Families of Functions

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will investigate the effects parameters *a*, *h*, and *k* have on a given function.
- Students will generalize the effects that parameters *a*, *h*, and *k* have on any function.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- function
- parameter
- vertical stretch and vertical compression
- horizontal translation and vertical translation
- transformation
- scale factor

About the Lesson

- This lesson involves changing the sliders for *a*, *h*, and *k* on each page and observing the effects each has on the graphs of the functions.
- As a result, students will:
  - Use the completed table to make generalizations about the effects of *a*, *h*, and *k* on the graphs of any function.
  - Describe the transformations to a parent function using their generalizations.

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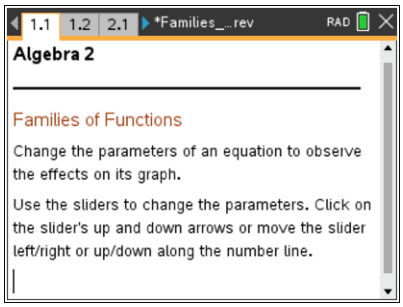
- Use Class Capture to see if students understand how *a*, *h*, and *k* affect the graph.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Families_of_Functions_Student.pdf
- Families_of_Functions_Student.doc

TI-Nspire document

- Families_of_Functions.tns

Discussion Points and Possible Answers

Tech Tip: If students have difficulty moving the point for *h* and *k*, make sure they have moved the cursor (arrow) until it becomes a hand (☜) getting ready to grab the point on the slider. Press ctrl ☜ to grab the point and close the hand (☜). Once the point is grabbed, use arrow keys to move it. When finished moving any slider or point, press esc to release.

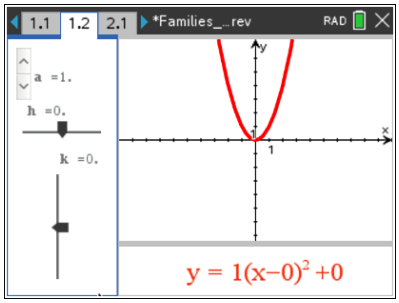
Tech Tip: If students experience difficulty changing the slider for *a*, check to make sure that they are tapping the up or down arrow. If they have difficulty moving the point for *h* and *k*, check to make sure that they are touching the point. If a student double taps off of a slider, the function entry line might open. If that happens, have them minimize the keyboard and tap elsewhere on the screen to close the function entry line.

Teacher Tip: Students should change the sliders for each variable to determine what effects that variable has on each graph. When moving the points for *h* and *k*, the slider for *a* should be set to any value except zero.

Page	Parent Function (Equation or Type)	Sketch of Parent Function	Effects of Parameter *a*	Effects of Parameter *h*	Effects of Parameter *k*
1.2	Quadratic $f(x)=a\cdot(x-h)^2+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
2.1	Absolute Value $f(x)=a\cdot\lvert x-h\rvert+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
3.1	Square Root $f(x)=a\cdot\sqrt{x-h}+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
4.1	Exponential $f(x)=a\cdot 2^{x-h}+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
5.1	Logarithmic $f(x)=a\cdot\log(x-h)+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
6.1	Cubic $f(x)=a\cdot(x-h)^3+k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*

7.1	Periodic (sine) $f(x) = a \cdot \sin(x - h) + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*

Move to page 1.2.



1. Given any function, describe the effects parameter *a* has on its graph when:
   a. $|a| > 1$

      Answer: The graph of the function is stretched vertically by that factor.

   b. $0 < |a| < 1$

      Answer: The graph of the function is vertically compressed by that factor.

   c. $a < 0$

      Answer: The graph of the function is reflected over a horizontal line.

   d. $a = 0$

      Answer: The graph of the function becomes a horizontal line.

2. Given any function, describe the effects parameter *h* has on its graph when:
   a. $h > 0$

      Answer: The graph of the function is translated horizontally to the right that number of units.

   b. $h < 0$

      Answer: The graph of the function translated horizontally to the left that number of units.

   c. $h = 0$

      Answer: The graph of the function does not translate horizontally.

3. Given any function, describe the effects parameter k has on its graph when

   a. $k > 0$

   Answer: The graph of the function is translated vertically upward that number of units.

   b. $k < 0$

   Answer: The graph of the function translated vertically downward that number of units.

   c. $k = 0$

   Answer: The graph of the function does not translate vertically.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

4. Given the following functions, describe the transformations on the parent function, $f(x)$.

   a. $f(x) = x^2$; $h(x) = 3(x - 4)^2 + 2$

   Answer: The graph of $f(x) = x^2$ is vertically stretched by a factor of 3. It is translated horizontally right 4 units and translated vertically up 2 units.

   b. $f(x) = x^3$; $g(x) = -(x - 1)^3$

   Answer: The graph of $f(x) = x^3$ is reflected over the x-axis and translated horizontally to the right 1 unit.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

5. Given the following transformations, write the equation of the function.

   a. The graph of $f(x) = \sqrt{x}$ is reflected over the x-axis, vertically stretched by a factor of 2, and translated vertically down 1 unit.

   Answer: $g(x) = -2\sqrt{x} - 1$

   b. The graph of $f(x) = |x|$ is translated horizontally to the left 3 units and translated vertically up 5 units.

   Answer: $g(x) = |x + 3| + 5$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The effects the parameters *a*, *h*, and *k* have on the graphs of functions.
- How to describe the transformations on a given parent function.

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Note 1

Question 3, *Class Capture*: After students have explored the effects of all three variables, ask them to display a quadratic that is translated down 3 units and right 5 units. Take a Class Capture when everyone has done so. All quadratics should have a vertex at (5, –3). Some students might change only *h* and *k*, while others might change all three. Discuss why each is correct.

Note 2

Question 4, *Quick Poll*: Use a multiple-choice *Quick Poll* for students to share their answers.

4. a. $h(x) = 3(x - 4)^2 + 2$ has been translated
   - A. left 4 units, down 2 units
   - B. left 4 units, up 2 units
   - C. right 4 units, down 2 units
   - D. right 4 units, up 2 units

Students should discuss why choice D is the correct answer.