NUMB3RS Activity: Hide and Seep
Episode: "Waste Not"

Topic: Visual mathematical modeling **Grade Level:** 9 - 12
Objective: Students create a visual model of liquid percolating through the soil with a cellular automata model, using probability and iteration
Time: 15 - 25 minutes
Materials: A calculator with random number generator

Introduction

In "Waste Not," an elementary school playground collapses into a sinkhole. Charlie and Larry work to figure out what caused the sinkhole – they suspect underground water flow and work to verify their hypothesis. Though the actual mathematics is not discussed, the maps around the office, as well as the equations displayed, come from the mathematical modeling of the seepage (percolation) of water through the ground. This activity employs one way to create a visual model of this process, namely a cellular automata model. The objective of this activity is for students to explore the use of a simple iterative process to create extremely complex results. There is no "punch line," except the appreciation of modeling as a powerful mathematical tool.

Discuss with Students

According to Wolfram's MathWorld, "A cellular automaton is a collection of 'colored' cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells. The rules are then applied iteratively for as many time steps as desired" (Weisstein, E. W., *Cellular Automaton.* [**http://mathworld.wolfram.com/CellularAutomaton.html**, 2006])

This activity uses a square grid rotated 45°, so that each row of cells (now shaped like "diamonds") represents spaces between soil particles and the intersections of the lines represent the particles themselves. The grid is a cross-section of the earth with the top row as the surface. Students should color about a third of the cells in the center of the top row to represent the initial "puddle" of water that will seep through the soil.

In order to simulate the seepage, an assigned probability is used to determine whether the water in a cell will seep into its neighbors in the next row. Because a "wet" cell has two neighbors, it is necessary to check each one individually, but always using the same probability. Conceptually, if the probability of seepage is too low, the water will not seep at all (representing very dense soil). On the other hand, if the probability is high enough, the water will flow freely. In between, there is a narrow range of probabilities that create a phenomenon called "fingering," which produces the highly complex patterns of actual seepage. The meaning of the term "fingering" becomes obvious when students generate their models.

Assign different probabilities to individual students in the range 0.55 – 0.65, in increments of 0.01. Instruct students to use the calculator to generate random numbers in the range 0 – 1. Use a random number for each wet cell in a row. Each time the random number is less than or equal to the assigned probability, water will seep from a wet cell into its neighbor in the next row.

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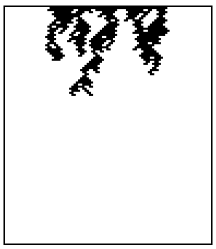
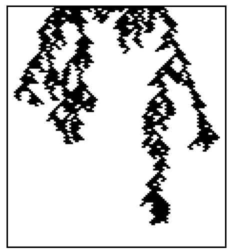
Student Page Answers:

Here are samples of finished models, made by the applet listed in “Extensions.” Note that the only difference is that the cells are in a “brick” pattern, instead of rotated squares. The overall images are consistent with this activity, and support the sample answer for question #1.

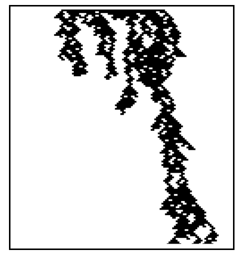
0.55



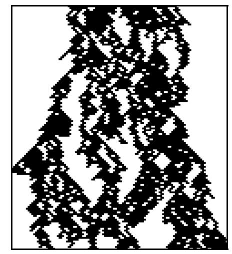
0.58



0.61



0.65



1. *Answers may vary. Generally, as the probability increases the seepage goes deeper. The seepage becomes denser, with thicker fingering.* **2.** *As indicated on this teacher page, smaller probabilities cause the seepage to decrease and ultimately stop, often quite quickly. With probabilities only slightly larger than 0.65, the water flows freely, with the fingering replaced by a solid flow.* **3.** *Certain areas can be “blocked out” for differing densities, obstacles, and air pockets. For denser soil, the probability can be lowered slightly. Similarly, a slightly larger probability can be used for less dense soil. Rock has a probability of zero, and air pockets a probability of one.*

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Name: ________________________________ Date: _______________

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This activity uses a square grid, rotated 45° so that each cell (now shaped like a "diamond") represents a space between soil particles, and the intersections of the lines represent the particles themselves. The grid is a cross-section of the earth with the top row as the surface.

Procedure

A. Set your calculator to create random numbers between 0 and 1. To do this, press [MATH], go to the **PROB** menu, and select **1:rand**. Then press [ENTER]. (As an option, you may want to set the [MODE] to show numbers with only two decimal places.)

B. In the center of the top row of cells, color in about a third of them. This represents the initial "puddle" of water that will seep through the soil.

C. Your teacher will assign you a probability between 0.55 and 0.65. Use this probability to determine if water seeps from the cells in one row to their neighbors in the next row. Record this probability in the space provided with the model on the next page.

D. Working from left to right for each "wet" cell in a row:
  1. Generate a random number.
  2. If the random number is *less than or equal to* your assigned probability, color in the left-hand neighbor in the next row.
  3. Repeat steps 1 and 2 for the right-hand neighbor.
  4. Repeat steps 1–3 for each wet cell in the row. (Note: if a wet cell seeps into its right-hand neighbor, there is no need to test that neighbor again if it becomes the left-hand neighbor of the next wet cell.)

E. Repeat step D for the new row of wet cells, and continue for each row until:
  - No new wet cells are generated and the seepage stops, or
  - You run out of rows.

When you and your neighboring classmates have finished, compare your models.

1. Describe how different probabilities affect the size and structure of the model.

2. What would happen if you used probabilities smaller than 0.55? Larger than 0.65?

3. How could you modify the grid and the procedure to account for soil that has different densities at different levels, obstacles such as rocks or air pockets?

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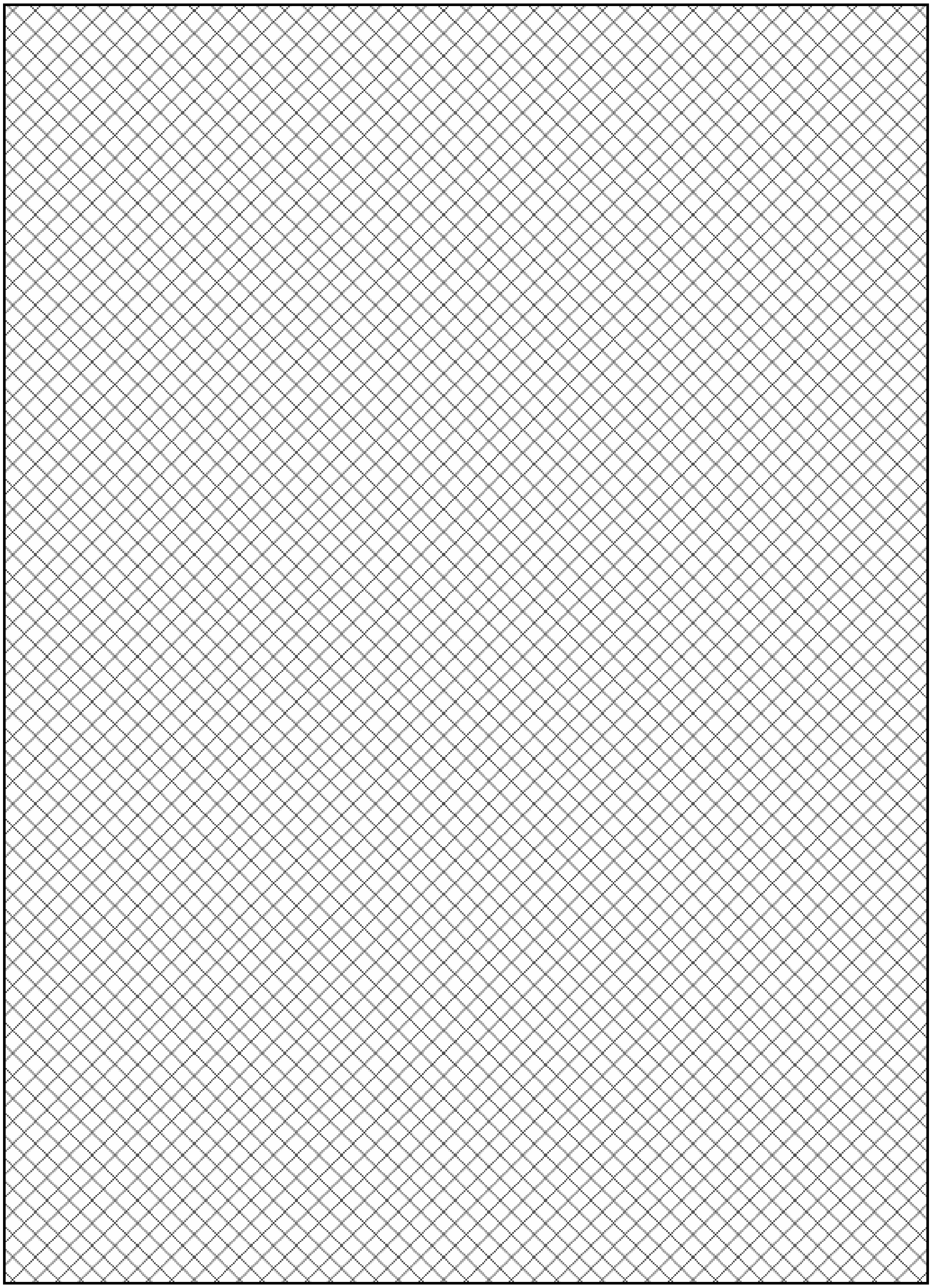
Use this grid to generate your seepage model:



Probability used: ________

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The goal of this activity is to give students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage teachers and students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Cellular automata (CA) models have many uses. This particular activity uses a one-dimensional model, since the cells are studied one row at a time. Many CA models are two-dimensional, and are used to study many topics, from urban development to artificial life.

For the Student

One very well studied two-dimensional CA is the "Game of Life", invented by Princeton mathematician John H. Conway in 1970. The complexity of its patterns generated by simple rules has inspired literally hundreds of research papers. For a very student-friendly article, that includes several examples and applets to try, see:
http://www.math.com/students/wonders/life/life.html

Related Topic

The CA model in this activity can be used to model many types of percolation and seepage. One important application is the study of seepage of dangerous materials such as oil or toxic waste, as in "Waste Not." In the range of probabilities that produce fingering, the irregularity of the underground pattern can be quantified by a measure called "fractal complexity". For a more detailed investigation of the fractal properties of percolation patterns, see *Mathematics Teacher* magazine (published by NCTM), Volume 91 #8 (November 1998) and Volume 92 #2 (February 1999). The activity is entitled "Forest Fires, Oil Spills, and Fractal Geometry: An Investigation in Two Parts". Part 1 focuses on CA models like the one in this activity. Part 2 explores the metric called fractal complexity. Part 2 is available for purchase, or for download by NCTM members, at:
http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MT1999-02-128a

In addition, this two-part activity also uses a two dimensional CA model for the spread of fire, featured in the *NUMB3RS* Season 2 activity called "Making Fireprints", Episode 211, "Scorched." To download this activity, go to **http://education.ti.com/exchange** and search for "6306."

Additional Resources

The author of this activity has a Java applet for the seepage (percolation) model. The only difference is that the cells are in a "brick" pattern, instead of rotated squares. See:
http://dimacs.rutgers.edu/~cbiehl/oil.html

For a gallery of interesting and colorful images generated by a wide variety of one- and two-dimensional CA models, see Mirek Wojtowicz's collection at:
http://psoup.math.wisc.edu/mcell/ca_gallery.html

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