EXPLORATIONS ™

Activity 5

Objectives

- Use linear programming to solve problems involving maximum and minimum values
- Use the Inequality Graphing application to solve linear programming problems

Maximizing Your Efforts

Introduction

During World War II, George Dantzig, a mathematician and statistician working for the U.S. Air Force, developed a mathematical process called linear programming as a means for improving military planning. Shortly thereafter, business and industry started using this method to plan and schedule production more economically. In the business world, where companies strive to minimize expenses and maximize profits, linear programming helps achieve these goals.

Today, linear programming is used in a wide range of fields to perform many complex tasks, from factory production to marketing and investing. Renowned Hungarian computer scientist Lazlo Lovasz has said, "If one would take statistics about which mathematical problem is using up the most computer time in the world, then the answer would probably be linear programming."

Problem

Games and More, Co. is a company that produces both video and DVD game players. The video player, Gamer Gallery, requires 1.5 hours for assembly and 0.25 hours for testing. The DVD player, Major Player, requires 1 hour for assembly and 0.5 hours for testing. Each month, the Games and More manufacturing facility has 45,000 available hours for product assembly and 20,000 available hours for product testing. The Games and More, Co. earns $60 profit from each Gamer Gallery and $75 profit from each Major Player that it sells.

How many of each type of player should the Games and More, Co. produce in order to obtain the greatest monthly profit?

Exploration

You can use linear programming to solve this manufacturing problem. Each step is described.

1. Define the variables in the problem.

Let g represent the number of Gamer Gallery players, and let m represent the number of Major Player players.

2. Write a statement, called the *objective function,* to find the value to be maximized.

For this problem, the objective function is expressed in terms of the variables g and m. The value to be maximized is the profit that the Games and More, Co. can make from selling their products.

a. Write an expression that represents the profit earned from selling Gamer Gallery and Major Player players.

$P(g, m) =$ _____

b. Determine the profit that would be earned if 50 Gamer Gallery players and 100 Major Player players were sold.

$P(50, 100) =$_____

3. Write a system of inequalities that represents the restrictions, or *constraints*, for the problem situation.

A table can be extremely helpful when identifying the constraints of a linear programming problem.

To help you write the system of inequalities you need, complete the following table.

	Gamer Gallery (hours per player)	Major Player (hours per player)	Available Hours
Assembly Time	1.5		45,000
Testing Time			

a. Write an inequality in terms of g and m that represents the assembly time constraint.

_____

b. Write an inequality in terms of g and m that represents the testing time constraint.

_____

You have written two of the four constraints needed to represent this Games and More manufacturing problem. While it may seem obvious, it is also necessary to state that Games and More cannot produce a negative number of video and DVD players.

c. Write two inequalities that express these two constraints.

_____ _____

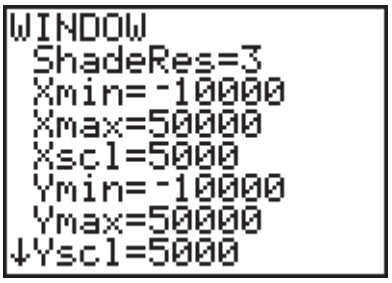
4. Graph the system of inequalities.

Since the objective function is written as $P(g, m) = 60g + 75m$, you can plot values of g along the horizontal axis, and values of m along the vertical axis.

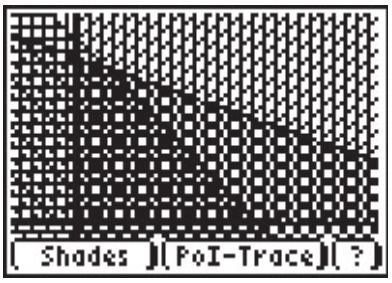
a. Enter the four inequalities in linear function form in the **Y=** editor.

Hint: *To shift from* **Y=** *to* **X=***, highlight* **X=** *in the upper left corner and press* [ENTER].

b. Press [WINDOW] and set the settings to the following:



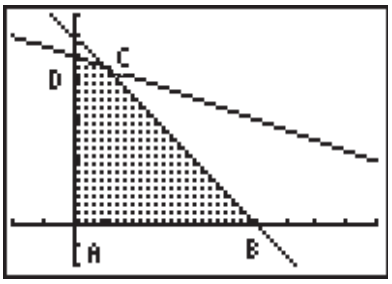
c. Graph the system of inequalities.



5. Find the vertices of the feasible region.

a. Use the **Shading** feature to show the feasible region. (Press [ALPHA] [F1] and select **Ineq Intersection**.)

Now only the intersecting regions are shaded.



b. Use **PoI-Trace**, the Point-of-Intersection Trace feature, to find the coordinates of the vertices. Record the coordinates of the vertices below. The vertex at the origin has been recorded for you.

Hint: *Press* [ALPHA] [F4] *and move the cursor to each vertex using the cursor keys.*

Vertices: (0, 0) _____ _____ _____

6. Substitute the coordinates of each vertex into the objective function to find the maximum value or maximum profit.

a. Complete the table below by substituting the coordinates of each vertex into the objective function. The value at the origin $P(0, 0)$ appears in the table.

Vertex	$P(g, m) = 60g + 75m$
(0, 0)	0

7. Based on these results, how many of each type of player results in the maximum monthly profit for the Games and More Company?

_____

8. What is the maximum possible monthly profit?

_____

Student Worksheet

Name _____

Date _____

Use linear programming to solve each of the following problems. A step-by-step process has been provided to assist you with the first problem.

Problem 1

Keshia is a college student who is majoring in mathematics education. She would like to earn some money to help with expenses. She has two part-time jobs. Keshia earns $8 an hour tutoring at the Academic Support Center on campus, and $10 an hour working at a local restaurant. Although she would like to work more hours at the tutoring center, she likes earning the higher pay at a local restaurant. She can tutor between 2 and 8 hours per week. What is Keshia's maximum earning potential if she works a total of no more than 20 hours per week?

1. Define the variables in this problem.

2. Write an objective function in terms of the variables in step **1** that represents Keshia's earning potential.

3. Identify the constraints.

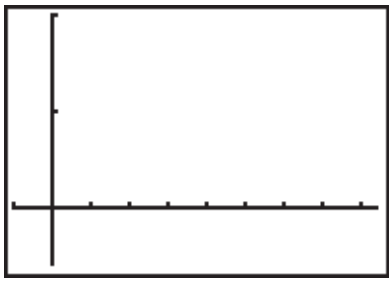
There are four constraints in this problem situation. The first two constraints pertain to the number of hours she wants to tutor. The third constraint relates to the number of hours she can work at a local restaurant and the fourth to the total number of hours she can work during one week.

Write the inequalities for all four constraints.

4. Graph the system.

Sketch its graph on the grid. Label the axes with the appropriate units.

Note: *Adjust the settings in the viewing window* (WINDOW) *so that the graph fits.*



5. Find the vertices of the feasible region.

6. Find Keshia's maximum weekly earnings by substituting the coordinates of the vertices into the objective function.

How many hours should Keshia tutor and how many hours should she work at a local restaurant to maximize her weekly earnings?

Problem 2

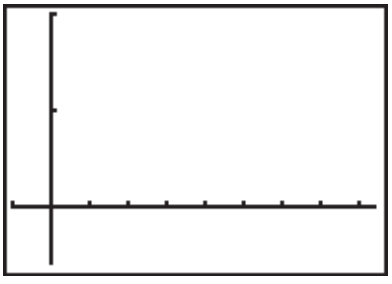
A local sports memorabilia shop is advertising two special trading card packages in an attempt to promote sales. These promotional card packages will showcase an assortment of vintage cards from several professional sports leagues. The shop has purchased 20 hockey cards, 50 basketball cards, and 40 baseball cards to use in the creation of these packs, advertised as the "Rookie" and "All Star" card collections. The Rookie card collection, which is designed to attract new customers, will contain 1 basketball card and 1 baseball card. The All Star card collection, which is designed to appeal to current collectors, contains 1 hockey card, 2 basketball cards, and 1 baseball card, all of which are in mint condition. If the sports memorabilia vendor makes a profit of $5 from each Rookie pack and $15 from each All Star pack, how many of each card pack does he need to create in order to maximize his profit potential?

Define the variables and write an objective function.

Complete the chart to help you identify the constraints and write the system of inequalities for the problem.

	Rookie Card Collection	**All Star Card Collection**	**Total**
Hockey Cards			
Basketball Cards			
Baseball Cards			

Use the grid to sketch a graph of the feasible region of the system of inequalities.



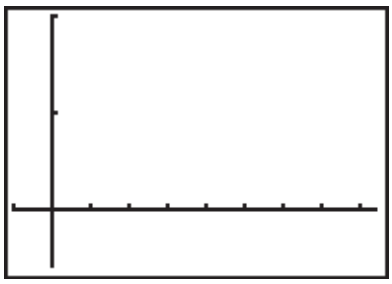
In the preceding problems, you used linear programming to calculate maximum profits. But this approach can also be used to find minimum values as well. Use linear programming to solve the following problem. You might want to set up a table similar to that in Question **2**.

Problem 3

Ms. Wilson needs to increase her daily intake of zinc and calcium. She wants to add a minimum of 10 mg of zinc and a minimum of 150 mg of calcium to her daily diet by taking vitamin supplement tablets. She finds two brands of vitamins, Health-Aid and X-tra Fit, from which to choose. Each Health-Aid tablet contains 3 mg of zinc and 25 mg of calcium. Each X-tra Fit tablet contains 2 mg of zinc and 50 mg of calcium. Single tablets of Health-Aid cost 7 cents, and single tablets of X-tra Fit cost 9 cents. How many tablets of each brand should Ms. Wilson take each day in order to obtain the desired amounts of zinc and calcium as inexpensively as possible?

Define the variables and write an objective function.

Use the grid to sketch a graph of the feasible region of the system of inequalities.



Teacher Notes

Activity 5

Maximizing Your Efforts

Objectives

- Use linear programming to solve problems involving maximum and minimum values
- Use the Inequality Graphing application to solve linear programming problems

Materials

- TI-84 Plus/TI-83 Plus
- Inequality Graphing application

Teaching Time

- 75 minutes

Prerequisite Skills

- Modeling problem situations using systems of linear inequalities
- Solving linear systems in two variables

Management

Many students will benefit from a teacher-led presentation of this material because of the complexity of the linear programming procedure. In addition, this lesson requires thorough knowledge of the features of the Inequality Graphing application. Students who are not that familiar with the features used should be encouraged to consult Appendix B.

Notes about Exploration

This Games and More maximization problem is an introduction to linear programming. The Exploration guides students through a step-by-step procedure. If students are not very familiar with function notation, give them some examples before they start the activity. Write these examples (or others) on the board and have students solve for different values.

$f(x) = 3x + 5$; $x = 10$

$g(x, y) = 3x + 5y$; $x = 10$; $y = 2$

Students often have difficulties writing the inequalities that represent the constraints in a linear programming problem and then calculating the coordinates of the vertices of the resulting feasible region. However, the Inequality Graphing application allows students to focus on writing the inequalities because the **Shading** and **Point-of-Intersection Trace** features make it easy to find the coordinates of the vertices in the feasible regions.

You may want to discuss how to choose the labels for the horizontal and vertical axes in a linear programming problem. Students could be asked if switching the variables affects the outcome. In the Games and More, Co. problem, does it make a difference if the objective function is written as $P(g, m) = 60g + 75m$ or $P(m, g) = 75m + 60g$? Since addition is commutative, it doesn't make a difference.

Answers to the Exploration Questions

2. **a.** $P(g, m) = 60g + 75m$

b. $P(50, 100) = 60(50) + 75(100) = \$10{,}500$

3.

	Gamer Gallery (hours per player)	Major Player (hours per player)	Available Hours
Assembly Time	1.5	1	45,000
Testing Time	0.25	0.5	20,000

a. $1.5g + 1m \leq 45,000$

b. $0.25g + 0.5m \leq 20,000$

c. $g \geq 0;\ m \geq 0$

5. **b.** Vertices: (0, 0), (30000, 0), (0, 40000), (5000, 37500)

6.

Ordered Pair	Value of $P(g, m) = 60g + 75m$
(0, 0)	0
(30,000, 0)	1,800,000
(0, 40,000)	3,000,000
(5,000, 37,500)	3,112,500

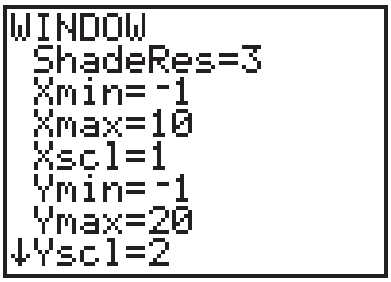
7. 5,000 Gamer Gallery video players and 37,500 Major Player DVD players

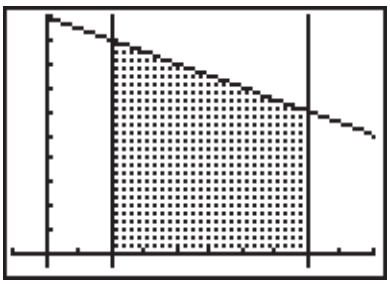
8. \$3,112,500

Answers to Student Worksheet

Problem 1

1. Answers will vary. In the following sample answers, x represents the number of hours that Keshia tutors, and y represents the number of hours that Keshia works at a local restaurant.
2. $P(x, y) = 8x + 10y$
3. $x \geq 2;\ x \leq 8;\ y \geq 0;\ x + y \leq 20$
4. Window settings and graph may vary.





5. (2, 0), (8, 0), (8, 12), (2, 18)
6. $196.00

Problem 2

Objective Function: $P(x, y) = 5x + 15y$

Constraints: $x \geq 0,\ y \geq 0,\ y \leq 20,\ x + 2y \leq 50,\ x + y \leq 40$

Vertices: (0, 0), (40, 0), (0, 20), (30, 10), (10, 20)

The sports memorabilia vendor will make a maximum profit of $350 when 10 "Rookie" card collections and 20 "All Star" card collections are sold.

Problem 3

Objective Function: $P(x, y) = 7x + 9y$

Constraints: $x \geq 0,\ y \geq 0,\ 3x + 2y \geq 10,\ 25x + 50y \geq 150$

Vertices: (0, 5), (6, 0), (2, 2)

Ms. Wilson will obtain the needed amount of zinc and calcium by taking 2 of each supplement. This combination of supplements, which costs 32 cents per day, is the least inexpensive for her.