Segments In Circles

ID: 9884

Time required
40 minutes

Activity Overview

In this activity, students will explore the relationships among special segments in circles. The special segments include tangent segments, segments created by two intersecting chords, and secant segments. Students will confirm relationships among segments to explore these "Power Theorems."

Topic: Circles

- *Prove and apply the Chord-Chord, Secant-Secant, and Tangent-Secant Product Theorems (also known as the "Power Theorems").*

Teacher Preparation and Notes

- *This activity is designed to be used in a high school or middle school geometry classroom with the Cabri Jr. application.*
- *This activity is designed to be* ***student-centered*** *with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.*
- ***Note:*** *Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:*
    1. Move the cursor over the value so it is highlighted.
    2. Press [+] to display additional decimal digits or [−] to hide digits.
- *The worksheet allows students to start with a new file, but you can also have students use the file CHORDSEC and start with the calculation of the products in Problem 1.*
- ***To download the CHORDSEC and TANSEC Cabri Jr files, go to education.ti.com/exchange and enter "9884" in the quick search box.***

Associated Materials

- *GeoWeek24_SegmentsCircles_worksheet_TI84.doc*
- *CHORDSEC.8xv and TANSEC.8xv*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Inscribed Angles Theorem (TI-84 Plus family) — 12437*
- *Chords and Circles (TI-84 Plus family) — 9773*
- *Angles Formed by Intersecting Chords, Secants, and Tangents (TI-84 Plus family) — 4065*

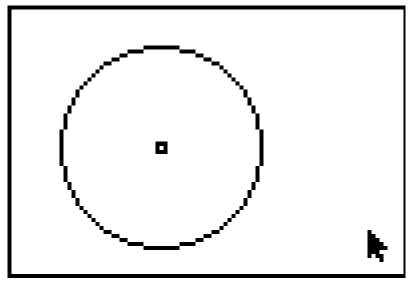
Theorems

- The Chord-Chord Product Theorem states, “When two chords intersect in a circle, the product of the segment lengths of one chord will equal the product of the segment lengths of the other chord.”
- The Secant-Secant Product Theorem states, “If two secants are drawn from an external point of a circle, the product of the lengths of one secant and its external part will equal the product of the lengths of the other secant and its external part.”
- The Tangent-Secant Product Theorem states, “If a tangent and a secant are drawn from an external point of a circle, the square of the length of the tangent segment will equal the product of the lengths of the secant and its external part.”

Problem 1 – Intersecting Chords

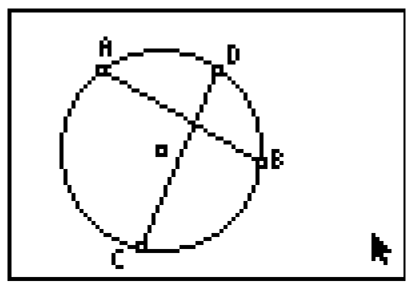
Students are to start the **CabriJr** app by pressing APPS, then choosing it from the list. Press any key to begin. Open a new file.

First they need to create a circle using the **Circle** tool. Hide the radius point using the **Hide/Show** tool.



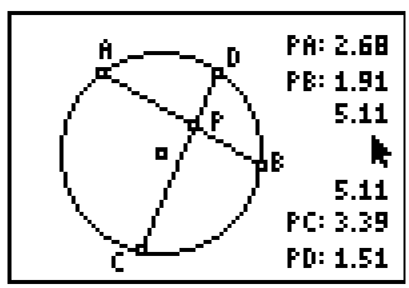
Then, students select the **Segment** tool and construct two chords in the circle. Using the **Alph-Num** tool they will label the chords $\overline{AB}$ and $\overline{CD}$ as shown.

Note: Press ENTER to start the label, then press ENTER again to end the label.



Using the **Point > Intersection** tool students are to create the intersection of the two chords and label this point *P*.

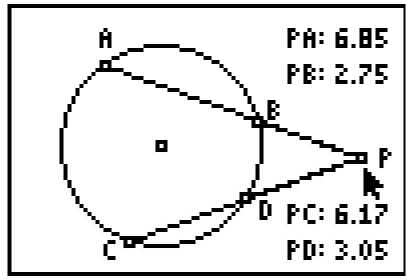
They will then measure the lengths of each segment using the **Measure > D. & Length** tool by selecting the endpoints of the segment.



Finally they will use the **Calculate** tool to calculate the product of the lengths of the segments of each chord. They should select one measurement, press the multiplication key, and then select the other measurement.

To complete the chart students need to drag an endpoint of either chord.

Problem 2 – Chords and Secants

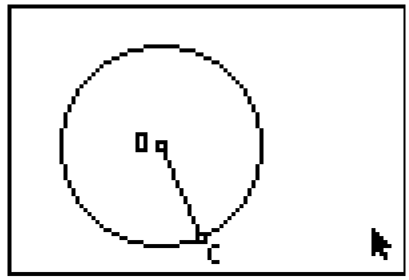


Students are to hide the two values of the products of the segments using the **Hide/Show** tool.

Then they need to drag point *P* outside of the circle as shown. They should see that $\overline{AP}$ and $\overline{CP}$ are now secants.

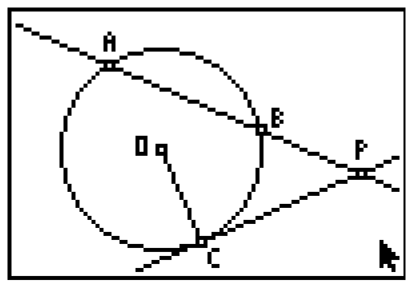
By moving point P to different location, students will make a conjecture about the relationship of the segments of the secants.

Problem 3 – Secants and Tangents



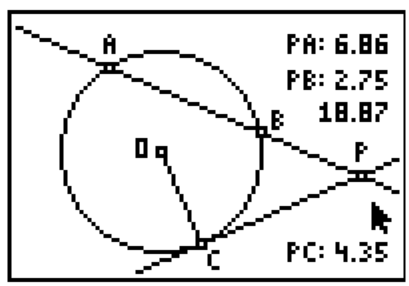
Now students will need to open a new Cabri Jr. file.

They are to create a circle using the **Circle** tool and then use the **Segment** tool to construct a radius labeling it $\overline{OC}$ as shown.



Students will need to select the **Perp.** tool to construct the line perpendicular to $\overline{OC}$ at point *C* and place a point on the tangent line and label it *P*.

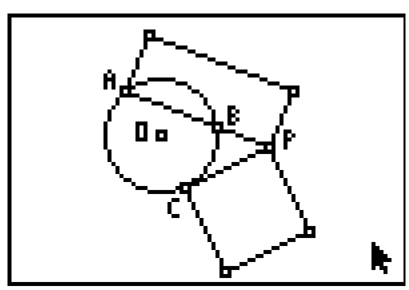
Using the **Line** tool students construct a secant line through point *P* that intersects the circle in two places. They should create the intersection points of the secant line with the circle and label them *A* and *B* as shown.



After measuring the lengths of segments $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$, students use the **Calculate** tool to calculate the product of the lengths of $\overline{PA}$ and $\overline{PB}$.

Students drag points *P*, *C* and *A* and record the data in the table on their worksheet.

Extension – Visualizing the Tangent-Secant Product Theorem

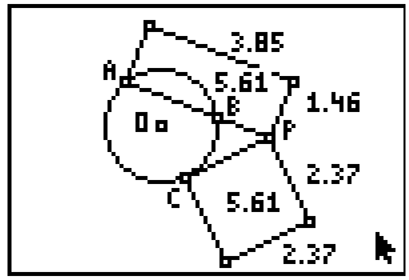


For this extension, students will need to open the Cabri Jr. file **TANSEC**.

The figure shows a secant and a tangent from the same external point, as was explored in Problem 3. Rectangles have been constructed on the segments.

Students are to measure the areas of the rectangles and their lengths and widths using the tools in the **Measure** submenu.

They will need to explain how the figure models the Tangent-Secant Product Theorem.



Drag point *P*, *C* and *A* and observe.

Answers – Student Worksheet

1–3. Check student's work.

4.

$\overline{PA}$ length	$\overline{PB}$ length	$PA \cdot PB$	$\overline{PC}$ length	$\overline{PD}$ length	$PC \cdot PD$
2.82	2.15	6.08	2.57	2.37	6.08
3.56	1.20	4.28	3.17	1.35	4.28
2.18	1.96	4.28	3.49	1.23	4.28

5. The products of the segments are equal.
6. The relationship holds.
7. Segments *AP* and *CP* are secants.

8–9. The product of the segments are equal. ($PA \cdot PB = PC \cdot PD$)

10. The relationship holds.

11–12. The line is tangent because it only touches one point on the circle.

13–15.

$\overline{PA}$ length	$\overline{PB}$ length	$PA \cdot PB$	$\overline{PC}$ length	PC^2
5.8	3.8	22.3	4.7	22.3
5.1	3.4	17.2	4.2	17.2
4.3	1.8	7.8	2.8	7.8

16–17. The product of the lengths of $\overline{PA}$ and $\overline{PB}$ is the length of $\overline{PC}$ squared.

