Infinite Geometric Series

ID: 11066

Time required
45 minutes

Activity Overview

In this activity, students will explore infinite geometric series. They will consider the effect of the value for the common ratio and determine whether an infinite geometric series converges or diverges. Students will numerically analyze infinite geometric series using spreadsheets. They will also consider the derivation of the sum of a convergent infinite geometric series and use it to solve several problems.

Topic: Sequences & Series

- *Explore geometric sequences*
- *Sum a geometric series*
- *Convergence of an infinite geometric series*

Teacher Preparation and Notes

- *This activity serves as an introduction to infinite geometric series. Students will need to have previously learned about finite geometric series.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11066" in the keyword search box.***

Associated Materials

- *InfiniteGeoSeries_Student.doc*
- *InfiniteGeoSeries.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Geometric Sequences and Series (TI-Nspire technology) — 8674*
- *Sum of an Infinite Geometric Series (TI-Nspire CAS technology) — 13471*
- *Spreading Doom (TI-84 Plus family) — 10073*

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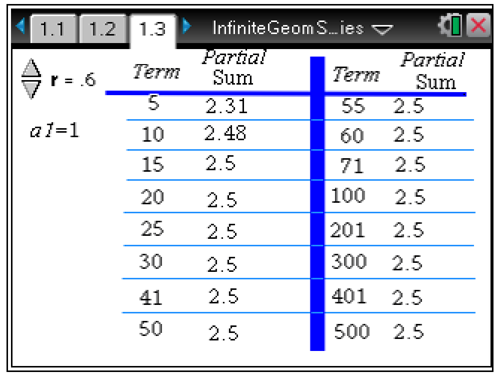
Problem 1 – Investigating Infinite Geometric Series

On page 1.3, students will explore what happens when the common ratio changes for an infinite geometric series. For each value of *r*, students will determine if the series converges or diverges.

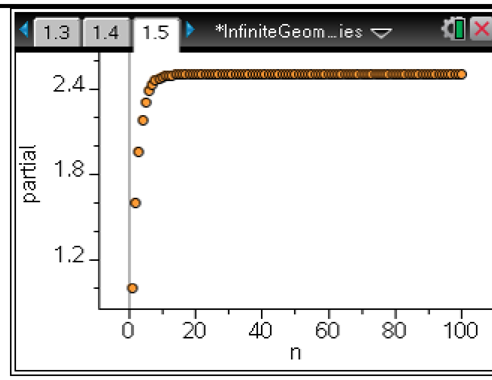


As an extension, students could change the initial value of the sequence by change the value of a1 to see if it affects the convergence or divergence of the series.

TI-Nspire Navigator Opportunity: ***Live Presenter***

See Note 1 at the end of this lesson.

On page 1.5, students will then observe a scatter plot of the term (**n**) vs. the partial sum (**partial**.) They can go back to page 1.3 and change the value of *r* and then return to the graph to see how it changes.



To change the window, students can click on the label for the vertical axis (**partial**) and then select **partial**.

1.

r	–2	–0.5	–0.2	0.2	0.5	2
Converges or Diverges	Diverges	Converges 0.667	Converges 0.833	Converges 1.25	Converges 2	Diverges

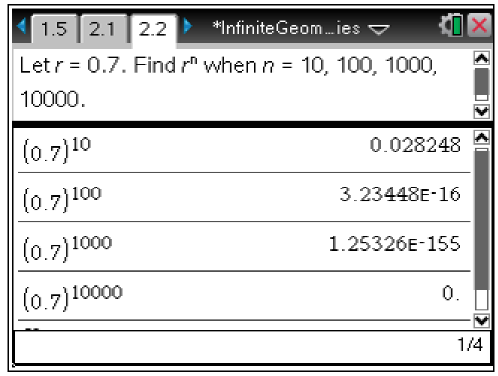
2. $|\, r \,| < 1$

3. There is a horizontal asymptote at the point of convergence.

Problem 2 – Deriving a Formula for the Sum of a Convergent Infinite Geometric Series

Students are to use the *Calculator* application on page 2.2 to determine the values of r^n when $r = 0.7$. They should see that as n gets very large r^n becomes zero when $|r| < 1$.

Students are given the formula for the sum of a *finite* geometric series. With the information found on page 2.2, they can determine the formula for the sum of an *infinite* geometric series using substitution.



TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

4.

n	10	100	1000	10000
$r^n = 0.7^n$	0.028248	3.23E–16	1.25E–155	0

5. $S_n = \dfrac{a_1(1-0)}{1-r} = \dfrac{a_1}{1-r}$

Problem 3 – Apply what was learned

In this problem, students are given a scenario relating to drug prescriptions and dosages. Students need to use the formulas shown in the previous problem to answer the questions.

They may get caught up on the first question. Explain to students that if 15% of the drug leaves the body every hour, then that means that 85% percent is still in the body.

a. 0.40 = 1 – 0.15(4)
b. 240 + 240(0.4) = 336 mg

c.

Hours	0 (1st dosage)	4 (2nd dosage)	8 (3rd dosage)	12	16
Amount in the Body	240	336	374.4	389.76	395.904

d. This is the 7th dosage, so $S_7 = \dfrac{240(1-.4^7)}{1-0.4} = 399.34464$.

e. This is the 19th dosage, so $S_{19} = \dfrac{240(1-.4^{19})}{1-0.4} = 399.999989$.

f. $S_t = \dfrac{240(1-.4^t)}{1-0.4}$

g. No, since $S = \dfrac{240}{1-0.4} = 400$, you will not reach the minimum lethal dosage.

h. Yes, since he/she waits 2 hours, only 30% of the drug is out of his/her system, so 70% remains. This is the common ratio *r* = 0.7 and $S = \dfrac{240}{1-0.7} = 800$

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*

This is a good place to have a class discussion on the effect the variable, *r*, and the effect the variable, *a*, has on the partial sum's.

Note 2
Problem 1, *Quick Poll*

Have the students submit their response as a quick poll and discuss the results.