Is It Rare?

ID: 9094

Activity Overview

In this activity, students use the Poisson distribution to determine the probabilities for various numbers of hurricanes hitting the United States in a given year.

Topic: Poisson Distributions

- *For cases involving n Bernoulli trials, determine when the Poisson distribution function is appropriate and use it to estimate the probability of exactly k successes.*
- *Derive the formula for the Poisson distribution function as the limiting case of the binomial distribution function when the number of trials is large and the mean value of the binomial distribution $\leq$ 10.*

Teacher Preparation and Notes

- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*

- *The Poisson distribution is most commonly used to model the number of random occurrences of some phenomenon in a specified unit of space or time.*

- *The Poisson distribution is defined by the equation $P(m, x) = \dfrac{m^x e^{-m}}{x!}$ where x represents the number of occurrences of the event, m is the average number of occurrences in the specified interval, and e is the base of the natural logarithm (approximately 2.718).*

- *Hurricane data can be found on the National Weather Service website, www.nhc.noaa.gov/pastall.shtml.*

- ***To download the student worksheet, go to education.ti.com/exchange and enter "9094" in the keyword search box.***

Associated Materials

- *IsItRare_Student.doc*

Hurricanes

Students are introduced to the formula used when there is a Poisson distribution. Make sure that students understand that *m* is the mean or average of the data set.

$$P(m, x) = \frac{m^x e^{-m}}{x!}, \quad x=0,1,2,3...$$

First students will calculate the probability of 11 hurricanes hitting the US.

Then they will use the same formula to find the probabilities of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 hurricanes hitting the US. Make sure that students record the probabilities, to at least 2 significant digits, on their worksheet or paper.
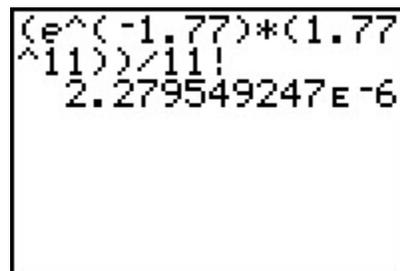
If using Mathprint OS:

Students can enter the formula as a fraction. To do this, press [ALPHA] [F1] and select n/d. Enter the numerator, $e^{-1.77}*1.77^{11}$. Press [⯆] to move to the denominator. Then enter 11!.

Note: Parentheses are not needed around the numerator or the denominator. Also when students press [⌃], the cursor will move to the exponent position. They should enter the value of the exponent and then press [▶] to move out of the exponent position.
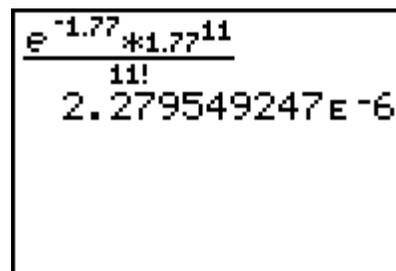
Students are to enter the probabilities they found into the List Editor in **L2**. **L1** is the number of hurricanes.

They will then graph the probabilities as a connected line plot. Pressing [ZOOM] and selecting **ZoomStat** will set the window appropriately.

Students should see that the graph is skewed to the right. This means that the greatest probabilities occur when the number of hurricanes is low.

The points form a line, so this data is linear. Use the **LinReg** command to draw a line through these points. (Adding **Y1** to the end of the command stores the equation of the line in **Y1**.)

```
LinReg
 y=ax+b
 a=-.0260839342
 b=.2267949389
```
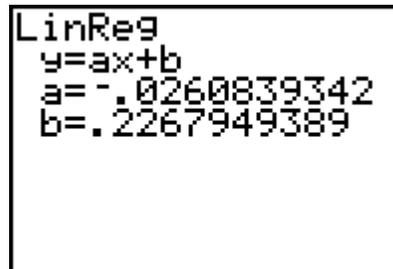
If using Mathprint OS:

Students can enter Y1 using the shortcut menu. To do this, press $\boxed{\text{ALPHA}}$ $\boxed{\text{F4}}$ and select **Y1**.

```
◄g(ax+b) L₁,L₂,
          ┌──────┐
          │Y1 Y6 │
          │Y2 Y7 │
          │Y3 Y8 │
          │Y4 Y9 │
          │Y5 Y0 │
 ┌────┬────┬────┬────┐
 │FRAC│FUNC│MTRX│YVAR│
 └────┴────┴────┴────┘
```
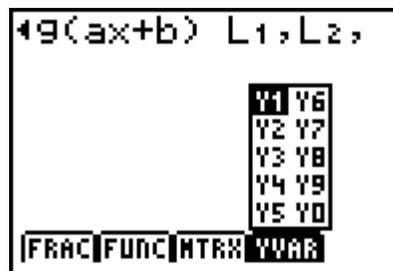
Cumulative Probabilities

Students are introduced to cumulative probabilities and how to calculate them using the complement of an event. Make sure that they understand the information before completing the problems in Question 4.

For any sample space, the sum of the probabilities of the outcomes is 1. So, for a given value of m,

$$P(0) + P(1) + P(2) + ... = 1$$

From this, you can use the **complement** of an event to help you find a probability. For example, given m, to find the probability that an event will happen more than 2 times ($x > 2$), subtract the probability that the event will happen 2 or fewer times ($x \le 2$) from 1.

$P(\text{2 or fewer times}) + P(\text{more than 2 times}) = 1$

$\qquad\qquad P(\text{more than 2 times}) = 1 - P(\text{2 or fewer times})$

$\qquad\qquad P(\text{more than 2 times}) = 1 - P(\text{0 times}) - P(\text{1 time}) - P(\text{2 times})$

$\qquad\qquad P(\text{more than 2 times}) = 1 - \left[P(\text{0 times}) + P(\text{1 time}) + P(\text{2 times})\right]$

Student activity solutions

1. P(1.77, 11) = 0.0000023   OR   6.28 x 10^{-6}

2. P(1.77, 0) = 0.1703, P(1.77, 1) = 0.3015
    P(1.77, 2) = 0.2668, P(1.77, 3) = 0.1574
    P(1.77, 4) = 0.0697, P(1.77, 5) = 0.0247
    P(1.77, 6) = 0.0073, P(1.77, 7) = 0.0018
    P(1.77, 8) = 0.00041, P(1.77, 9) = 0.00008
    P(1.77,10) = 0.000014, P(1.77, 11) = 0.0000023

3. One hurricane per year has the greatest probability of occurring because the average
    number of hurricanes per year is 1.77.

4   **a.** 0.738
    b. 0.261
    c. they are complementary

Exercise solutions

1. P(9, 0) = 0.0001, P(9, 1) = 0.001, P(9, 2) = 0.005 , P(9, 3) = 0.015, P(9, 4) = 0.034,
    P(9, 5) = 0.061, P(9, 6) = 0.091, P(9, 7) = 0.117, P(9, 8) = 0.132, P(9, 9) = 0.132 ,
    P(9, 10) = 0.119

2. 0.116

3. 0.884