Teacher Notes

Activity 9

The World Population: Linear and Exponential Models

Objective

- Students will develop an understanding of the exponential function and use it to model real world data

Applicable TI InterActive! Functions

- List Editor
- Graph
- Table
- Browser

Problem

Students will collect world population data from the Internet and find linear and exponential models for the data and investigate the appropriateness of each model.

Pre-Activity

Investigate the parameters a and b of the function $f(x) = ab^x$.

1. Students should open a new TI InterActive! document.

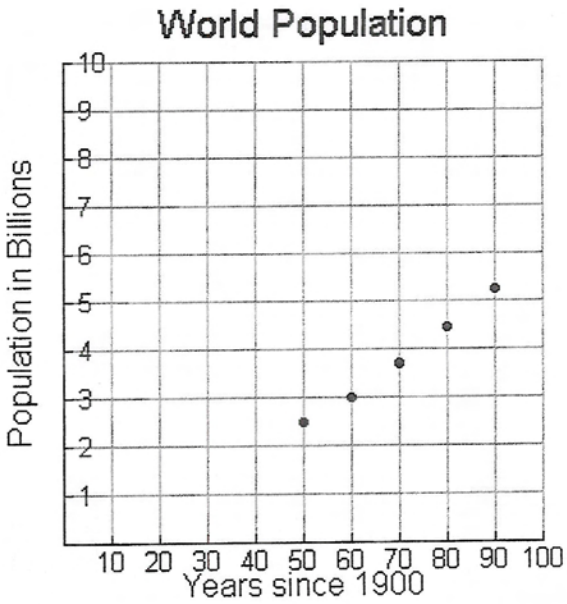
2. Select Graph and define y1(x): = 1 * 2^x, y2(x): = 2 * 2^x, y3(x): = 3 * 2^x and y4(x): = 4 * 2^x. Then graph the functions on the same grid. Have students compare and contrast the four functions. In particular, they should note the effect that a has on the graph of $f(x) = ab^x$. After closing the Graph window, students should type a statement about their findings in the TI InterActive! document.

3. Students should open a new graphing window. Select Graph and define y1(x): = 1 * 2^x, y2(x): = 1 * 3^x, y3(x): = 1 * 4^x, y4(x): = 1 * 0.5^x, and y5(x): = 1 * 0.25^x. Then graph the functions on the same grid. Have students compare and contrast the four functions. In particular, they should note the effect that b has on the graph of $f(x) = ab^x$. After closing the Graph window, students should type a statement about their findings in the TI InterActive! document.

Exploration

Steps 1 through 10 are details for the students to set up the problem and extract the data from the Internet. The following solutions are based on data from the United Nations, Population Division, Department of Economic and Social Affairs posted on September 1, 1999. When students have completed step 10, their graph should appear as shown.

1. through 10.



Analysis

1. Linear and exponential.

2. Answers will vary. One model is $y = .07x - 1.1$. If students have difficulty, have them change the slope until the line is parallel to the set of data. Then shift the line to match the data.

3. Based on our model, the slope is 0.07 billion people/year. This implies that for each year, the population increases by 0.07 billion people.

4. Based on our model, the y-intercept is -1.1 billion people. This implies that in 1900 the world's population was -1.1 billion people.

5. No. The population of the world in 1900 could not have been a negative amount.

6. Answers will vary. One model is $y = 1.065 * 1.018^x$. This is a more difficult fit. The teacher may need to suggest value of a and allow students to guess the value of b.

7. Based on our model, 1.065 billion is the number of people in the world in 1900.

8. Based on our model, the growth factor is 1.018. So the growth rate per is 1.018 – 1 = 0.018 or a growth of 1.8% each year.

9. The exponential model could be appropriate since the population in 1900 is 1.065 billion rather than the negative number as predicted by the linear model.

11. The linear model only fits the data since 1950. This model predicts the world's population to be 0 around 1916 and negative prior to that year.

12. The exponential model fits better but it still underestimates the population from 1900 until 1950.

13. Answers will vary. One model is $f(x) = 0.4 * 1.026^x + 1.25$. The teacher should suggest to students that they first estimate the vertical shift by choosing a value lower than the population in 1900.

15. Answers will vary. The following answers are based on the models stated above.

		Population in Billions		
Year	**Years since 1900**	**Linear Model**	**Exponential Model**	**Shifted Exponential Model**
1850	-50	-4.6	0.44	1.36
1916	16	0.02	1.42	1.85
Now	99	5.83	6.23	6.33
2050	150	9.4	15.47	20.05

16. Answers will vary. Based on our models, in 1850 the population was 1.26 and the shifted exponential is the best fit. In 1916, the population would be between 1.75 and 1.86 billion which is close to the value predicted by the shifted exponential model. The current population (1999) is close to 6 billion which is closest to the exponential prediction. For the year 2050, the exponential models predict a much higher population.