Math Objectives
- Students will investigate the effects parameters $a$, $h$, and $k$ have on a given function.
- Students will generalize the effects that parameters $a$, $h$, and $k$ have on any function.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary
- function
- parameter
- vertical stretch and vertical compression
- horizontal translation and vertical translation
- transformation
- scale factor

About the Lesson
- This lesson involves changing the sliders for $a$, $h$, and $k$ on each page and observing the effects each has on the graphs of the functions.
- As a result, students will:
  - Use the completed table to make generalizations about the effects of $a$, $h$, and $k$ on the graphs of any function.
  - Describe the transformations to a parent function using their generalizations.

Tech Tips:
- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.

Activity Materials
Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Lesson Files:
Student Activity
- Families_of_Functions_Student.pdf
- Families_of_Functions_Student.doc
TI-Nspire document
- Families_of_Functions.tns
### Discussion Points and Possible Answers

**Tech Tip:** If students have difficulty moving the point for \( h \) and \( k \), make sure they have moved the cursor (arrow) until it becomes a hand \( \text{hand} \) getting ready to grab the point on the slider. Press \( \text{ctrl} \) to grab the point and close the hand \( \text{hand} \). Once the point is grabbed, use arrow keys to move it. When finished moving any slider or point, press \( \text{esc} \) to release.

**Tech Tip:** If students experience difficulty changing the slider for \( a \), check to make sure that they are tapping the up or down arrow. If they have difficulty moving the point for \( h \) and \( k \), check to make sure that they are touching the point. If a student double taps off of a slider, the function entry line might open. If that happens, have them minimize the keyboard and tap elsewhere on the screen to close the function entry line.

**Teacher Tip:** Students should change the sliders for each variable to determine what effects that variable has on each graph. When moving the points for \( h \) and \( k \), the slider for \( a \) should be set to any value except zero.

<table>
<thead>
<tr>
<th>Page</th>
<th>Parent Function (Equation or Type)</th>
<th>Sketch of Parent Function</th>
<th>Effects of Parameter ( a )</th>
<th>Effects of Parameter ( h )</th>
<th>Effects of Parameter ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>Quadratic ( f(x) = a \cdot (x - h)^2 + k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
<td>translates the graph left or right depending on the sign of ( h )</td>
<td>translates the graph up or down depending on the sign of ( k )</td>
</tr>
<tr>
<td>2.1</td>
<td>Absolute Value ( f(x) = a \cdot</td>
<td>x - h</td>
<td>+ k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
</tr>
<tr>
<td>3.1</td>
<td>Square Root ( f(x) = a \cdot \sqrt{x - h} + k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
<td>translates the graph left or right depending on the sign of ( h )</td>
<td>translates the graph up or down depending on the sign of ( k )</td>
</tr>
<tr>
<td>4.1</td>
<td>Exponential ( f(x) = a \cdot 2^{x-h} + k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
<td>translates the graph left or right depending on the sign of ( h )</td>
<td>translates the graph up or down depending on the sign of ( k )</td>
</tr>
<tr>
<td>5.1</td>
<td>Logarithmic ( f(x) = a \cdot \log(x - h) + k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
<td>translates the graph left or right depending on the sign of ( h )</td>
<td>translates the graph up or down depending on the sign of ( k )</td>
</tr>
<tr>
<td>6.1</td>
<td>Cubic ( f(x) = a \cdot (x - h)^3 + k )</td>
<td>Teacher Observation</td>
<td>stretches or compresses the graph vertically</td>
<td>translates the graph left or right depending on the sign of ( h )</td>
<td>translates the graph up or down depending on the sign of ( k )</td>
</tr>
</tbody>
</table>
7.1 Periodic (sine) 
\[ f(x) = a \cdot \sin(x - h) + k \]

<table>
<thead>
<tr>
<th>Teacher Observation</th>
<th>stretches or compresses the graph vertically</th>
<th>translates the graph left or right depending on the sign of ( h )</th>
<th>translates the graph up or down depending on the sign of ( k )</th>
</tr>
</thead>
</table>

**Move to page 1.2.**

1. Given any function, describe the effects parameter \( a \) has on its graph when:
   a. \(|a| > 1\)
      
      **Answer:** The graph of the function is stretched vertically by that factor.
   
   b. \(0 < |a| < 1\)
      
      **Answer:** The graph of the function is vertically compressed by that factor.
   
   c. \(a < 0\)
      
      **Answer:** The graph of the function is reflected over a horizontal line.
   
   d. \(a = 0\)
      
      **Answer:** The graph of the function becomes a horizontal line.

2. Given any function, describe the effects parameter \( h \) has on its graph when:
   a. \(h > 0\)
      
      **Answer:** The graph of the function is translated horizontally to the right that number of units.
   
   b. \(h < 0\)
      
      **Answer:** The graph of the function translated horizontally to the left that number of units.
   
   c. \(h = 0\)
      
      **Answer:** The graph of the function does not translate horizontally.
3. Given any function, describe the effects parameter $k$ has on its graph when
   a. $k > 0$
      
      **Answer:** The graph of the function is translated vertically upward that number of units.
   
   b. $k < 0$
      
      **Answer:** The graph of the function translated vertically downward that number of units.
   
   c. $k = 0$
      
      **Answer:** The graph of the function does not translate vertically.

4. Given the following functions, describe the transformations on the parent function, $f(x)$.
   a. $f(x) = x^2; h(x) = 3(x - 4)^2 + 2$
      
      **Answer:** The graph of $f(x) = x^2$ is vertically stretched by a factor of 3. It is translated horizontally right 4 units and translated vertically up 2 units.
   
   b. $f(x) = x^3; g(x) = -(x - 1)^3$
      
      **Answer:** The graph of $f(x) = x^3$ is reflected over the x-axis and translated horizontally to the right 1 unit.

5. Given the following transformations, write the equation of the function.
   a. The graph of $f(x) = \sqrt{x}$ is reflected over the x-axis, vertically stretched by a factor of 2, and translated vertically down 1 unit.
      
      **Answer:** $g(x) = -2\sqrt{x} - 1$
   
   b. The graph of $f(x) = |x|$ is translated horizontally to the left 3 units and translated vertically up 5 units.
      
      **Answer:** $g(x) = |x + 3| + 5$
Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The effects the parameters \(a\), \(h\), and \(k\) have on the graphs of functions.
- How to describe the transformations on a given parent function.

**TI-Nspire Navigator**

**Note 1**

**Question 3, Class Capture:** After students have explored the effects of all three variables, ask them to display a quadratic that is translated down 3 units and right 5 units. Take a Class Capture when everyone has done so. All quadratics should have a vertex at \((5, \, -3)\). Some students might change only \(h\) and \(k\), while others might change all three. Discuss why each is correct.

**Note 2**

**Question 4, Quick Poll:** Use a multiple-choice Quick Poll for students to share their answers.

4. a. \(h(x) = 3(x - 4)^2 + 2\) has been translated
   - A. left 4 units, down 2 units
   - B. left 4 units, up 2 units
   - C. right 4 units, down 2 units
   - D. right 4 units, up 2 units

Students should discuss why choice D is the correct answer.