Irrational Thoughts
Teacher Answers

7   8   9   10   11   12

TI-Nspire CAS   Investigation   Student   45min

Aim

Identify strategies and techniques to express irrational numbers in surd form.

Equipment

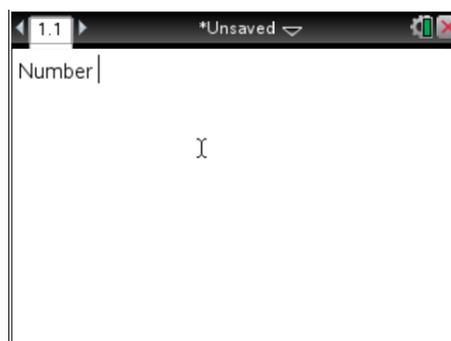For this activity you will need:

- TI-Nspire CAS

Irrational Investigation:

Start a new TI-nspire document and insert a **Notes** application.
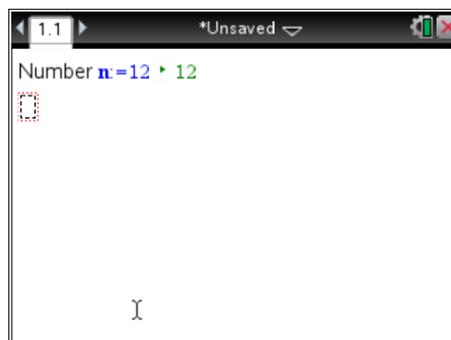
Type the heading:

Number

This text is meant as a label only.

Press **CTRL + M** to insert a maths box next to the text.
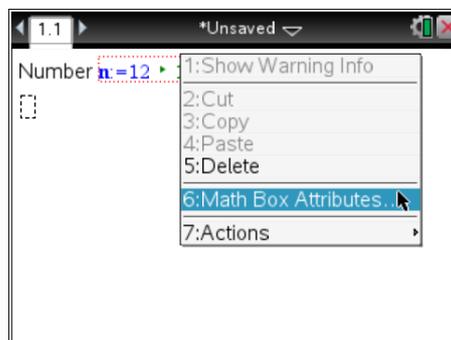
In the maths box type:

n:= 12

Note: The ":=" is located on the keyboard for CX model calculators or can be input using a combination of keys on earlier models or the computer software.

The next task is to hide the output of the maths box. Place the cursor back over the maths box and press **Ctrl + Menu** (Equivalent to a right-mouse click).

Select **Maths Box Attributes...**

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Use the first option to **Hide** the **Output** of this first entry. The output in this case simply confirms that n has been assigned a value.

Press **[Enter]** to confirm the selection, the again to exit the maths box.
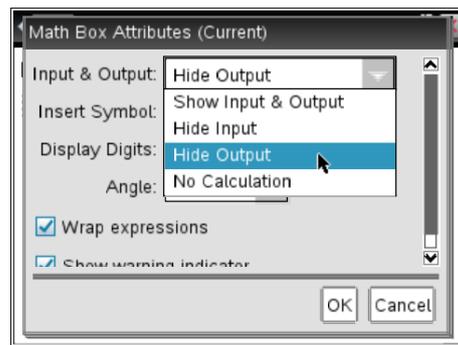
Navigate to the next line. Make sure the cursor is not 'inside' the maths box. Type the following:
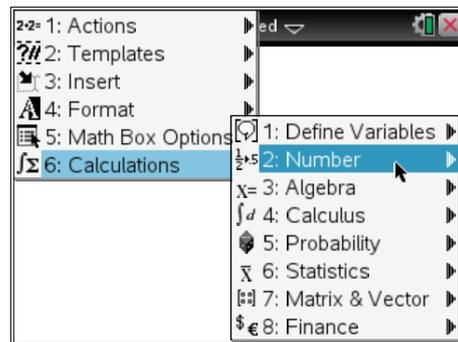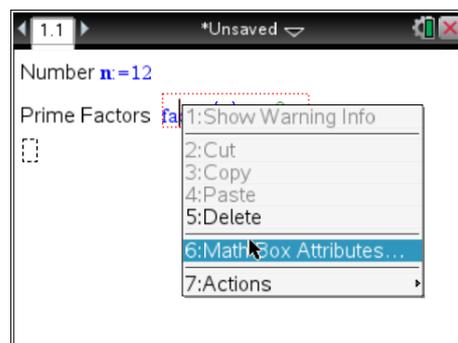
> Prime Factors
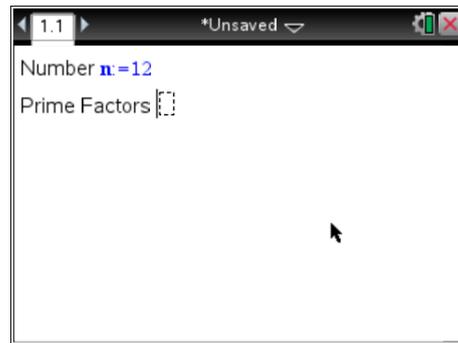
Insert another maths box (Ctrl + M) and press the **[Menu]** key and select **Calculations**, followed by **Number** then **Factor**.

Factor the number stored in n by typing n in the factor command.

This command will express the number currently stored in n as a product of its prime factors.

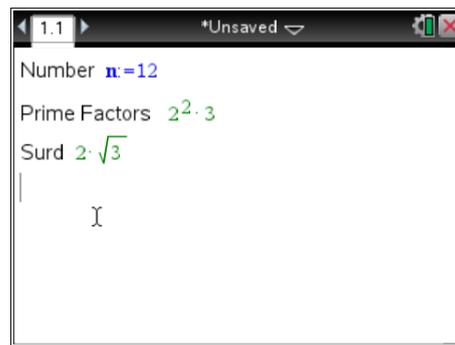Change the attributes of this maths box to **hide** the **input**.

Navigate to the next line and type:

    Surd

Add a maths box and use it to calculate the square root of *n*.

$$\sqrt{n}$$

Select the maths box and use the attributes to once again hide the input.


If the answer for surd form is not displayed as shown above, the document settings will need to be adjusted. The best way to achieve this is to set the Calculation Mode to "Auto" or "Exact".

Press the [**Home**] key and select **Settings** followed by **Document Settings**.

Use the [**Tab**] key to navigate to **Calculation Mode** and select **Auto** followed by **OK.**

Once the calculation mode has been set, return to the current document.
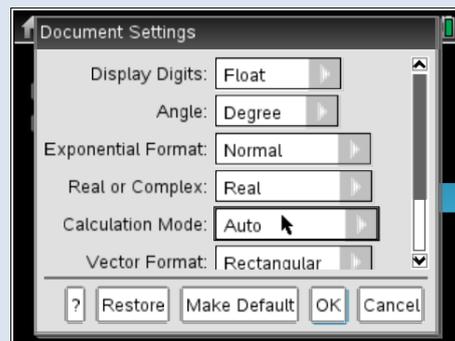
The prime factorisation of 12 is: $12 = 2 \times 2 \times 3$

                                              $12 = 2^2 \times 3$

This is displayed on the calculator: $12 = 2^2 \cdot 3$

The surd form is displayed on the calculator as: $12 = 2\sqrt{3}$

1. The value for *n* can be changed by editing the value in the maths box. Change the value for *n* for each of the following:

	Prime Factorisation	Surd Representation
a)	$45 = \ 3^2 \times 5$	$\sqrt{45} = \ 3\sqrt{5}$
b)	$28 = \ 2^2 \times 7$	$\sqrt{28} = \ 2\sqrt{7}$
c)	$175 = \ 5^2 \times 7$	$\sqrt{175} = \ 5\sqrt{7}$
d)	$539 = \ 7^2 \times 11$	$\sqrt{539} = \ 7\sqrt{11}$

2. Explain how the surd representation (square root) relates to the prime factorisation for each number in question 1.

Square root is the inverse operation to squaring; it follows that squared factors will have an integer value for the square root. Prime factors that are not perfect squares result in an irrational result when square rooted, therefore when expressed as a surd, remain within the square root sign. Students should refer in some way to the square root 'undoing' (inverse) the squared factor.

3. Change the value for *n* for each of the following:

|	Prime Factorisation		Surd Representation
a)	$50 = 5^2 \times 2$		$\sqrt{50} = 5\sqrt{2}$
b)	$147 = 7^2 \times 3$		$\sqrt{147} = 7\sqrt{3}$
c)	$242 = 11^2 \times 2$		$\sqrt{242} = 11\sqrt{2}$
d)	$507 = 13^2 \times 3$		$\sqrt{507} = 13\sqrt{3}$

4.   Complete the following statement:

"If $n = a^2 b$ and *a* and *b* are both positive prime numbers then $\sqrt{n} = a\sqrt{b}$ "

Note: In the absence of domain restrictions for *a* the calculator returns: $|a|\sqrt{b}$. While this is a more accurate answer students are unlikely to have considered negative values of *a*, nor are they likely to have used the absolute value notation previously. Students writing their answer in this format have most likely taken their answer directly from the calculator rather than through thoughtful reflection of previous questions and answers.

The CAS calculator can work with general rules such as the one above. Insert a calculator application by pressing **Ctrl + I** and select **calculator**.
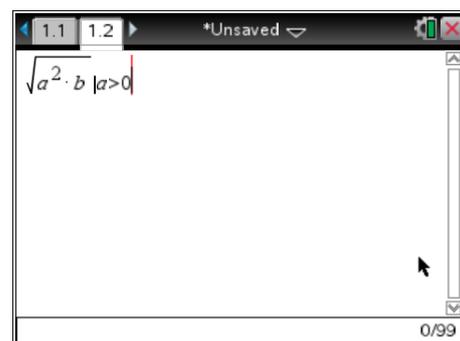

Type the following:

$\sqrt{a^2 b}$
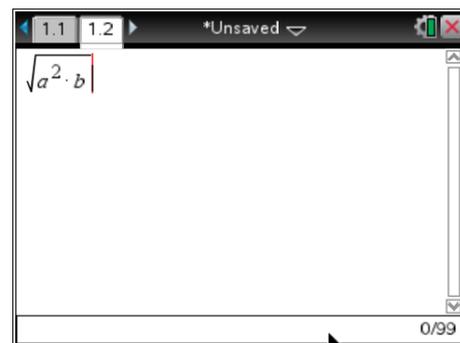
5. Write down the answer produced by the calculator for $\sqrt{a^2 b}$ Answer: $|a|\sqrt{b}$

The CAS calculator treats variables as 'any number'. The value of a number can be restricted using " | ". This symbol can be read as 'given that'.

Repeat the above question using the restriction that '*a*' is greater than zero.

$\sqrt{a^2 b}\ |a > 0$

6. Write down the answer produced by the calculator for $\sqrt{a^2 b}\ |a > 0$ Answer: $a\sqrt{b}$

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Not all numbers can be written in the form $n = a^2b$ the next set of numbers explore other forms of a natural number.

7. Change the value of *n* for each of the following:

	Prime Factorisation	Surd Representation
a)	$24 = 2^3 \times 3$	$\sqrt{24} = 2\sqrt{6}$
b)	$135 = 3^3 \times 5$	$\sqrt{135} = 3\sqrt{15}$
c)	$56 = 2^3 \times 7$	$\sqrt{56} = 2\sqrt{14}$
d)	$875 = 5^3 \times 7$	$\sqrt{875} = 5\sqrt{35}$

8. **Based on your results from questions** 7a) to 7d), complete the following statement:
   "If $n = a^3b$ and *a* and *b* are both positive prime numbers then $\sqrt{n} = a\sqrt{a \cdot b}$ "
   Note: In the absence of domain restrictions for *a* the calculator returns: $\sqrt{a^3b}$. The possibility of a negative value for *a* makes this answer simpler to read than: $|a|\sqrt{a \cdot b}$. With the domain restrictions the calculator produces the answer: $a^{\frac{3}{2}}\sqrt{b}$, understandable from an algebraic perspective but not particulalry favourable as general expression applicable to numerical results.

9. Change the value of *n* for each of the following:

	Prime Factorisation	Surd Representation
a)	$31 = 31$ (Prime number)	$\sqrt{31} = \sqrt{31}$
b)	$105 = 3 \times 5 \times 7$	$\sqrt{105} = \sqrt{107}$
c)	$273 = 3 \times 7 \times 13$	$\sqrt{273} = \sqrt{273}$
d)	$154 = 2 \times 7 \times 11$	$\sqrt{154} = \sqrt{154}$

10. Explain the result for questions 9a) to 9d).
    There are no repeated (or squared) prime factors, therefore there is no simplication for the surd expression.

11. Change the value of *n* for each of the following:

	Prime Factorisation	Surd Representation
a)	$200 = 2^3 \times 5^2$	$\sqrt{200} = 10\sqrt{2}$
b)	$252 = 2^2 \times 3^2 \times 7$	$\sqrt{252} = 6\sqrt{7}$
c)	$800 = 2^5 \times 5^2$	$\sqrt{800} = 20\sqrt{2}$
d)	$1512 = 2^3 \times 3^3 \times 7$	$\sqrt{1512} = 6\sqrt{42}$

12. Explain what happens when a prime factor has a power (or index) greater than 2.

Example: $8 = 2^3$. How is the surd expressed and why?

Since $2^3 = 2^2$ x 2, the squared term can be extracted from the square root.

Large powers can be reduced: $2^5 = 2^2$ x 2^2 x 2 so $\sqrt{32} = 4\sqrt{2}$