# *NUMB3RS* Activity: Birthday Surprise
# Episode: “Traffic”

**Topic:** Probability using complementary events **Grade Level:** 9 - 12
**Objective:** To use the complement of an event to find probabilities with respect to the “birthday problem” and similar applications.
**Materials:** TI-83 Plus/TI-84 Plus graphing calculator
**Time:** 15 - 20 minutes

## Introduction

As Charlie teaches his class about randomness and probability, he refers to the famous “birthday problem.” This problem often takes the form of a bet among guests at a party that at least two of them have the same birthday. It can come as a surprise that only 23 people are needed to make it an even bet (0.5 probability). With 60 people, it is a virtual certainty that at least two people will have the same birthday. This activity enables students to understand the concept by working with the probability that there is *no match* and then using the concept of the *complement* of an event to quickly compute the probability of a match.

## Discuss with Students

Review the meaning of probability and how it is expressed. Specifically, review that the probability of a number of independent events is found by taking the product of the individual probabilities of each event.

Throughout the activity, assume that a year is 365 days, and assume that birthdays are equally distributed among the days.

If time allows, a good approach to this lesson is to ask the students how many people they think would be necessary to have in the room to have a probability of at least 0.5. A common answer is 183 (why?). After they have recorded their opinions, do an experiment with the class. If the class is very large, the sample might be large enough to have a match. In the case of a smaller class, have the students write their birthdays as well as those of their parents on a small piece of paper. Collect these and put them on a spreadsheet, the board, etc. (This adds to the dramatic effect and protects the anonymity of any student who may not know the birthday of a parent or who does not want his or her birthday made public.)

Some of the other examples in this activity can also be used in this way. For serial numbers of dollar bills, get each bill from a different student in order to make sure they are not new bills in sequential order (that would negate the randomness requirement).

Note that while some students may find it beneficial to actually key in the BIRTHDAY program on their calculators, the program file can also be downloaded for free by going to **http://education.ti.com/exchange** and searching for “7463.”



Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ

**Student Page Answers:**

**1.**

| **Number of Cars** | ***P*(no match)** | ***P*(match)** |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 0.9 | 0.1 |
| 3 | 0.72 | 0.28 |
| 4 | 0.504 | 0.496 |
| 5 | 0.3024 | 0.6976 |
| 6 | 0.1512 | 0.8488 |
| 7 | 0.06048 | 0.93952 |
| 8 | 0.018144 | 0.981856 |
| 9 | 0.0036288 | 0.9963712 |
| 10 | 0.00036288 | 0.99963712 |

**2.** *5 cars* **3.** *5 people (0.61)* **4.** *8 people (0.95)* **5.** *13 bills (0.55)* **6.** *22 bills (0.91)*
**7.** *23 people (0.507)* **8.** *41 people (0.903)*



Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ

 

Name: ___________________________________ Date: ________________

# *NUMB3RS* Activity: Birthday Surprise

As Charlie teaches his class about randomness, he refers to the famous "birthday problem." This problem frequently takes the form of a bet among guests at a party that at least two of them have the same birthday. How many people should be at the party in order for this to be an even bet (just as likely to have a match or not have one)? Before continuing, make a guess at the number.

Most people overestimate how many are necessary. It is not easy to compute directly, but is much less difficult if it is calculated using the *complement* of a match. That is, first find the probability of there **not** being a match (which is the complement of having a match). Let $P(\sim M)$ represent the probability of not having a match. Because the sum of the probability of a match $P(M)$ and not having a match $P(\sim M)$ is 1, $P(M) = 1 - P(\sim M)$.

If there is one person in the room, clearly there is no chance of two matching. But if a second person is in the room, there will be no match if the second person was born on any of the 364 days other than that when the first person was born. The probability of no match is $\frac{364}{365}$, so the probability of a match is $1 - \frac{364}{365} = \frac{1}{365}$. If there are three people, the third will not match as long as the third birthday does not match either of the first two people. Thus there are 363 "favorable" days, so to find the probability of no matches, multiply the previous probability by $\frac{363}{365}$; the probability of no match for three people is $\left(\frac{364}{365}\right)\left(\frac{363}{365}\right) \approx$ 0.99 and the probability of a match is about 0.01. Because these fractions are cumbersome, the following problems use smaller numbers in order to make it easier to understand the principle before coming back to the actual birthday problem.

**1.** Suppose the license plates in a certain state are randomly created and all end in a digit from 0 to 9. Complete the following table to find the probability that certain numbers of cars have license plates that end with the same digit.

| **Number of Cars** | ***P*(no match)** | ***P*(match)** |
|---|---|---|
| 1 | 1 | 1 – 1 = 0 |
| 2 | 0.9 | 1 – 0.9 = 0.1 |
| 3 | (0.9)(0.8) = 0.72 | 1 – 0.72 = ? |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

**2.** From the table, what is the fewest number of cars (called the *sample size*) necessary for the probability of a match to be greater than 0.5?

 
Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ

**3.** Instead of birthdays, consider the simpler problem of birth months. Assume that a person is equally likely to be born in each month (which in reality is not exactly accurate for many reasons including the fact that some months have more days than others). Using the same reasoning, what is the smallest sample of people necessary so that the probability of two people having the same birth month is greater than 0.5?

**4.** What is the fewest number of people needed so that the probability of a match is greater than 0.9?

The following program can be used to simulate these types of problems on your graphing calculator.

```
PROGRAM: BIRTHDAY
:ClrHome
:Input "TOTAL POSSIBLE?",T
:1→A
:1→N
:Lbl 1
:ClrHome
:While N≠T
:A∗(T+1−N)/T→A
:Disp "NUMBER",N,"PROB NO MATCH",A,"PROB MATCH",1−A
:N+1→N
:Pause
:Goto 1
:End
```

When you run this program, it will ask for the total possible number of events (e.g., 365 for birthdays, 12 for birth months, 10 for license plate digits, etc.). Each time you press [ENTER], the number in the sample increases by one and the probability of no match and the probability of a match are displayed. This program will not only make it easier to answer the next four questions, but will also demonstrate how the probability of a match starts small and then rapidly increases as the sample size grows.

**5.** Use the same reasoning to determine the fewest number of random dollar bills so that there is at least a 0.5 probability that the final two digits of the serial numbers on at least two of the bills match (note that there are 100 possible – from 00 to 99).

**6.** In the previous problem, determine the fewest number of random dollar bills so that there is at least a 0.9 probability that the final two digits of the serial numbers on at least two of the bills match.

**7.** Now return to the original problem that Charlie used in the classroom. What is the minimum number of people needed in the room so that the probability of two people having the same birthday is at least 0.5?

**8.** What is the minimum number of people needed in the room so that the probability of two people having the same birthday is at least 0.9?

 
Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ

***The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.***

# Extensions

## Introduction

As Charlie notes in his class, people make many errors and poor choices because they do not understand the mathematics of probability. Besides teaching a technique, problems like this should make students cautious when presented with propositions that seem "obvious." For example, since 42 people have held the office of president of the United States, should a person be surprised if two shared a birthday? Even though there are fewer than 42 presidents who have died, would it be likely that two have died on the same date? In general, events that termed "coincidences" must be checked mathematically. For more information, see the Web site below.
**http://www.cut-the-knot.org/do_you_know/coincidence.shtml**

## Related Topics

- There are many other topics (particularly in the fields of combinatorics and probability) in which it is easier to calculate the complement and then subtract to find the desired answer. For example, suppose someone deals two cards from an ordinary deck. What is the probability of getting a pair (both cards have the same rank, e.g., queens, 7's etc.)? Computing directly, there are 13 different ranks and the number of pairs of each is ${}_4C_2 = 6$, so there would be 13(6) = 78 favorable and ${}_{52}C_2 = 1{,}326$ pairs. Thus the probability is $\frac{78}{1{,}326} \approx 0.0588$. By using the complement, you only need to consider that the second card is NOT a match, namely 48 out of the 51 remaining cards. Thus the probability is $1 - \frac{48}{51} \approx 0.0588$. Of course, this is a simple case. As the number of cards increases, so does the complexity of the mathematics. Try applying this technique to any card game with which you are familiar.
- The birthday problem can be extended. For example, it can consider the probability of having *n* matches instead of just 2. For a more extensive (and advanced) mathematical treatment of this problem, see **http://mathworld.wolfram.com/BirthdayProblem.html**.

## Additional Resources

- There is an interesting picture using beans in boxes that can help the visualization of the concepts of this problem in the Life Science Library book <u>Mathematics</u> by David Bergamini (Time-Life Books, New York, 1972, pp. 142–3).
- For an applet that simulates the birthday problem and includes a cumulative graph of trials of different sizes, go to **http://www.mste.uiuc.edu/reese/birthday**.
- For an entire lesson starting with the history of the birthday problem, go to **http://www.teacherlink.org/content/math/interactive/probability/lessonplans/birthday/home.html**.

 
Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ