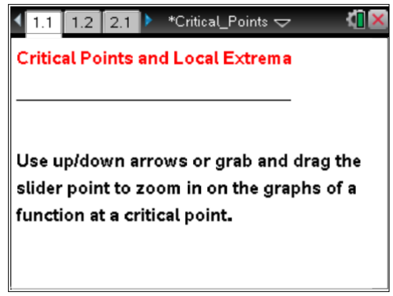
Critical Points and Local Extrema

Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *Critical_Points.tns*.



A function **f** has a critical point at c if

- the value c is in the domain of the function **f** (in other words, $\mathbf{f}(c)$ is defined) and
- either $\mathbf{f}'(c) = 0$ or $\mathbf{f}'(c)$ is undefined.

A function has a local maximum at c if $\mathbf{f}(c) \geq \mathbf{f}(x)$ when x is near c (that is, if $\mathbf{f}(c) \geq \mathbf{f}(x)$ for all x in some open interval containing c). Similarly, **f** has a local minimum at c if $\mathbf{f}(c) \leq \mathbf{f}(x)$ when x is near c (if $\mathbf{f}(c) \leq \mathbf{f}(x)$ for all x in some open interval containing c).

In this activity, you will see several different examples of critical points and local extrema (maxima or minima).

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Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. The graph of the differentiable function shown in the left window has a box centered around the point (1, 2). You can use the up/down arrows at the top of the screen (or drag the point on the line segment) to see a "zoomed in" view of this boxed area of the graph in the right window.

   a. This function has a local minimum at $x = 1$. Using the graph and the definition of local minimum above, explain why.

   b. What appears to happen to the graph as you zoom in on the point (1, 2)?

   c. What is $\mathbf{f}'(1)$? Explain your answer. Why is $c = 1$ a critical point of **f?**

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2. This is the graph of a function having a local maximum at $x = –2$.

   a. What appears to happen to the graph as you zoom in on the point (–2, 1)?

   b. What is the value of $\mathbf{f}'(–2)$? Explain your answer. Why is $c = –2$ a critical point of **f?**

c. What value could the derivative of a function have at the location of a local maximum or minimum? Explain your answer.

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3. This is the graph of a function having a local minimum at $x = -1$.

   a. What happens to the graph as you zoom in on the point (–1, –2)?

   b. Assuming this behavior persists no matter how far you zoom in, is this function differentiable at $x = -1$? Why or why not?

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4. This is the graph of a function having a local maximum at $x = 2$.

   a. What happens to the graph as you zoom in on the point (2, –1)?

   b. Assuming this behavior persists no matter how far you zoom in, is this function differentiable at $x = 2$? Why or why not?

   c. Do these examples support your answer to question 2c? If so, explain why. If not, how might you modify your previous answer?

   d. Could a function have a local maximum or minimum at a non-critical point? Why or why not?

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5. The graph of this increasing function has a horizontal tangent at the point $x = 2$.

    a. Is $x = 2$ a critical point? Why or why not?

    b. Does **f** have either a local minimum or local maximum at $x = 2$?

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6. The graph of this increasing function has a vertical tangent at the point $x = –2$.

    a. Is $x = –2$ a critical point? Why or why not?

    b. Does **f** have either a local minimum or local maximum at $x = –2$?

    c. Does this contradict the statement you made in question 3d? Explain why or why not.