Let's add another sine wave with a phase shift of **240°**.

12. Press [◆] [HOME]
13. Type **240** [2nd] **D** [STO▸] [θ] **3** [ENTER]
14. Press [◆] [Y=]
15. Press [ENTER] at the **y3**= prompt
16. Type **v** [×] [SIN] **2** [×] [π] [×] **f** [×] **x** [+] [θ] **3** [)] [ENTER]

17. Press ⓤ to highlight **y3**
18. Press [F6] **2**
19. Press [◆] [GRAPH]

The display shows all three waveforms.
See Figure 19.



(Figure 19)

Capacitive and Inductive Reactance

Capacitive reactance is defined by the formula:

$$\mathbf{X_c = \frac{1}{2\pi fc}}$$

where

f = frequency in hertz
c = capacitance in farads.

Inductive reactance is defined by:

$$\mathbf{X_l = 2\pi fl}$$

where

f = frequency in hertz
l = inductance in henries.

Capacitive and inductive reactance are vectors. In Polar mode, vectors contain a magnitude and angle.

The vectors for inductance, capacitance, and resistance are shown below:

Xl = Xl∠90°

Xc = Xc∠-90°

R=R∠0°

Often an AC circuit gives the capacitance in farads and the inductance in henries, where reactance is needed. Short functions are useful to convert capacitance and inductance to reactance. You can even include the phase angle in the result. We will create two functions, one named **ind()** to calculate inductive reactance, and another named **cap()** to compute capacitive reactance.

A function lets you enter variables and calculates the answer based upon a given formula. Functions are ideal for repetitive calculations. You only need to write the function once. Refer to page 303 of the TI-92 users manual for more information.

Function :

Inductive Reactance (ind() Function)

1. Set Mode to **Degree**

Now the function can be created.

2. Press [APPS]
3. Press **7: Program Editor**
4. Press **3: New**
   See Figure 20.
5. Press [▶]
6. Press **2 Type: Function**
7. Press [▼] twice
8. Type **ind** in the **Variable** box
   See Figure 21.
9. Press the [ENTER] key twice to display the framework for the **ind()** function.
   See Figure 22.
10. Add **l,f** inside the parentheses on the first line.

Note that the l and f variables are separated by a comma. These are two variables that are passed to the function.

11. Type the equation [[] **2** [×] [π] **f** [×] **l** [,] [2nd] [F] **90** []]
12. Press [ENTER]
    See Figure 23.
13. Press [◆] [HOME]

The link version of **ind()** function is:

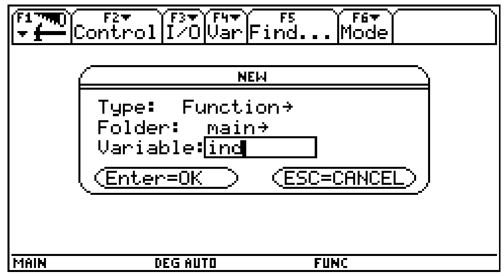
```
ind(L,f)
Func
[2*π*f*l,∠90]
EndFunc
```

In the **ind()** function, the line

[2 ∗ π ∗ f ∗ l, ∠90]

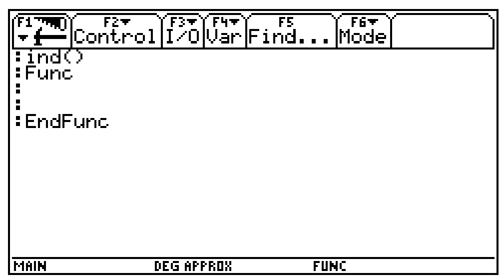
denotes a vector. The left part of the equation is the magnitude of the inductive reactance. The right part (after the comma) is the angle. The ∠ after the comma indicates that the inductive reactance is a vector.

(Figure 20)

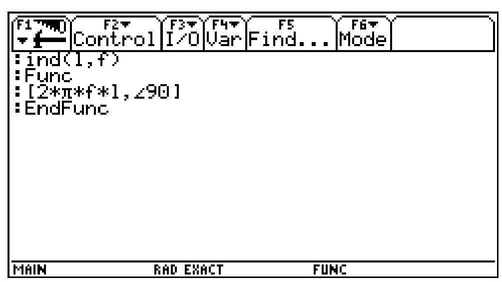


(Figure 21)

*Note that if you have tried to type this function in before and are now trying to alter the code, you need to OPEN the function. Repeat the process above, except press 2 in step 4 and select **ind** from the list box to re-enter the function.*



(Figure 22)



(Figure 23)

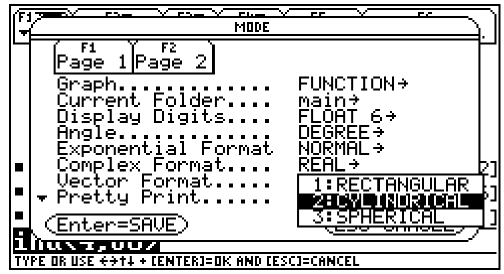
Example:

ind() Function

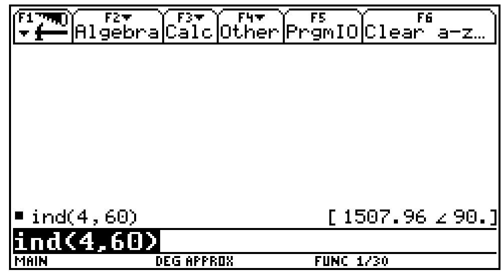
Let's use the **ind()** function to compute the reactance of a **4 henry** inductor at **60 Hz.**

1. Press [◆] [HOME]
2. Set Mode to **Degree** and **Approximate**
3. Press [MODE]
4. Press [▼] to **Vector Format**
5. Press [▶]
6. Press **2:CYLINDRICAL**
   See Figure 24.
7. Press [ENTER]
8. Type **ind** [(] **4** [,] **60** [)] [ENTER]
   See Figure 25.

The display shows **[1507.96 ∠90]**.



(Figure 24)



(Figure 25)

Function:

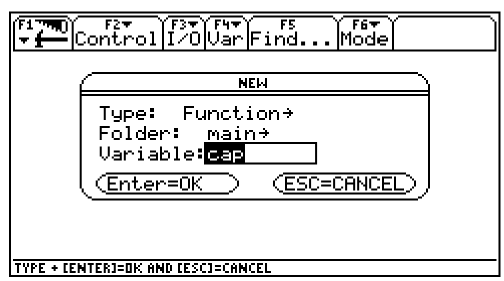
Capacitive Reactance (cap() Function)

1. Set Mode to **Degree**

Create the **cap()** function:

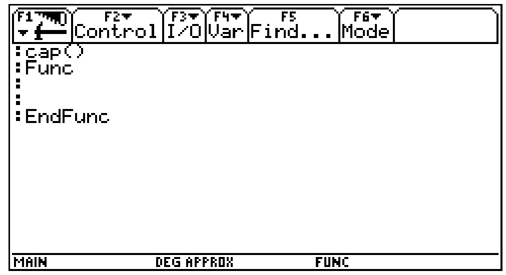
2. Press [APPS] key
3. Press **7: Program Editor**
4. Press **3: New**
   See Figure 26.
5. Press [▶]
6. Select **2: Function**
7. Press [▼] twice
8. Type **cap** in the **Variable** box
   See Figure 27.

(Figure 26)



(Figure 27)

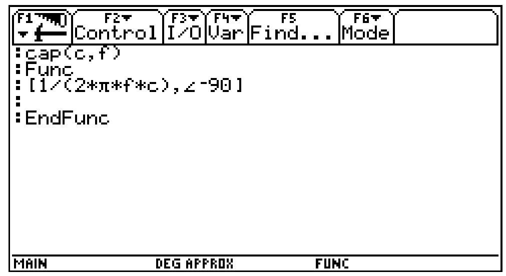
9. Press [ENTER] twice to display the framework for the **cap()** function
   See Figure 28.
10. Add **c,f** inside the parentheses on the first line.
11. Type the equation [[] **1** [÷] [(] **2** [×] [π] [×] **f** [×] **c** [)] [:] [2nd] [F] [(-)] **90** []]
    See Figure 29.
12. Press [ENTER]
13. Press [♦] [HOME]



(Figure 28)

The link version of the **cap** function is:

```
cap(c,f)
Func
[1/(2*π*f*c),∠-90]
EndFunc
```



(Figure 29)

Example:

cap() Function

The expression

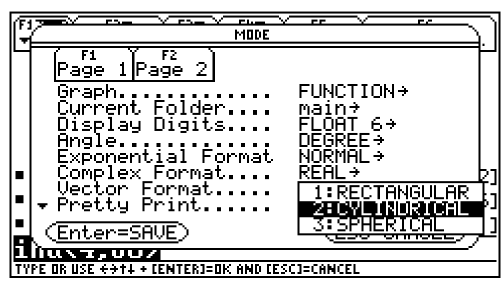
[1/ (2 ∗ π ∗ f ∗ c), ∠-90]

is a vector. The left part of the equation is the magnitude of the capacitive reactance. The right part (after the comma) is the angle. The ∠ after the comma indicates that the capacitive reactance is a vector.

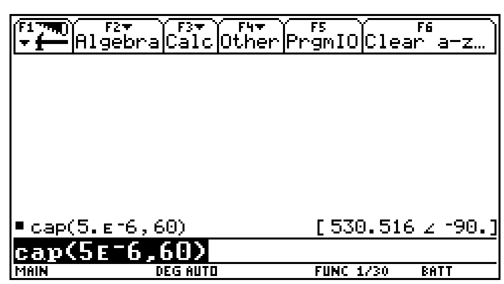
Note when entering EE press ***2nd*** *and* ***EE*** *on the keypad. Refer to the section on exponents in Chapter 1 for more information.*

1. Set Mode to **Degree**
2. Press [MODE]
3. Press [▼] to **Vector Format**
4. Press [▶]
5. Press **2**
   See Figure 30.
6. Press [ENTER]
7. Type **cap** [(] **5** [EE] [(-)] **6** [,] **60** [)]
   Note the - in this equation is the negative key (-).
8. Press [ENTER]

The display in Figure 31 reads **[530.516∠-90]**.



(Figure 30)



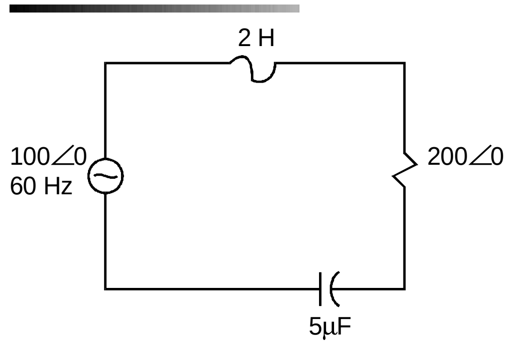
(Figure 31)

Example:

Solving Series AC Circuits

*This example uses the functions **ind()** and **cap()** that were created earlier in this chapter*

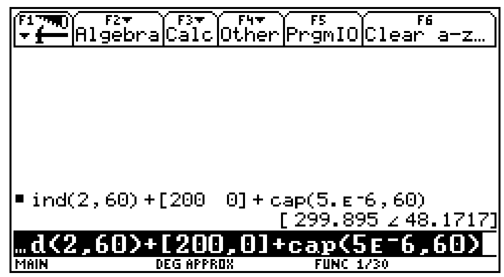
The circuit in Figure 32 has a 2 henry coil, a 200 ohm resistor, and a 5 micro-farad capacitor.



(Figure 32)

To find the total impedance of the circuit:

1. Type **ind** [(] **2** [,] **60** [)] [+] [[] **200** [,] **0** []] [+] **cap** [(] **5** [EE] [(-)] **6** [,] **60** [)]
2. Press [ENTER]
   See Figure 33.



(Figure 33)

The display reads **299.895 ∠ 48.17**.

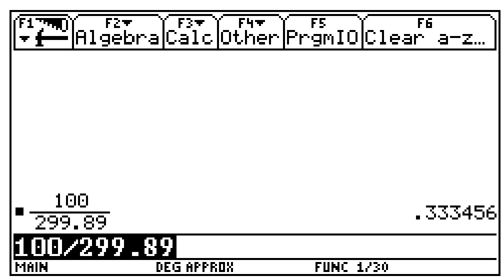
The total impedance (AC resistance) **Z** equals **299.895** ohms, and the impedance angle equals **48.17°**.

By Ohm's Law, the total current equals the total voltage divided by total impedance, or

I = V/Z

To find the current:

3. Type **100** [÷] **299.89**
4. Press [ENTER]
   See Figure 34.



(Figure 34)

The display reads **.333456**.

The phase shift for the current is

0 - 48.17 = -48.17°

The current is equal to **.333456 ∠ -48.17** amps.

Example :

Solving Parallel AC Circuits

The TI-92 is capable of solving circuits with complex parallel impedance. Assume a parallel combination of an inductor **0+12*i*** and a capacitor of **0-4*i***. Both have a phase shift of **90** degrees. The inductor is shifted +**90°**, and the capacitor -**90°**.

The formula for parallel reactance is the same as parallel resistance. $1/(1/r_1)+(1/r_2)+...$

Paralleling two reactances with the TI-92 can be a problem. This is because you can add and subtract vectors but not multiply or divide. We will develop a function **parac()** to compute the combined reactance of two AC devices in parallel. **parac()** uses a function called **div()** that does division of vectors in polar coordinates.

Because the **div()** function is called from **parac()**, we must enter it first.

Function:

div() Function

To enter the **div()** function, follow these steps:

1. Set Mode to **Degree**
2. Press [APPS]
3. Press **7: Program Editor**
4. Press **3: New**
5. Press (right arrow)
6. Select **2: Function**
7. Press (down arrow) twice
8. Type **div** in the **Variable** box
9. Press [ENTER] twice

Now enter the code for **div()**.

```
div(aa,bb,cc,dd)
Func
Local xx,yy,ff
aa/cc→xx
bb-dd→yy
[[xx,∠yy]]→ff
Return ff
EndFunc
```

Function:

parac() Function

Now we can enter **parac()**:

1. Press [APPS]
2. Press **7: Program Editor**
3. Press **3: New**
4. Press (right arrow)
5. Select **2: Function**

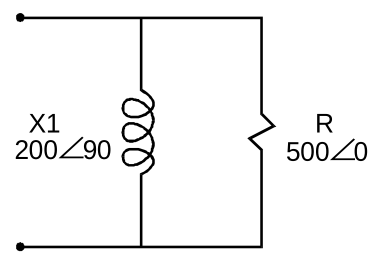
6. Press ⓥ twice
7. Type **parac** in the **Variable** box
8. Press [ENTER] twice

Next enter the program.

```
parac(a,b,c,d)

Func
Local a,b,c,d,e,f,g,h

1/a→a
1/c→c
0-b→b
0-d→d
[[a,∠b]]+[[c,∠d]]→e
mat►list(e)→f
√(f[1]*f[1]+f[2]*f[2])→g
tan⁻¹(f[2]/f[1])→h
Return div(1,0,g,h)

EndFunc
```

Example:

parac() Paralleling a 200 Ohm Inductor and 500 Ohm Resistor

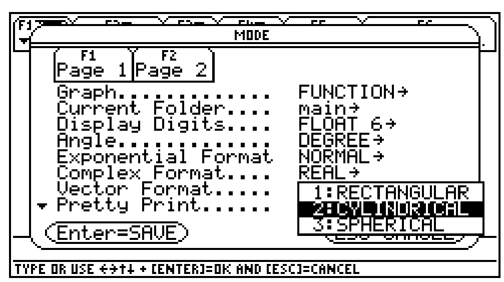


(Figure 35)

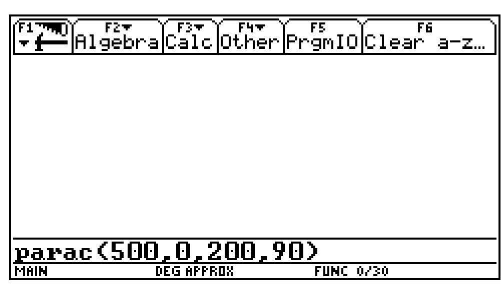
Now we have the tools we need to compute the total reactance of two AC devices in parallel. Let's do the calculation for the circuit shown in Figure 35. This example has a 500Ω resistor in parallel with a 200 henry inductor.

To find the total reactance:

1. Press [♦] [HOME]
2. Press [MODE]
3. Press ⓥ to **Vector Format**
4. Press ⓡ
   See Figure 36.
5. Press **2:CYLINDRICAL**
6. Press [ENTER]
7. Type **parac** [(] **500** [,] **0** [,] **200** [,] **90** [)]
   See Figure 37.



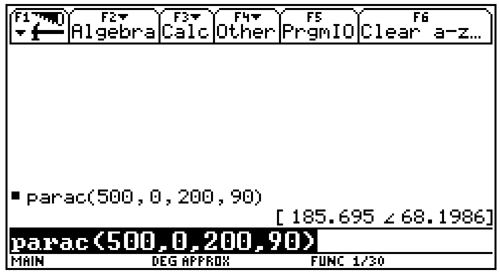
(Figure 36



(Figure 37)

8. Press [ENTER]
   See Figure 38.

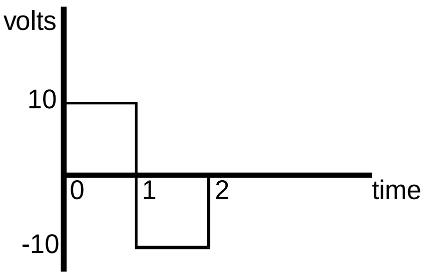
The TI-92 displays **[185.695∠ 68.199]**.



(Figure 38)

Example :
Average Value (method)

The average value of an AC waveform is calculated by taking the area under the curve and dividing it by the period.

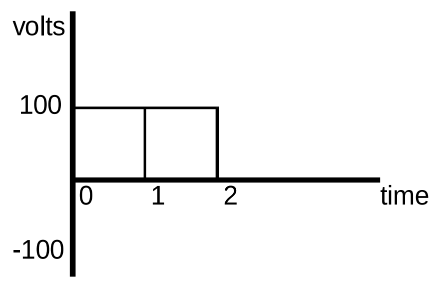


(Figure 39)

For the example in Figure 39, the average value is:

(10∗1 + -10∗1)/2 = 0/2 = 0

As you might imagine, any AC waveform that has the same positive area as negative area has an average value of **0**. It would be very useful to have a value similar to the average value that we could use to characterize a sine wave. Let's try squaring the waveform. A squared version of the wave is shown in Figure 40.



(Figure 40)

Note that the negative portion to the right has now become positive. This is because **-10∗-10 = 100**. Let's find the area under the curve.

100∗1+ 100∗1 = 200

Now divide the area by the period:

200/2 = 100

The average value of the new curve is **100**.

You can find the average value of continuous functions (including the sine wave) by integrating it and dividing by the period. By integrating you obtain the area under the curve. The TI-92 has a key for integrating. It is labeled above the number 7 on the numeric key pad and is denoted by [∫]**.** You must press [2nd] **7** to access this key.

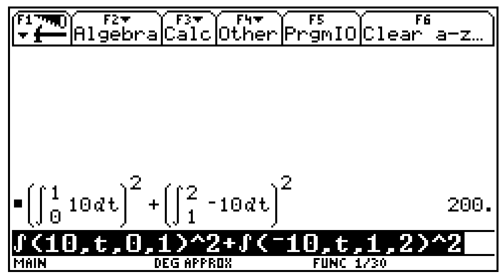
It is beyond the scope of this text to teach integrals. They are useful for finding the area under a curve. To use the integral function you must specify the variable that you are integrating with respect to, and the lower and upper limits.

Example:

Average Value

In this example, integration is with respect to time **t**. Split the curve into two parts representing the positive and negative parts of the curve. The first part is integrated from **0** to **1** (the positive part), and the second from **1** to **2** (the negative part).

1. Press 2nd [∫] (the 7 on the numeric keypad)
2. Type **10** , **t** , **0** , **1**) ^ **2**
3. Press +
4. Press 2nd [∫]
5. Type (-) **10** , **t** , **1** , **2**) ^ **2**
6. Press ENTER
   See Figure 41.



(Figure 41)

The TI-92 displays **200**. This is the area under the curve. To find the average, divide the area by the period.

200/2 = 100

Example:

Average Value (Sine Wave)

The average value of a sine wave can be found by integrating it and dividing by the period. We begin by integrating a sine wave. As stated at the beginning of the chapter, the sine wave is defined by:

v(t) = Vp sin(wt+θ)

Let the frequency **f** equal **60**.

The angular velocity then equals:

w = 2*π*60.

Then, let the phase shift **θ** be **0**.

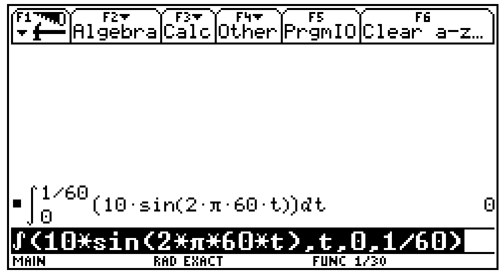
The equation becomes:

v(t) = Vp sin(2*π*60*t + 0), or

v(t) = Vp sin(2*π*60*t)

For one period of the sine wave, time **t** runs from **0** to **2π**.

1. Set Mode to **Radian**
2. Set Mode to **Exact**
3. Press [2nd] [∫]
4. Type **10** [×] [SIN] **2** [×] **π** [×] **60** [×] **t** [)] [,] **t** [,] **0** [,] [(] **1** [÷] **60** [)] [)]
5. Press [ENTER]
   See Figure 42



(Figure 42)

*It is interesting to note that if **Approximate** mode is used in the previous example, the result will not quite equal zero due to approximations made during the calculation. The TI-92 recognizes this condition and displays the warning **Questionable accuracy** at the lower left of the display.*

The TI-92 displays **0** on the screen.

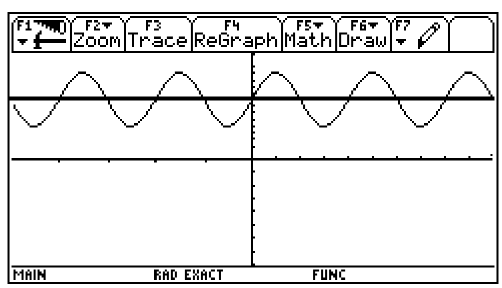
Thus we see that the area under a sine wave over one period is zero. This is consistent with our understanding of a sine wave as a function that is symmetric with respect to zero.

Finding the average value of a sine wave by dividing by the period **1/60** is now a trivial exercise. Since the area under the curve is zero, the average value is also zero.

Example:

Average Value (Sine Wave plus DC Content)

The waveform in Figure 43 is lifted up off the x axis. In this case there is a DC content. If the sine wave is centered on 5 volts, the average value will be 5 volts. In order to find the area under the curve, integrate the equation for the sine wave with 5 added to it.



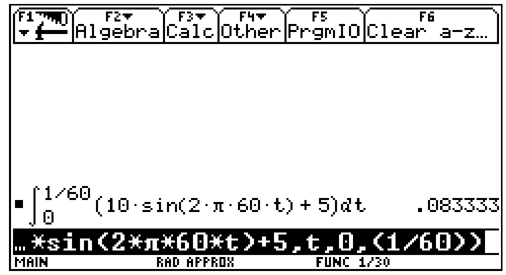
(Figure 43)

1. Set Mode to **Radian**
2. Set Mode to **Approximate**
3. Press [2nd] [∫]
4. Type **10** [×] [SIN] **2** [×] [π] [×] **60** [×] **t** [)] [+] **5** [,] **t** [,] **0** [,] [(] **1** [÷] **60** [)] [)]
5. Press [ENTER]
   See Figure 44.

The display reads **.083333**.

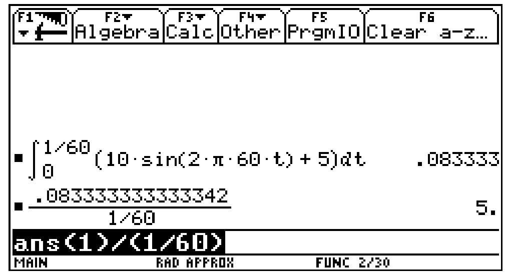
This is the area under the curve. To find the average value, divide the area under the curve by the period **1/60**.

6. Type [(] **1** [÷] **60** [)]
7. Press [ENTER]
   See Figure 45.



(Figure 44)

The screen reads **5**. This is the average value of the waveform.



(Figure 45)

RMS Calculations

Since the average value of a sine wave is always equal only to its DC content, a calculated value that is similar to the average value is used for AC. It is called the RMS value. RMS stands for root mean square. To find the rms value, first square the function, find its average (or mean), then take the square root of the average. The formula to do this is shown below:

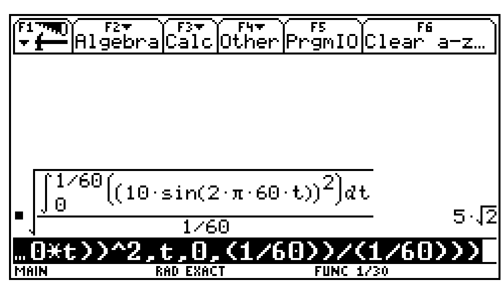
$$\mathbf{V_{rms}} = \sqrt{\frac{\int(10*\sin(2\pi*60*t))^2}{1/60}}$$

The **^2** part of the equation squares the sine wave function. It is then integrated and divided by the period **1/60** to determine the mean. Finally, the square root of the mean is found.

Example:

RMS Integration

1. Set Mode to **Radian**
2. Set Mode to **Exact**
3. Press [2nd] [√]
4. Press [2nd] [∫]
5. Type [(] **10** [×] [SIN] **2** [×] [π] [×] **60** [×] **t** [)] [)] [^] **2** [,] **t** [,] **0** [,] **1** [÷] **60** [)] [÷] [(] **1** [÷] **60** [)] [)]
6. Press [ENTER]
   See Figure 46.



(Figure 46)

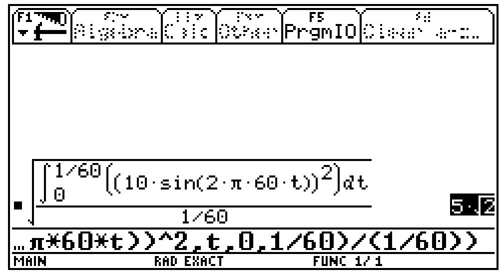
7. Press ⬆
8. Press [ENTER]

The previous solution, **5√2**, will now appear in the entry line, as shown in Figure 47.

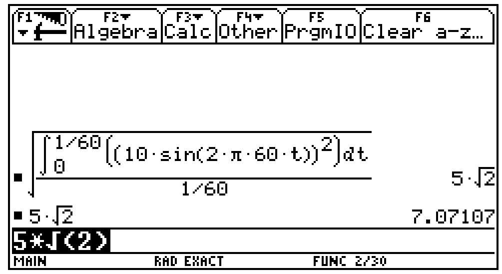
9. Press [◆] [ENTER]

This forces an approximate answer of **7.07**, as shown in Figure 48.

The method described above finds the root mean square of any continuous function.



(Figure 47)



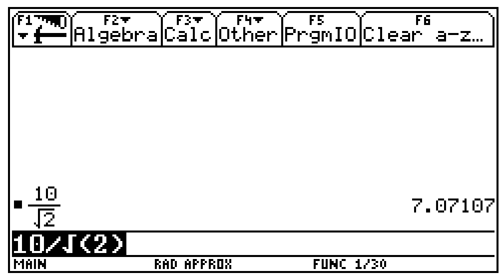
(Figure 48)

Example:

RMS Calculated

If you want to find the RMS value of a sine wave you can divide the peak value by **√2**. In the previous example, the sine wave had a peak value of **10**.

1. Set Mode to **Approximate**
2. Set Mode to **Radian**
3. Type **10** [÷] [√] **2** [)] [ENTER]
   See Figure 49.



(Figure 49)

The TI-92 displays **7.07** as the answer.

