Math Objectives

- Students will develop a basic understanding of the polar coordinate system and locate points given in polar form.
- Students will convert points between polar and rectangular coordinates.
- Students will sketch graphs of polar equations and compare them to their rectangular coordinate function counterpart.
- Students will use appropriate technological tools strategically, and look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- polar coordinates
- rectangular coordinates
- absolute value
- polar axis
- pole
- argument

About the Lesson

- This lesson involves a brief introduction to the polar coordinate system.
- As a result, students will:
  - Determine the location (quadrant) of various points given in polar form.
  - Recognize cases in which a point lies on an axis.
  - Convert points between polar and rectangular form.
  - Discover that polar coordinates for a point are not unique.
  - Use their calculators to check graphs sketched using paper and pencil.

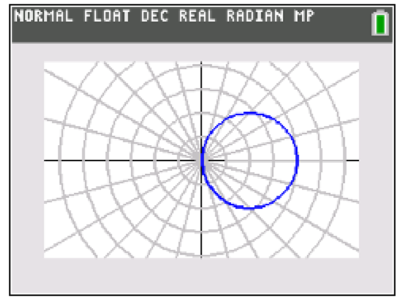
Teacher Preparation and Notes.

This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint^{TM} functionality.*



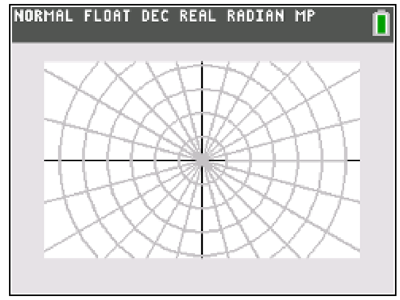
Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Polar_Coordinates_84CE_Student.pdf
Polar_Coordinates_84CE_Student.doc
POLARGRD.8xp
Special_Types_of_Polar_Functions.pdf

Open the document *POLARGRD.8xp.*

In this activity, you will be introduced to the polar coordinate system. You will plot points given in polar form, convert polar coordinates to the rectangular coordinate system, and sketch the graphs of polar equations.



The polar coordinate system is a two-dimensional coordinate system defined by a point, called the pole, and a ray from the pole, called the polar axis. In a rectangular coordinate system, the **pole** is usually placed at the origin, and the **polar axis** is represented by the positive x-axis. A point in the polar coordinate system is represented by the ordered pair (r,θ) where r is the distance from the pole and θ is the angle (in radians) measured counterclockwise from the polar axis.

Tech Tip: This activity uses a program that opens a polar grid on the graph screen. This is meant to help students locate polar coordinates and graph polar functions. Make sure that the students have changed the handheld to polar mode and radian measure under the mode screen.

Problem 1 – Identifying The Quadrants in a Polar System

1. **Press graph.** On this screen, you will see a polar grid. Use this grid to aid in completing the table below. Starting at the polar axis and rotating counterclockwise, each slant line increases by a unit of $\frac{\pi}{12}$ (or $15°$) and each ring increases by one unit.
   a. Complete the following tables by finding the quadrant in which the point (r,θ) lies.

   __Solution:__

r	1.7	1.3	–0.6	–4.2	–3.2	3.1	–1.5	–2.5
θ	$\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{4\pi}{3}$	$-\frac{13\pi}{4}$	$\frac{13\pi}{12}$	$-\frac{7\pi}{4}$
Quadrant	**II**	**III**	**IV**	**IV**	**IV**	**II**	**I**	**III**

r	0.8	2.1	2	–2.7	4	3.5	–1.4	–3
θ	$\frac{19\pi}{6}$	$\frac{\pi}{4}$	$-\frac{17\pi}{12}$	$\frac{11\pi}{4}$	$\frac{7\pi}{6}$	$-\frac{\pi}{3}$	$\frac{11\pi}{3}$	$\frac{\pi}{3}$
Quadrant	**III**	**I**	**II**	**IV**	**III**	**IV**	**II**	**III**

r	-4	2.7	1	3.9	-5	-2	-1	1.5
θ	$-\frac{\pi}{6}$	$-\frac{11\pi}{3}$	$\frac{23\pi}{12}$	$-\frac{11\pi}{6}$	$-\frac{9\pi}{4}$	$\frac{11\pi}{6}$	$-\frac{7\pi}{6}$	$\frac{9\pi}{4}$
Quadrant	**II**	**I**	**IV**	**I**	**II**	**II**	**IV**	**I**

b. Describe the location of the point with the following polar coordinates:
(i) $r > 0$ and $\theta = 0$

Sample Solution: The point lies on the polar axis, or the positive x-axis.

(ii) $r < 0$ and $\theta = \frac{3\pi}{2}$

Sample Solution: The point lies on the positive y-axis.

(iii) $r < 0$ and $\theta = \frac{\pi}{2}$

Sample Solution: The point lies on the negative y-axis.

(iv) $r > 0$ and $\theta = -3\pi$

Sample Solution: The point lies on the negative x-axis.

Problem 2 – Matching Polar Coordinates with Rectangular Coordinates

If a point has polar coordinates (r,θ), then the rectangular coordinates are given by $x = r\cos\theta$ and $y = r\sin\theta$. Similarly, if a point has rectangular coordinates (x, y), then the polar coordinates are (r,θ) such that $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x},\ x \neq 0$.

2. Complete each of the following tables. Remember that there are an infinite number of polar coordinates that can be plotted in a single location. For example, $\left(1,\frac{\pi}{6}\right)$ is equivalent to $\left(-1,\frac{7\pi}{6}\right)$.

a. For each given polar coordinate, find two different polar coordinates that represent the given point.

Sample Solutions:

(r_1,θ_1)	$\left(2,\frac{\pi}{4}\right)$	$\left(3,\frac{7\pi}{4}\right)$	$\left(6,\frac{2\pi}{3}\right)$	$\left(1,\frac{7\pi}{6}\right)$	$\left(-2,\frac{5\pi}{4}\right)$	$\left(\frac{3}{4},\frac{17\pi}{6}\right)$
(r_2,θ_2)	$\left(2,\frac{9\pi}{4}\right)$	$\left(3,-\frac{\pi}{4}\right)$	$\left(6,\frac{8\pi}{3}\right)$	$\left(1,\frac{19\pi}{6}\right)$	$\left(2,\frac{\pi}{4}\right)$	$\left(\frac{3}{4},\frac{5\pi}{6}\right)$
(r_3,θ_3)	$\left(-2,\frac{5\pi}{4}\right)$	$\left(3,\frac{15\pi}{4}\right)$	$\left(-6,\frac{5\pi}{3}\right)$	$\left(-1,\frac{\pi}{6}\right)$	$\left(2,-\frac{7\pi}{4}\right)$	$\left(-\frac{3}{4},-\frac{\pi}{6}\right)$

b. For each point given in polar coordinates below, determine the rectangular coordinates.

Sample Solutions:

(r,θ)	$\left(3,\frac{7\pi}{3}\right)$	$\left(1,\frac{\pi}{6}\right)$	$\left(-2,-\frac{4\pi}{3}\right)$	$\left(\sqrt{5},-\frac{3\pi}{2}\right)$	$\left(-8,\frac{3\pi}{4}\right)$	$\left(\frac{13}{4},-\frac{\pi}{3}\right)$
x	$\frac{3}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$4\sqrt{2}$	$\frac{13}{8}$
y	$\frac{3\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$\sqrt{5}$	$-4\sqrt{2}$	$-\frac{13\sqrt{3}}{8}$

c. For each point given in rectangular coordinates below, determine two representations in polar coordinates.

Sample Solutions:

(x,y)	$(3,4)$	$(-\sqrt{2},2)$	$(4,-7)$	$(-\sqrt{3},-1)$	$(-5,5)$	$(7,24)$
r_1	5	$\sqrt{6}$	$\sqrt{65}$	2	$5\sqrt{2}$	25
θ_1	0.927	2.186	-1.052	$-\frac{5\pi}{6}$	$\frac{3\pi}{4}$	1.287
r_2	-5	$-\sqrt{6}$	$-\sqrt{65}$	-2	$-5\sqrt{2}$	-25
θ_2	4.069	5.328	2.090	$\frac{\pi}{6}$	$\frac{7\pi}{4}$	4.429

Problem 2 – Graphing Polar Functions

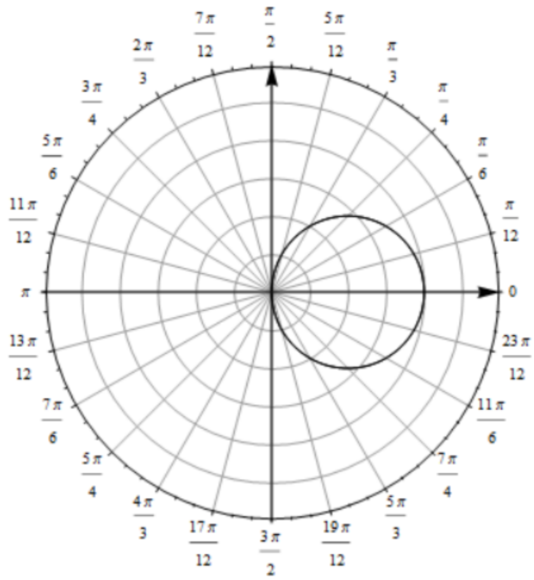
3. Over the next two worksheet pages, carefully sketch a complete graph of each given polar equation. Create a table of values, search for patterns, and sketch the graph on the axes provided. Check your results on the handheld.

Note: Make sure the Graph Mode is set to Polar. Press mode and use the down/right arrows to move and change from function to polar form.

a. $r = 4\cos\theta$.

Sample Solution:

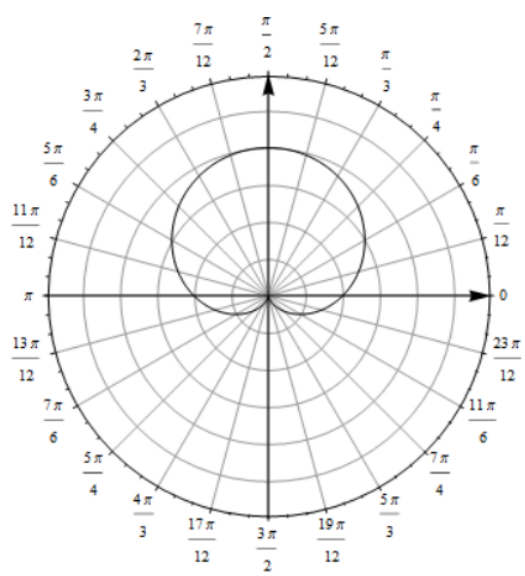
θ	r
0	4
$\frac{\pi}{6}$	$2\sqrt{3}$
$\frac{\pi}{4}$	$2\sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-2
$\frac{3\pi}{4}$	$-2\sqrt{2}$
$\frac{5\pi}{6}$	$-2\sqrt{3}$
π	-4



b. $r = 2 + 2\sin\theta$.

Sample Solution:

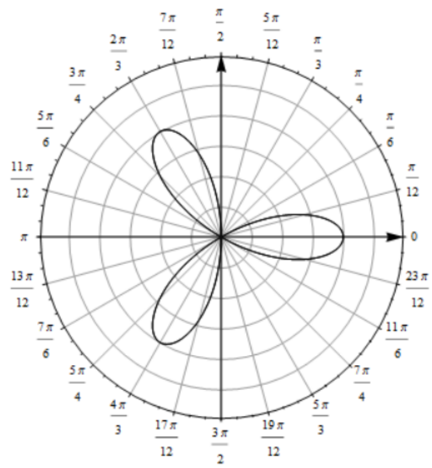
θ	r
0	2
$\frac{\pi}{6}$	3
$\frac{\pi}{4}$	$2+\sqrt{2}$
$\frac{\pi}{3}$	$2+\sqrt{3}$
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	$2+\sqrt{3}$
$\frac{3\pi}{4}$	$2+\sqrt{2}$
$\frac{5\pi}{6}$	3
π	2



c. $r = 4\cos 3\theta$.

Sample Solution:

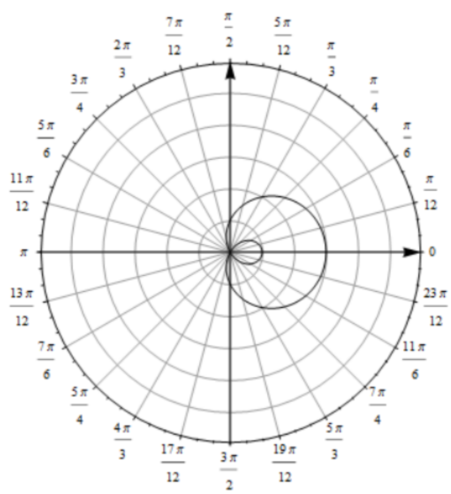
θ	r
0	4
$\frac{\pi}{6}$	0
$\frac{\pi}{4}$	$-2\sqrt{2}$
$\frac{\pi}{3}$	-4
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	4
$\frac{3\pi}{4}$	$2\sqrt{2}$
$\frac{5\pi}{6}$	0
π	-4



d. $r = 1 + 2\cos\theta$.

Sample Solutions:

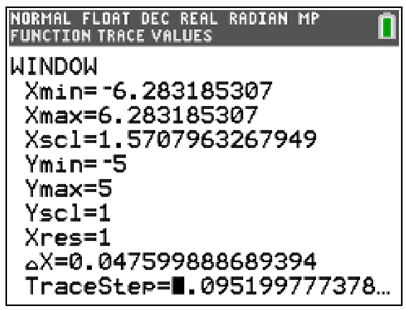
θ	r
0	3
$\frac{\pi}{6}$	$1+\sqrt{3}$
$\frac{\pi}{4}$	$1+\sqrt{2}$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	$1-\sqrt{2}$
$\frac{5\pi}{6}$	$1-\sqrt{3}$
π	-1



Problem 4 – Comparing Trigonometric Functions in the Polar and Rectangular Systems

Teacher Tip: Problem 4 makes a connection to the rectangular function equation of the four previous graphs. There is another document you can use with your students to prepare them for this problem, entitled *Special Types of Polar Functions*. Part (b) for each question might require a lot of group discussion and teacher guidance.

In this problem, you will compare the four polar graphs from **Problem 3** with their rectangular system counterparts. Graph each of the following on the handheld (remember to change the graph mode back to function, clear the POLARGRD program, and set your window to the screen shot below) and answer each corresponding question.



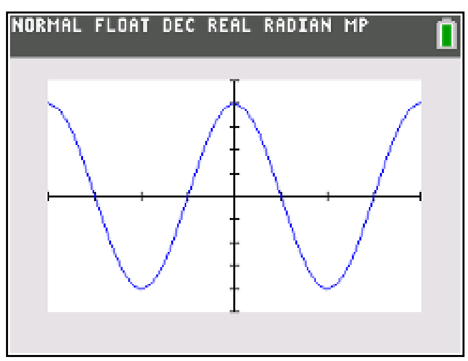
4. $f(x) = 4\cos(x)$

(a) What shape was created on the polar graph from question 3 (a)?

Solution: Circle along the positive x-axis.

(b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a circle along the x-axis. The amplitude of the cosine curve is 4, this also is the distance from the pole to the farthest ring, which is also the diameter of the circle of the polar graph. The period of the cosine function is 2π, therefore the circle will be completed twice as it only took a distance of π to create it on the polar graph.

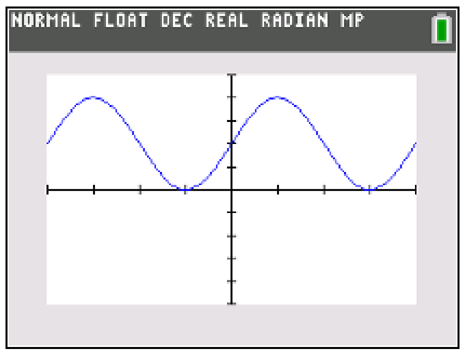
5. $f(x) = 2 + 2\sin(x)$

   (a) What shape was created on the polar graph from question 3(b)?

Solution: Cardioid (Limaçon) along the positive y-axis.

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude, period, and vertical shift in the explanation.

   Possible Solution:



Since this is a sine curve, and sine is represented by y in the unit circle, the shape is a limaçon (cardioid) along the y-axis. The amplitude of the sine curve is 2 and the vertical translation is 2, therefore their sum is also the distance from the pole to the farthest ring of the polar graph. The period of the sine function is 2π, therefore the cardioid will be completed once as it took a distance of 2π to create it on the polar graph.

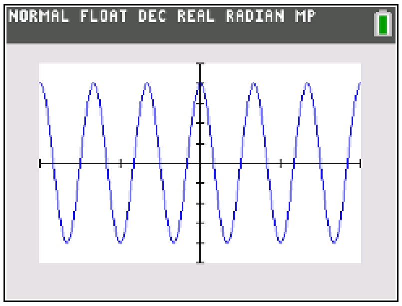
6. $f(x) = 4\cos(3x)$

   (a) What shape was created on the polar graph from question 3(c)?

   Solution: Rose curve with 3 petals, one centered on the positive x-axis.

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

   Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a rose curve with 3 petals, one centered along the x-axis. The amplitude of the cosine curve is 4, this also is the distance from the pole to the farthest end of each petal. The period of the cosine function is $\frac{2\pi}{3}$ (3 complete cycles from $0\ to\ 2\pi$), therefore the 3 rose petals will be completed twice on the polar graph.

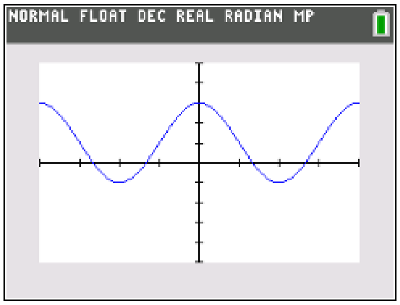
7. $f(x) = 1 + 2\cos(x)$

   (a) What Shape was created on the polar graph from question 3(d)?

   Solution: Limaçon with an inner loop along the positive x-axis.

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

   Possible Solution:



Since this is a cosine curve, and cosine is represented by x in the unit circle, the shape is a limaçon with an inner loop along the x-axis. The amplitude of the cosine curve is 2 and the vertical translation is 1, therefore their sum is also the distance from the pole to the farthest ring of the polar graph. The period of the cosine function is 2π, therefore the limaçon will be completed once as it took a distance of 2π to create it on the polar graph.

Extensions

Here are some possible extensions to this activity:
1. Create a list of polar equations and graphs and ask students to match each equation with its corresponding graph.
2. Ask students to convert specific equations from rectangular form to polar form.
3. Ask students to sketch the graphs of other polar equations.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The polar coordinate system and how to plot points given in polar form.
- How to convert points between rectangular and polar form.
- How to sketch the graph of a polar equation.