

Activity 1

Objectives

- Develop an intuitive understanding of the limiting process
- Estimate limits from graphs and tables of values

Materials

- TI-84 Plus / TI-83 Plus

Approaching Limits

Introduction

The idea of a limit is one of the fundamental concepts covered in calculus courses. In this activity, you will investigate, both graphically and numerically, the limit of a function at a point.

The limit describes the behavior of a function *near* a point. It represents how the output value would behave as an input value gets very close to a value of interest. Although it may, in fact, reach that value, the concern is more for what happens as the input value is getting nearer and nearer the value being examined. In some cases, the output value may not get near any value at all.

Exploration

Take a look at the numerical behavior of the function $f(x) = \frac{x^2 - 9}{x - 3}$, and consider the function value as the value of x gets close to 3.

1. Store the function in **Y1** in the **Y=** editor, and set up the table as shown.

```
TABLE SETUP
TblStart=0
ΔTbl=1
Indent:  [Auto] Ask
Defend:  [Auto] Ask
```

2. Display the table, and record the function values when $x = \{2, 3, 4\}$.
3. Were the function values what you were expecting? Explain why or why not.

4. Change the table setup to begin at 2, and make the table increment by 0.5.

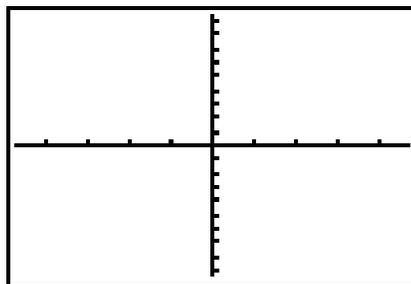
```
TABLE SETUP
TblStart=2
ΔTbl=.5
Indent:  Ask
Defend:  Ask
```

5. What values do you get for $x = \{2.5, 3, 3.5\}$?
6. Repeat the process, having the table begin at 2.5 and making the table increment by 0.25. Record the function values for $x = \{2.75, 3, 3.25\}$.
7. Repeat the process, having the table begin at 2.9 and making the table increment by 0.1. Record the function values for $x = \{2.9, 3, 3.1\}$.
8. Repeat the process one more time. This time start the table at 2.99, and make the table increment by 0.01. Record the function values for $x = \{2.99, 3, 3.01\}$.
9. What conclusion can you draw at this point? (You may want to repeat the process with smaller and smaller increments until you feel comfortable arriving at a conclusion.)

Now examine the graph of the function to see the behavior.

10. With the function still stored in **Y1**, set the viewing window as shown, and sketch the graph.

```
WINDOW
Xmin=-4.7
Xmax=4.7
Xscl=1
Ymin=-9.3
Ymax=9.3
Yscl=1
Xres=1
```

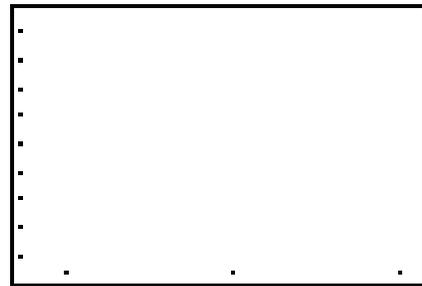


11. Select **4:SetFactors** from the **ZOOM MEMORY Menu**, and set your factors as shown.

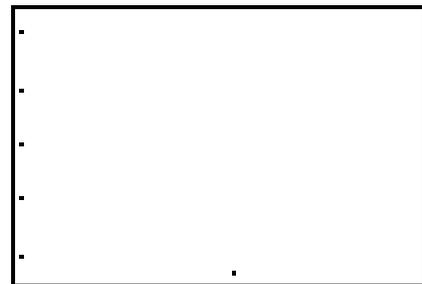
```
ZOOM [MODE]
1:ZPrevious
2:ZoomSto
3:ZoomRcl
4:SetFactors...
```

```
ZOOM FACTORS
XFact=4
YFact=2
```

12. Graph the function, and **TRACE** to the point (2.9, 5.9). Select **2:Zoom In** from the **ZOOM Menu**, and press **ENTER**. On the axes, sketch what you see and describe it.



13. **TRACE** to the point (2.975, 5.975), and repeat the zoom process from Question 12. Record and describe what you see.



14. **TRACE** toward the value $x = 3$ from values above 3 and from values below 3. Get as close as you can. You can zoom in more if necessary. What can you conclude?
15. If values come closer and closer to a particular output value, the value that the function is *approaching* is called the *limit*. The notation is shown below.

$$\lim_{x \rightarrow \text{some value}} f(x) = \text{LIMIT}, \text{ OR } \lim_{x \rightarrow c} f(x) = L$$

Express, in limit notation, the results of Questions 1–14.

16. Use

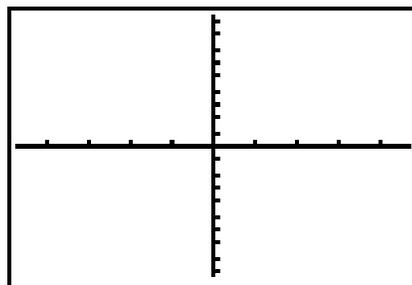
$$g(x) = \frac{x^2 - 9}{x - 2}$$

and use the table setup from Question 1. Record the values for $x = \{1, 2, 3\}$.

17. Change the increments as before: first to 0.5, then to 0.25, then to 0.1, and then to 0.01. Record the values directly above and below 2 in each case.

18. What conclusion, if any, can you draw about this function as x gets closer and closer to a value of 2?

19. Using the same viewing window as in Question 10, display the graph. Record and describe what you see.



20. Try zooming in to view the function. What can you conclude?

21. How do the graphical results compare to the numerical exploration on this function?

22. If the input approaches a single value and the output does NOT approach a single value, the function is said to have no limit. Using the notation from Question 15, express the result to the limit of $g(x)$ as x approaches a value of 2.

Find the limit, if it exists, for each of the following. If it does not exist, explain why.

23. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

24. $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$

25. $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

26. $\lim_{x \rightarrow 1} \frac{x^2 - 2}{x - 1}$

27. $\lim_{x \rightarrow 2} \frac{\sqrt{5x^2 - 15}}{x^3}$