EXPLORATIONS™

Activity 4

Perpendicular Bisector Theorem

Objectives

- To investigate the relationship between the points on a perpendicular bisector of a segment and the endpoints of the segment
- To make logical statements based on the results of the investigation

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Introduction

This exploration uses distance measures to investigate the relationship between a point in the plane and the distance to the endpoints of a segment. When are the two distances equal? When are they not equal? What implications do these relationships have for the position of the point in the plane?

This activity makes use of the following definition:

Perpendicular bisector—a line (or segment) that is perpendicular to and passes through the midpoint of the segment.

Construction

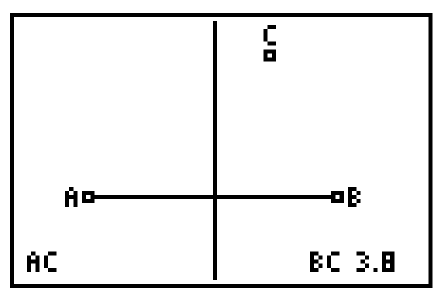
Construct a segment, its perpendicular bisector, and a point not on the segment.

Draw a horizontal segment $\overline{AB}$ on the bottom half of the screen.

Construct the perpendicular bisector of $\overline{AB}$.

Draw a point *C* anywhere on the screen. Do not attach point *C* to $\overline{AB}$ or its perpendicular bisector.

Measure the distance from *C* to the endpoints of $\overline{AB}$. Label the measurements.



Note: *Not all measurements are shown.*

Exploration

Observe the relationship between the distances *AC* and *BC* when point *C* is in various locations both on and off the perpendicular bisector.

Questions and Conjectures

1. Consider the following and form a conjecture:
   - What appears to be true when point *C* is on the perpendicular bisector?
   - What appears to be true when point *C* is not on the perpendicular bisector?
   - State your conjecture and explain your reasoning.
2. Write a **conditional statement** pertaining to *AC* and *BC* with respect to the location of point *C* and the perpendicular bisector.
   - Write the **converse** of your conditional statement.
   - Write the **inverse** of your conditional statement.
   - Write the **contrapositive** of your conditional statement.
3. For the statements in Question 2, determine which are always true, sometimes true, or never true. Explain your reasoning.

Teacher Notes

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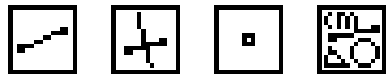
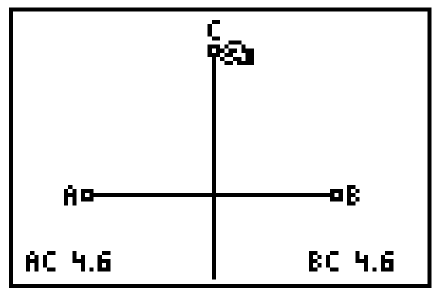


Additional Information

Point *C* is on the perpendicular bisector of $\overline{AB}$ if and only if *AC* = *BC*.

One way to explain this theorem is to first consider when point *C* is located on the intersection point of $\overline{AB}$ and the perpendicular bisector. This point is, of course, the midpoint of $\overline{AB}$ and therefore is equidistant from both points *A* and *B* by definition. Moving point *C* up or down the perpendicular bisector moves it away from points *A* and *B* equally, making the distances *AC* and *BC* change by equal amounts. Formal proof of the theorem follows from a Side-Angle-Side (SAS) triangle congruence argument.



Some Applications or Extensions of the Perpendicular Bisector Theorem.

- Constructing isosceles triangles
- Constructing kites
- Locating centers of circles
- Finding circumcenters of triangles
- Finding the location of the reflection line, given an image and a preimage of an object

Answers to Questions and Conjectures

1. Consider the following and form a conjecture:
   - What appears to be true when point *C* is on the perpendicular bisector?
   - What appears to be true when point *C* is not on the perpendicular bisector?
   - State your conjecture and explain your reasoning.

   The perpendicular bisector is the set of points that are equidistant from points *A* and *B*. As point *C* moves along the perpendicular bisector, *AC* and *BC* change but they are always equal. When *C* is to the left of the perpendicular bisector, as point *C* moves, *AC* and *BC* change but *AC* is always less than *BC*. When *C* is to the right of the perpendicular bisector, as point *C* moves, *AC* and *BC* change but *AC* is always greater than *BC*.

2. Write a **conditional statement** pertaining to *AC* and *BC* with respect to the location of point *C* and the perpendicular bisector.
   - Write the **converse** of your conditional statement.
   - Write the **inverse** of your conditional statement.
   - Write the **contrapositive** of your conditional statement.

   Students could choose any of these statements below as their conditional statement. Be sure they have the correct statements based on their conditional statement.

   Statement: If point *C* is on the perpendicular bisector of $\overline{AB}$, then $AC = BC$. $(p \to q)$

   Converse: If $AC = BC$, then point *C* is on the perpendicular bisector of $\overline{AB}$. $(q \to p)$

   Inverse: If point *C* is not on the perpendicular bisector of $\overline{AB}$, then $AC \neq BC$. $(\sim p \to \sim q)$

   Contrapositive: If $AC \neq BC$, then point *C* is not on the perpendicular bisector of $\overline{AB}$. $(\sim q \to \sim p)$

3. For the statements in Question 2, determine which are always true, sometimes true, or never true. Explain your reasoning.

   A conditional statement and its contrapositive are logically equivalent since the contrapositive is true when the statement is true and false when the statement is false. In this case, the converse and inverse statements are also true since the hypothesis and conclusion of the statement are both true. A theorem with these properties is usually stated as an “if and only if” proposition.