According to the Standards:

Instructional programs from preK-grade 12 should enable students to:

- Recognize and use connections among mathematical ideas
- Use the language of mathematics to express mathematical ideas precisely
- Select, apply and translate among mathematical representations to solve problems

In grades 9-12 students should

1. Students should develop an increased capacity to link mathematical ideas and a deeper understanding of how more than one approach to the same problem can lead to equivalent results.

Calculus Scope and Sequence: Infinite Series
Keywords: convergence, integral test, improper integral
Description: This activity will demonstrate the use of the integral test to discuss the convergence of an infinite series.

The Integral Test allows us to decide the convergence or divergence of a series by comparing it to an improper integral.

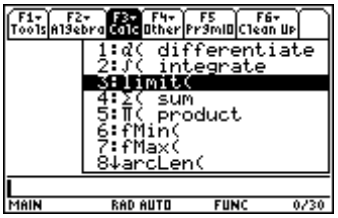
The Integral Test can be applied to any series whose terms: $a_n = f(n)$***. Where f can be considered a*** *continuous, positive,decreasing* ***function of a real variable, x.***

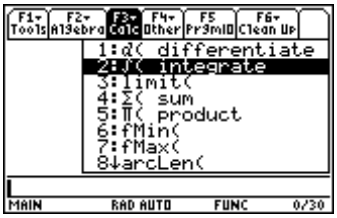
- ***If*** $\sum_{n=1}^{\infty} \frac{1}{4n^2+9}$ $\int_1^{\infty} f(x)$ ***converges, then*** $\sum_{n=1}^{\infty} a_n$***, converges***
- ***If*** $\int_1^{\infty} f(x)$ ***diverges, then*** $\sum_{n=1}^{\infty} a_n$***, diverges***

Determine whether the harmonic series: $\sum_{n=1}^{\infty} \frac{1}{n}$ converges:

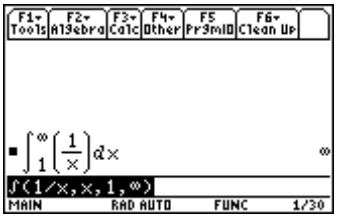
(Note that it fits all requirements for the integral test)

- The limit function is found in F3-Calc-#3 and requires three arguments: lim(function, variable, limiting value)
- The integral function is also found in F3-Calc-#2 and requires four arguments: (function, variable, upper bound, lower bound)
- ∞ key is found on the keyboard: <diamond> - catalog





Set up and evaluate the improper integral: $\int_1^{\infty} \frac{1}{x}\,dx$



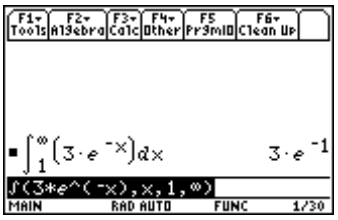
Note that the the improper integral diverges, therefore the harmonic series is proven to diverge by the integral test.

For a full review of improper integrals see the activity titled: Improper Integration

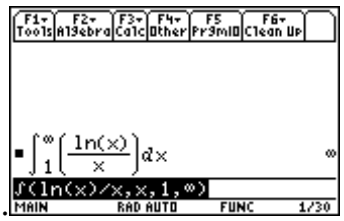
Try These:

1. $\sum_{n=1}^{\infty} 3e^{-n}$ 2. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ 3. $\sum_{n=1}^{\infty} \frac{1}{4n^2+9}$

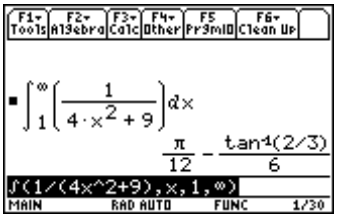
Answers:

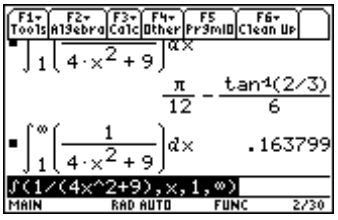


Converges



DIVERGES





Converges