F Distributions

ID: 9781

Activity Overview

*In this activity, students study the characteristics of the F distribution and discuss why the distribution is not symmetric (skewed right) and only has positive values. Students then use the **FCdf** and **invF** commands to find probabilities and percentiles, verifying their values on a graph by using the **Integral** tool. They move on to find critical values and then compute a confidence interval.*

Topic: Continuous Distributions and their Properties

- *Graph the probability density function*
- *Probabilities*
- *Percentiles*
- *Confidence intervals*

Teacher Preparation and Notes

- *Students should already be familiar with the normal distribution and it characteristics, as well as finding and interpreting confidence intervals for normal distributions (with known population standard deviation or sample size larger than 30).*

- *Making a decision based on a confidence interval, as done in Problem 4, is a precursor to hypothesis testing.*

- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*

- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9781" in the keyword search box.***

Associated Materials

- *Fdist_Student.doc*
- *Fdist.tns*
- *Fdist_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Central Limit Theorem (TI-Nspire technology) — 9998*
- *Chi-Square Distributions(TI-Nspire technology) — 9738*
- *Percentiles(TI-Nspire technology) — 9539*
- *Is It Rare? (TI-Nspire technology) — 9095*

**Problem 1 – Characteristics of the *F* Distribution**

Step 1: Use pages 1.2 and 1.3 to introduce the *F*
distribution. Have students then advance to pages
1.4 and 1.5 and compare this distribution to others
they have seen.

Allow students to change the degrees of freedom
for one or both samples.

Students should note that as with the chi-square
distribution, the values of the *F* distribution cannot
be negative, and the graph is skewed to the right.
Unlike the chi-square distribution, the graph does
not become more symmetric as the number of
degrees of freedom increases.

Step 2: On page 1.6, students are to determine if the
graph changes when the degrees of freedom are
interchanged. They can choose to graph **f1(*x*)**
from the previous page and then enter **f2(*x*)** as
FPdf(*x*, 10, 6), or type two new functions.

Students will see that the distributions are different
when the degrees of freedom are interchanged
(unless, of course, the degrees of freedom are the
same for each sample).
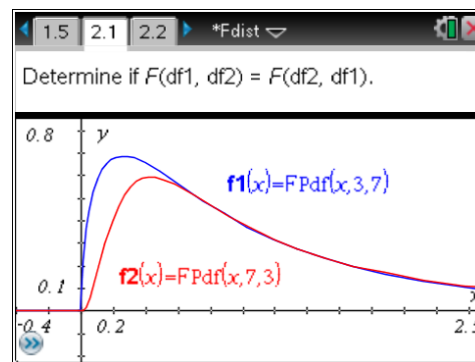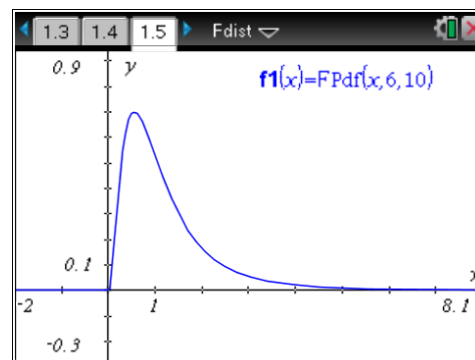
TI-Nspire Navigator Opportunity: *Live Presenter, Quick Poll*

See Note 1 at the end of this lesson

Problem 2 – Probabilities and Percentiles

Step 3: Ask students what would be true about the
variances of two samples if *F* was near 1 and why.

The variances would be almost the same. When
the numerator and denominator of a fraction are
the same, the fraction has a value of 1. The value
gets further from 1 as the numerator and
denominator have greater differences.

Discuss why the graph is skewed right. Students can look at graphs in Problem 1 during the discussion. Zero is the leftmost limit, as a fraction of two nonnegative numbers must be nonnegative. There is, however, no limit to how large the ratio can be.

TI-Nspire Navigator Opportunity: *Quick Poll*

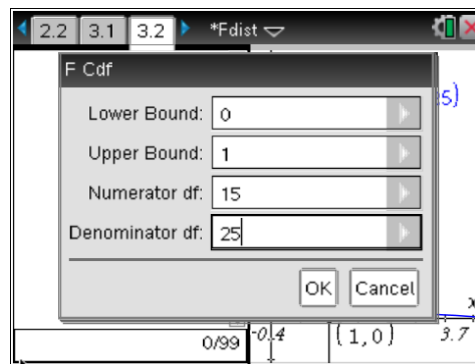**See Note 2 at the end of this lesson**

Step 4: On page 3.2, students are to find the probability that F is less than 1 for two samples, where $n_1 = 16$ and $n_2 = 26$.

FCdf finds the area under the curve, to the left of the given F value. The command can be typed, selected from the catalog, or selected by pressing menu **> Statistics > Distributions > F Cdf**. If typing the values in parentheses, be sure to enter them in this order: lower bound, upper bound, d.f. for numerator, d.f. for denominator.
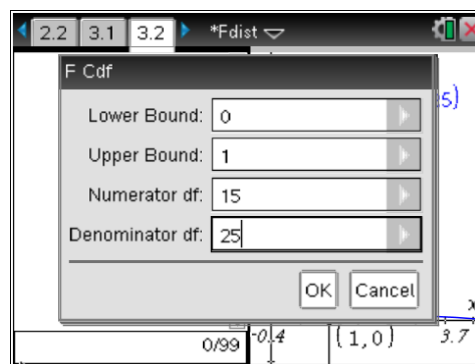
Students may think to use negative infinity for the lower bound, but since the values must be nonnegative, 0 can also be used.

Step 5: To check the area under the graph, students should use the **Integral** tool (menu **> Analyze Graph > Integral**). To use the tool, first select the curve, then the lower bound, and then the upper bound.

**TI-Nspire Navigator Opportunity:** *Live Presenter*

See Note 3 at the end of this lesson

Step 6: If needed, explain or review the notion of a percentile. Students are to find the F value at the 90th percentile on page 3.4.

As with **FCdf**, the **invF** command can be typed, selected from the catalog, or selected from the menu. Enter the percent to the left, d.f. for the numerator, and d.f. for the denominator.
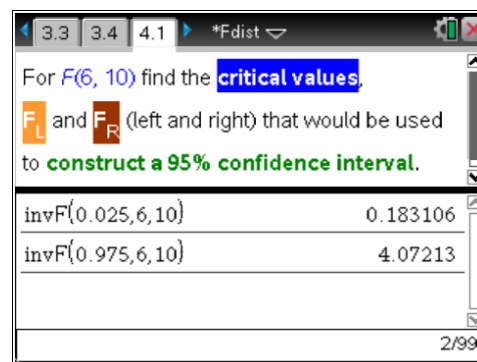
Students can check their value on the graph by changing the x-coordinate of the plotted point on the x-axis to their F value and then using the **Integral** tool.
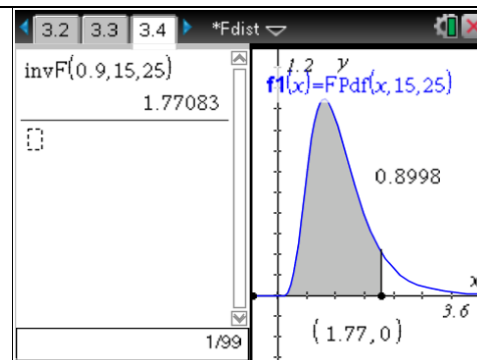
**Problem 3 – Critical Values for an F Distribution**

Step 7: Now that students can find an F value for a given area to the left of it, students can construct confidence intervals. As with the chi-square distribution, the critical values are denoted as left and right with subscripts L and R.

On page 4.1, have students find the critical values that would be used for $F(6, 10)$ at the 95% level.

Step 8: Students can check their critical values by moving to page 4.2 and changing the x-coordinates and finding areas under the curve.

Because the right tail is skewed, the graph would need to be extended to get 0.025 exactly. An option is to check the left tail and the area between the tails. If these sum to 0.975, then the right tail is 0.025.
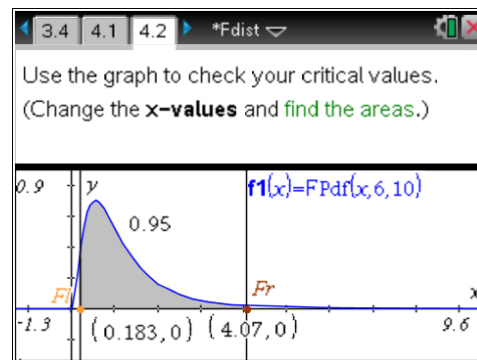
Step 9: On page 4.4, students are asked to find the critical values for $F(12, 19)$ at the 99% level and then verify that taking the reciprocal of the area to the left of the other critical value with the degrees of freedom interchanged gives that critical value.

Because of this, most tables in statistics books will give only the values for F_R and let students compute F_L when they need it.

Problem 4 – Constructing a Confidence Interval

Step 10: Now that students can find critical values, they can construct confidence intervals. Introduce the expression for the confidence interval for a ratio of variances.

Step 11: A scenario is displayed on page 5.3. Students are to construct a 95% confidence interval for the ratio of the population variances on page 4.3.
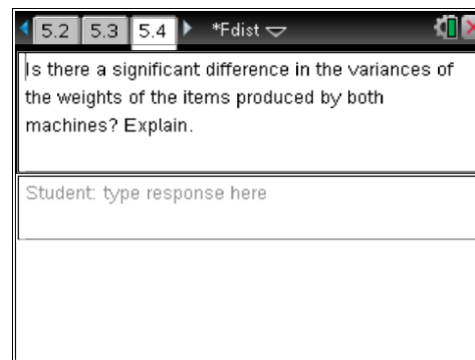
We are 95% confident that the ratio of the population variances is between 1.42739 and 5.087

Step 12: Have students discuss their answer to the
question posed on page 5.4. Students should say
that because $F = \frac{(6.7)^2}{(4.1)^2} = 2.67043$ is in the
confidence interval, there is no significant
difference in the variances.

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◀ 5.2  5.3  5.4 ▶  *Fdist ▽                    ◱✕
Is there a significant difference in the variances of
the weights of the items produced by both
machines? Explain.

Student: type response here
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TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter, Quick Poll*

Use *Live Presenter* and *Quick Poll* to ask students what effect the degrees of freedom have
on the graph besides switching them.) Some discussion questions could be:

What happens when the second value gets larger and larger?

What happens when the second value gets smaller and smaller?

Note 2
Problem 2, *Quick Poll*

Ask students this question using *Quick Poll*. Then use the results to guide the discussion

Note 3
Problem 2, *Live Presenter*

Try using *Live Presenter* to show students how to use the **Integral** tool for finding the area.
Also, show students how the area calculation result can be moved to another location on
the screen as well as how the endpoints of the integration can be moved as well.