7-1 Areas of Parallelograms and Triangles

Lesson Preview

What You'll Learn

OBJECTIVE 1 To find the area of a parallelogram

OBJECTIVE 2 To find the area of a triangle

. . . And Why

To find the force of wind against the side of a building, as in Example 5

Check Skills You'll Need

(For help, go to Lesson 1-7.)

Find the area of each figure. 1. 25 cm^2 2. 28 $in.^2$ 4. $\frac{3}{2}$ ft^2

1. a square with 5-cm sides

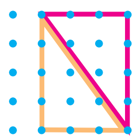
2. a rectangle with base 4 in. and height 7 in.

3. a 4.6 m-by-2.5 m rectangle 11.5 m^2

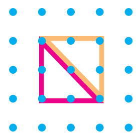
4. a rectangle with length 3 ft and width $\frac{1}{2}$ ft

Each rectangle is divided into two congruent triangles. Find the area of each triangle.

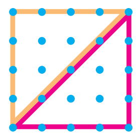
5. 6 $units^2$



6. 2 $units^2$



7. 8 $units^2$



New Vocabulary

- base of a parallelogram
- altitude of a parallelogram
- height of a parallelogram
- base of a triangle
- height of a triangle

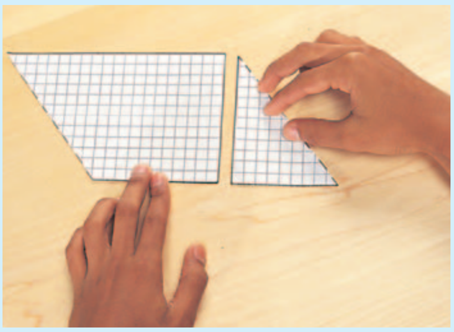
Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE 1

Area of a Parallelogram

Investigation: Area of a Parallelogram

- Cut a rectangle out of centimeter grid paper by cutting along grid lines.
- Record the base, height, and area of the rectangle.
- Cut a right triangle from one end of the rectangle. Tape the triangle to the opposite end to form a parallelogram as shown below.
- Compare the original rectangle with the parallelogram formed. List the ways the rectangle and the parallelogram are the same and the ways they are different. See left.



The rectangle and ▱ have the same base length, height, and area. Their shapes are different.

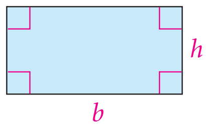
The picture on page 348 shows that a parallelogram with the same base and height as a rectangle has the same area as the rectangle.

Key Concepts

Theorem 7-1 **Area of a Rectangle**

The area of a rectangle is the product of its base and height.

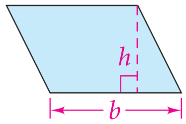
$A = bh$



Theorem 7-2 **Area of a Parallelogram**

The area of a parallelogram is the product of a base and the corresponding height.

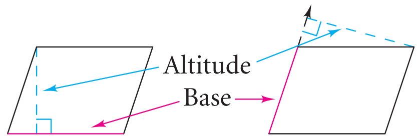
$A = bh$



Reading Math

The term *base* is used to represent both a segment and its length.

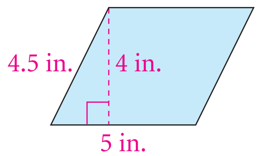
A **base of a parallelogram** is any of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base drawn from the side opposite the base. The **height** is the length of an altitude.



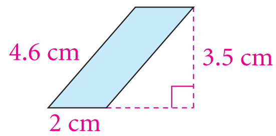
1 EXAMPLE Finding the Area of a Parallelogram

Find the area of each parallelogram.

a.



b.



You are given each height. Choose the corresponding side to use as the base.

$A = bh$ | $A = bh$

$= 5(4) = 20$ ←**Substitute.**→ $= 2(3.5) = 7$

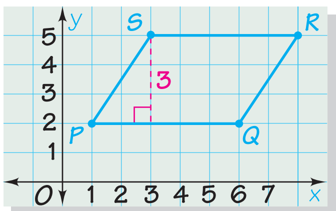
The area is 20 in.^{2}. | The area is 7 cm^2.

Check Understanding **1** Find the area of a parallelogram with base 12 m and height 9 m. **108 m^2**

2 EXAMPLE Finding Area in the Coordinate Plane

Find the area of ▱$PQRS$ with vertices $P(1, 2)$, $Q(6, 2)$, $R(8, 5)$, and $S(3, 5)$.

Graph ▱$PQRS$. If you choose $\overline{PQ}$ as the base, then the height is 3.



$b = PQ = 5$

$h = 3$

$A = bh = 5(3)$

$= 15$

The area of ▱$PQRS$ is 15 square units.

Check Understanding **2** Find the area of ▱$EFGH$ with vertices $E(-4, 3)$, $F(0, 3)$, $G(1, -2)$, and $H(-3, -2)$. **20 units^2**

3 EXAMPLE Finding a Missing Dimension

For ▱$ABCD$, find CF to the nearest tenth.

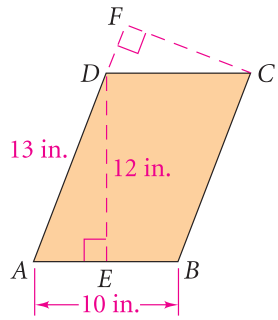
First, find the area of ▱$ABCD$. Then use the area formula a second time to find CF.

$$A = bh$$
$$= 10(12) = 120 \quad \textbf{Use base } AB \textbf{ and height } DE.$$

The area of ▱$ABCD$ is 120 in.^{2}.

$$A = bh$$
$$120 = 13(CF) \quad \textbf{Use base } AD \textbf{ and height } CF.$$
$$CF = \frac{120}{13} \approx 9.2$$

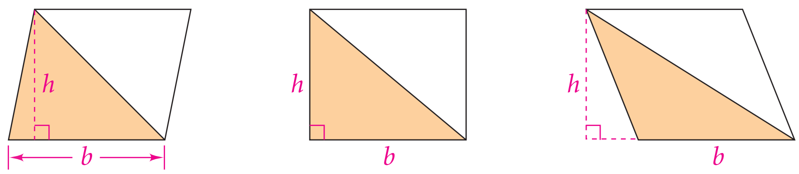
CF is about 9.2 in.



Check Understanding 3 A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. Find the height corresponding to an 18-cm base. **7.5 cm**

OBJECTIVE 2 Area of a Triangle

A diagonal divides any parallelogram into two congruent triangles.



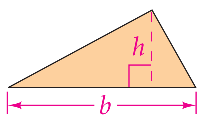
Therefore, the area of each triangle is half the area of the parallelogram.

Key Concepts

Theorem 7-3 **Area of a Triangle**

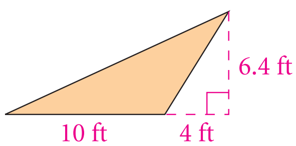
The area of a triangle is half the product of a base and the corresponding height.

$$A = \frac{1}{2}bh$$



A **base of a triangle** is any of its sides. The corresponding **height** is the length of the altitude to the line containing that base.

4 EXAMPLE Finding the Area of a Triangle

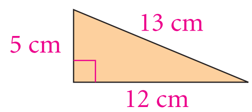


Find the area of the shaded triangle at the left.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(6.4) = 32 \quad \textbf{Substitute and simplify.}$$

The area of the shaded triangle is 32 ft^2.

Check Understanding 4 Find the area of the triangle at the right. **30 cm^2**



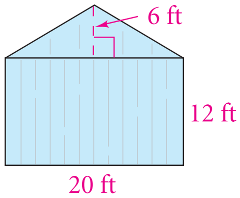
5 EXAMPLE Real-World Connection

Real-World Connection

In 1992 this building in Homestead, Florida, succumbed to the 145 mi/h winds of Hurricane Andrew.

Structural Design When designing a building, you must be sure that the building can withstand hurricane-force winds, which have a velocity of 73 mi/h or more. The formula $F = 0.004Av^2$ gives the force F in pounds exerted by a wind blowing against a flat surface. A is the area of the surface in square feet, and v is the wind velocity in miles per hour.

How much force is exerted by a 73 mi/h wind blowing directly against the side of the building shown here?



Find the area of the side of the building.

triangle area $= \frac{1}{2}bh = \frac{1}{2}(20)6 = 60 \text{ ft}^2$

rectangle area $= bh = 20(12) = 240 \text{ ft}^2$

area of the side $= 60 + 240 = 300 \text{ ft}^2$

Use the area of the side of the building and the velocity of the wind to find the force.

$F = 0.004Av^2$	**Use the formula for force.**
$= 0.004(300)(73)^2$	**Substitute 300 for *A* and 73 for *v*.**
$= 6394.8$	

The force is about 6400 lb, or 3.2 tons.

Check Understanding 5 **Critical Thinking** Suppose the bases of the rectangle and triangle in the building above are doubled to 40 ft, but the height of each figure remains the same. How is the force of the wind against the side of the building affected?
The force is doubled.

EXERCISES

For more practice, see *Extra Practice.*

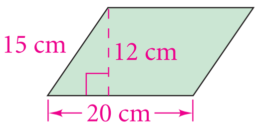
Practice and Problem Solving

A Practice by Example

Example 1 (page 349)

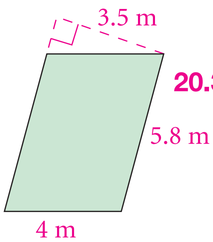
Find the area of each parallelogram.

1.



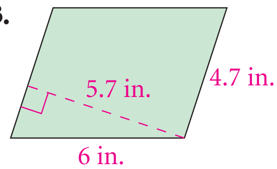
240 cm²

2.



20.3 m²

3.



26.79 in.²

Example 2 (page 349)

Coordinate Geometry **Find the area of the parallelogram with the given vertices.**

4. $A(2, 0), B(7, 0), C(8, 4), D(3, 4)$ **20 units²**

5. $E(-4, 0), F(-1, 0), G(1, -3), H(-2, -3)$ **9 units²**

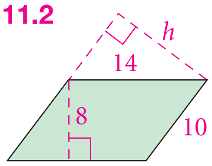
6. $I(2, 2), J(4, 2), K(2, -3), L(0, -3)$ **10 units²**

7. $M(-6, -1), N(-5, 0), P(1, 0), Q(0, -1)$ **6 units²**

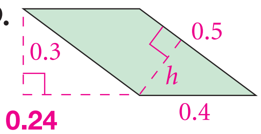
Example 3 (page 350)

Find the value of *h* for each parallelogram.

8. **11.2**

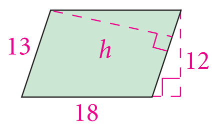


9.



0.24

10.



$16\frac{8}{13}$