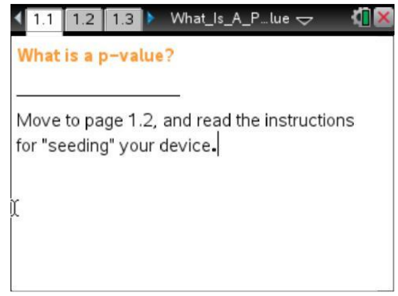
What is a *P*-value?



Math Objectives

- Students will recognize that a *p*-value only has meaning if the null hypothesis is true (a conditional probability).
- Students will interpret a *p*-value in given contexts as the relative frequency for sample statistic values at least as extreme as that from the observed sample, assuming that the null hypothesis is true.
- Students will recognize the relationship between sample size and *p*-values: in general, for sample means giving *p*-values less than 0.5, increasing the sample size decreases the *p*-value.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

Vocabulary

- alpha value
- sample size
- population
- relative frequency
- sample mean
- sampling distribution
- sample standard deviation

About the Lesson

This lesson involves beginning with a null hypothesis specifying the mean of a normally distributed population with a given standard deviation.

As a result, students will:

- Generate an observed outcome (a sample of fixed size), which determines a *p*-value. That value is represented by a shaded region in the sampling distribution.
- Generate additional samples of the same size from the hypothesized population and observe where the means of these samples fall.
- Estimate the likelihood of getting by chance an outcome at least as extreme as the original observed sample mean from the graph of the simulated sampling distribution of sample means that were generated if the null hypothesis is true.

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- Use Class Capture to compare student results.
- Send .tns file to students.
- Use Quick Poll to determine student understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- What_Is_A_P-value.pdf
- What_Is_A_P-value.doc

TI-Nspire document

- What_Is_A_P-value.tns

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Prerequisites

- Students should have knowledge of random sampling and of sampling distributions.
- The students are asked to compare p-values and alpha levels. If they do not have experience with alpha values, Question 7 should be omitted.
- **Teacher Note**: It is *highly recommended* that students work through *The Meaning of Alpha* prior to this lesson.

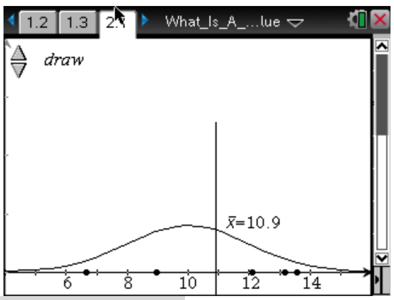
Discussion Points and Possible Answers

Tech Tip: Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a *Calculator* page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

Tech Tip: To enter their random seed value on page 1.3, students will need to press the space immediately after the RandSeed command to have the keyboard appear. (Syntax: RandSeed #, where # is a number unique to each student.) Ensuring that students carry out this step will prevent students from generating identical data.

Teacher Tip: Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or with a new device, as otherwise the "random" numbers will all be the same as those on other similarly cleared devices.

Move to page 2.1.



Consider the following hypothesis test scenario:

H_0: $\mu = 10$ and H_a: $\mu > 10$

Page 2.1 represents a population whose mean is 10, as assumed in the null hypothesis.

Teacher Tip: The lesson assumes H_a: $\mu > 10$ but could be adapted for H_a: $\mu < 10$.

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1. Use the arrow labeled “draw” to select a random sample of size 5 ($n = 5$) from this population.

   a. What do the points in the graph represent?

   Sample Answers: The points in the graph represent elements of the population that were selected in the sample.

   b. What does the vertical line in the graph represent?

   Sample Answers: The vertical line in the graph is the mean of the sample shown on the top graph.

   c. Estimate the values of the elements in the sample and give the sample mean.

   Sample Answers: Answers will vary depending on the sample that is chosen: for example: Sample = {9.3, 10.1, 10.9, 12.6, 13.1}; sample mean = 11.2.

   d. Considering the alternative hypothesis stated above, does it seem likely that your sample came from the hypothesized population or from a population whose mean is noticeably larger than 10? Explain your reasoning.

   Sample Answers: Students with sample values far to the right might respond that it seems as if their sample came from a population whose mean is larger than 10. Students with sample values clustered around 10 are likely to believe that their sample could have come from the hypothesized population.

2. Predict the sampling distribution of the sample means from this population.

   Sample Answers: Student answers will vary. They should recognize that if the null hypothesis, H_o: $\mu = 10$, is correct, then the sampling distribution of sample means should be approximately symmetric around 10 and narrower than the population distribution.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of the lesson.

Use the random sample you have for Question 1 to answer the next set of questions.

Move to page 2.2.

The graph in the top work area is from Page 2.1 with the vertical axis rescaled; the one in the lower work area is the sampling distribution of sample means of size 5 from that population.

The term *p-value* describes the probability that the mean of a new sample will be at least as extreme in the direction of the alternative hypothesis as the one from the random sample you drew if the null hypothesis is correct. The shaded area in the lower screen indicates the *p*-value.

> **Teacher Tip:** This activity does not deal with the calculation of the *p*-value but focuses on the interpretation of what a *p*-value actually represents. Standard texts will describe how to do the related calculations and how to account for knowledge about the standard deviation of the population. The concept of *p*-value developed in the activity applies to proportions as well as to sample means.

3. a. Interpret the *p*-value you have on your screen.

Sample Answers: Answers will vary. For a sample mean of 11.2, the *p*-value is about 0.09, so 9% of any future samples of size 5 from the given population will have means of 11.2 or greater.

b. Explain why this area represents a probability.

Sample Answers: The relative frequency of values occurring in an interval is equal to the area between the density curve and the horizontal axis on that interval.

c. Based on your observed *p*-value, does it seem likely that your sample came from a population with mean = 10? Why or why not?

Sample Answers: Answers will vary. Students might suggest that their *p*-value was very small, and so it seems likely that their observed sample mean corresponding to that *p*-value would be unlikely to occur by chance in that population; or they might have a *p*-value that was large (i.e., 0.45), and so it seems very likely that their sample mean would occur in a sample from the hypothesized population.

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Teacher Tip: Discuss student answers to Question 3 part c. The main idea behind deciding whether a sample could have come from the given population is to decide whether that sample is "typical" or "unusual." Focus on student responses that indicate this line of reasoning. The remainder of this activity is designed to help students see that low *p*-values indicate unusual samples.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of the lesson.

Move to page 2.3, without changing the *p*-value you found above.

The distribution curve on Page 2.3 is a rescaled plot of the curve on the lower screen of Page 2.2. However, the values generated by the draw arrow here are not the sample values as on Page 2.1. Instead, the means of samples are plotted, with five sample means displayed for each click.

4. Use the draw arrow to select 100 sample means.

   a. How does the simulated sampling distribution of the sample means compare to your predictions in Question 2? Explain any differences.

   Sample Answers: Student answers will vary. They will likely get a distribution somewhat similar to the one described in Question 2. Any differences can be explained by random variation among samples.

   b. From your simulation of sample means, estimate the likelihood of getting a sample from the given population so that the new sample mean is at least as extreme as the original observed sample mean.

   Sample Answers: Student answers will depend on their simulated distribution of the sample means. Ideally, most student answers will be near their displayed *p*-values.

Teacher Tip: Be sure students recognize that it is not possible in real applications of statistics to draw more than one sample. But to decide whether an individual sample is likely or unlikely to have come from the hypothesized population, it is necessary to understand the distribution of all samples from that population.

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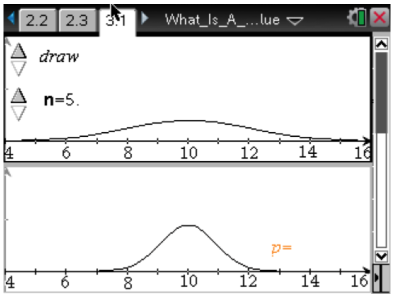
Teacher Tip: You might want to have your class repeat question 4 with new samples. If so, they should reset this page by clicking the reset arrow.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of the lesson.

Teacher Tip: Before you go to Question 6, you might want students to return to Page 2.1 in the .tns file, generate another "original" sample mean for $n = 5$ (i.e., a new p-value), and have them answer Question 4 again, contrasting their answers with the ones they had the first time through. This is also a good opportunity for students to share their results and talk about any general patterns they observe.

Move to page 3.1.



5. Assume H_0: $\mu = 10$ and H_a: $\mu > 10$.

   a. Draw samples from the population for $n = 5$ until you find a p-value less than 0.1. What are your sample mean and the corresponding p-value?

   Sample Answers: One answer might be for an observed sample mean of 11.5, the p-value for $n = 5$ is about 0.05.

   b. Change the sample size to $n = 10$, and draw samples until you get a sample mean close to the sample mean you found in part a. What is the corresponding p-value? Repeat the process for sample size $n = 15$.

Teacher Tip: Be sure students are looking for the "same" sample mean, not the "same" p-value.

Sample Answers: A sample answer might be: for an observed sample mean of 11.6, when $n = 10$, the p-value is about 0.01; for $n = 15$, a sample mean of 11.5 gives a p-value of about 0.00, which means the value is very close to 0.

Teacher Tip: The .tns file permits only 100 samples to be selected using the "draw" slider. If students reach a point where they can no longer draw samples but need more, they should press the down arrow (▼) to continue.

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c. Jordan found an observed sample mean for a sample of size $n = 5$ and claimed that a sample of size $n = 20$ with the same sample mean would have the same *p*-value. Explain whether you agree with Jordan and why.

Sample Answers: The *p*-values would not be the same. For $n = 20$, the *p*-value would be much smaller. As the sample size increases, the standard deviation of the sampling distribution decreases, and the curve becomes taller and narrower. Thus, a given value for a sample mean will have less area beyond that sample mean as the sample size increases.

6. All of the work in this activity has assumed that H_0: $\mu = 10$ and H_a: $\mu > 10$. How would your thinking change if H_a: $\mu < 10$?

Sample Answers: The only difference would be the shaded region determined by the *p*-value would be in the lower tail of the sampling distribution of the sample means. All of the reasoning about what a *p*-value represents would be the same. In particular, note that the words "at least as extreme" could apply to values either less than or greater than a given sample mean.

Teacher Tip: You might want students to consider how to interpret two-sided *p*-values at this point as well.

7. In earlier work, you studied alpha levels. Describe similarities and differences between alpha levels and *p*-values.

Sample Answers: Both alpha levels and *p*-values are probabilities that future sample means would fall beyond a given point. Both begin by assuming the null hypothesis is true.

An alpha level is the probability that a sample statistic value will lead to a “reject the null” conclusion based on a fixed rejection criterion selected by the researcher before a sample is drawn, given that the null hypothesis is actually true.

The *p*-value is based on the observed statistic—in this case, the sample mean—from a single random sample and is the probability that another sample will have a mean at least as extreme as the one from the random sample you drew, if the null hypothesis is correct. The observed sample mean marks the boundary of the shaded region, and the *p*-value describes the area of the shaded region.

Wrap Up

At the conclusion of this lesson, teacher should ensure that students are able to understand:

- A *p*-value as the probability that the means of other samples from the hypothesized population will be at least as extreme as the mean of the observed sample, provided the null hypothesis is true.
- The connection between a *p*-value and the null hypothesis.
- The size of the sample drawn from a given population will affect the magnitude of the *p*-value. In general, for sample means giving *p*-values less than 0.5, increasing the sample size decreases the *p*-value.

Assessment

Use Quick Poll or an exit quit to assess students' understanding of the key concepts of the lesson. Identify each statement as true or false, and be prepared to give a reason for your choice.

1. The decision to reject or not reject the null hypothesis is based on the size of the *p*-value.

   Sample Answers: True. Small *p*-values indicate a low probability that an observed sample mean would occur by chance from the hypothesized population.

2. If the null hypothesis is true, a *p*-value of 0.05 would mean that on average one out of 20 samples would result in a mean at least as big as that observed in our random sample just by chance.

   Sample Answers: True, provided that H_a is a "greater than" hypothesis. This is the meaning of a *p*-value.

3. If a *p*-value is 0.04 and H_a is a "greater than" hypothesis, there is a 96% chance that the sample mean you observed is from a sample from a population with a mean equal to the null hypothesis.

   Sample Answers: False. Since the sample has already been selected, it is nonsense to speak of the probability that it *will* have any particular property. It is what it is. What you can say is that future random sampling from the population would lead to sample means smaller than you observed in 96% of those future samples.

4. Small *p*-values suggest that the null hypothesis is unlikely to be true.

   Sample Answers: True. The smaller the *p*-value is, the more convincing is the rejection of the null hypothesis. It indicates the strength of evidence for rejecting the null hypothesis.

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Note 1

Question 2, Name of Feature: Class Capture

This is an opportunity to use Class Capture to display the different samples students have generated and to discuss how their answers to Question 2 part a might be different because of the variability in their samples.

Note 1

Question 3, Name of Feature: Class Capture

A Class Capture could be used to discuss why those with small *p*-values might believe their observed sample mean was unlikely to have occurred by chance.

Note 3

Question 4, Name of Feature: Class Capture

A Class Capture will show the different results for 100 samples. Looking at how these results are actually similar across the class can be a good opportunity to make very clear what p-values measure and how to interpret them by looking across different sample means that occurred by chance. You might make a student with an interesting *p*-value a Live Presenter to show his or her screens for 2.1 and 2.3 and discuss how these pages relate to his or her answer to Question 4.

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