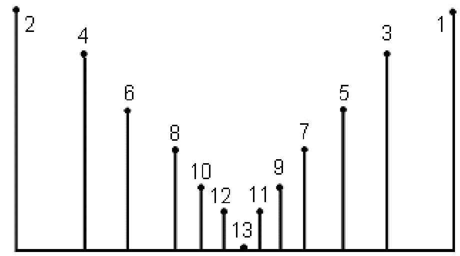
Alternating Series

Name ______________________

Class ______________________

Problem 1 – Introduction to an alternating series

With your pencil, touch the top of each vertical line in numerical order. Then answer the questions that follow.



1. What do you notice about the path of your pencil point?

2. Relate your illustration to a number line with both positive and negative values. What can you now say about the path of the pencil point?

3. If the center is 0 and each line is a term belonging to a series, what can you say about the series and its terms?

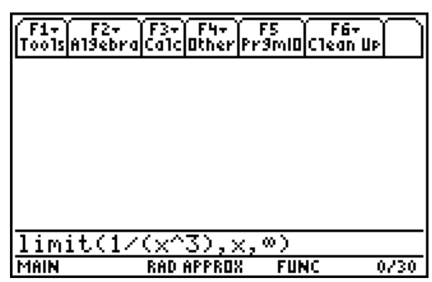
Problem 2 – Alternating Series Test

If an alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges, then these conditions must hold.

(1) $\lim_{x\to\infty} a_n = 0$ (2) $a_{n+1} \le a_n$ for all n

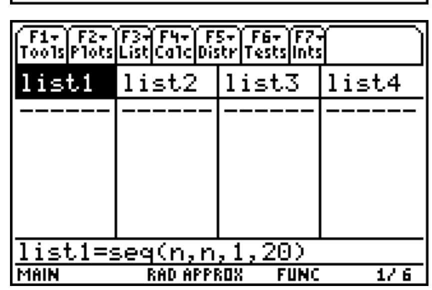
Use the Home screen to find the limit of a_n.

Press **F3: Calc > 3:Limit** to enter the **limit** command followed by the expression, the variable, and the direction the variable approaches.



Use the Stats & List Editor to test the second condition.

At the top of **list1**, enter the formula **seq(n,n,1,20)** to generate the number of the term. Use formulas to generate the terms of a_n in **list2** and the terms of a_{n+1} in **list3**.



Compare list2 (a_n) and list3 (a_{n+1}) using the formula **list3 ≤ list2**.

Determine the convergence or divergence of the following series by testing the two conditions of the Alternating Series Test.

4. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{\sqrt{n}}$

6. $\sum_{n=1}^{\infty} \frac{n}{(-3)^{n-1}}$

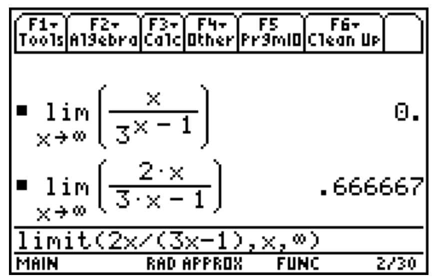
7. $\sum_{n=1}^{\infty} \frac{(-1)^{n}\,2n}{3n-1}$

Problem 3 – Alternating Series Estimation

Clear all lists *except* list1.

At the top of **list2**, use the **seq** command to generate the terms of the alternating series, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n!}$.

Use the **sum** command to find the partial sums of the alternating series. For example, in cell list3[2] enter the formula **sum(list2, 1, 2)** to find the sum of the first two terms of the series.



8. Approximate the sum of an alternating series

i) by its first three terms

ii) by its first six terms

To see the partial sums of the first 20 terms enter the formula **Σ((–1)^(n–1)/(2n!),n,1,1)** in cell list4[1].

Calculate the partial sums for the remaining terms by copying/pasting the formula and changing the last number appropriately.

9. What do you notice about the change in the sum as the value of *n* increases?

10. What do you think will occur with the approximation as the *n* approaches infinity?