Tangent Challenge

Teacher Notes and Answers

7 8 9 10 11 **12**

TI-Nspire™ Investigation Student 20 min

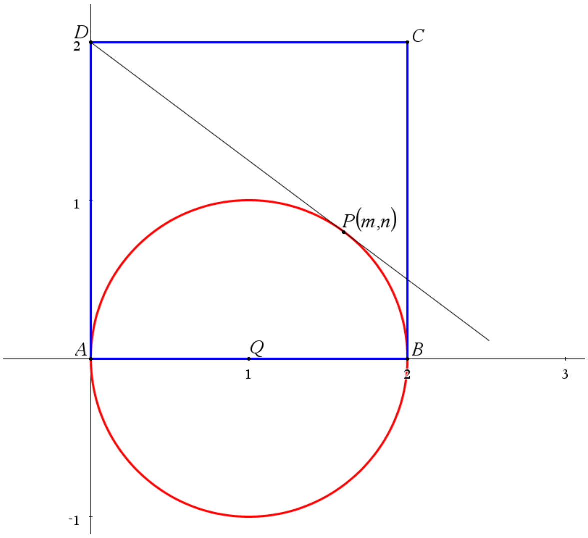
Problem Statement

A circle of radius 1 unit is drawn such that it is centred at point Q(1, 0).

A square is also drawn with vertices A(0, 0); B(2, 0); C(2, 2) and D(0, 2).

The line DP passes through D and is tangent to the circle at point P. (Shown opposite)

Aim: Determine the equation to the line DP.



Teacher:

One of the amazing oppportunities when students are using a CAS platform is to have them solve problems like these, without any clues or scaffolding. For this reason a TI-nspire file has been provided that can be used in-lieu of the following questions. Leaving the entire solution process to students allows teachers to see how students explore and solve the problem. What connections can they make? Will they use calculus? A complete geometrical solution exists for the problem, students that recognise the geometrical approach should NOT be surprised that length AD = PD.

The solutions provided below use the scaffolded approach, students are lead through a particular journey. The advantage of this approach is that it allows students to practice specific skills. One interesting approach is to give half the class the scaffolded version and the others the opportunity to discuss and explore.

Question: 1.

Determine the equation to the circle.

Answer: $(x-1)^2 + y^2 = 1$

Question: 2.

Use your circle equation to determine a relationship between m and n. [Equation 1]

Answer: $(m-1)^2 + n^2 = 1$

Question: 3.

Determine the gradient of the circle in terms of m and n at the point P.

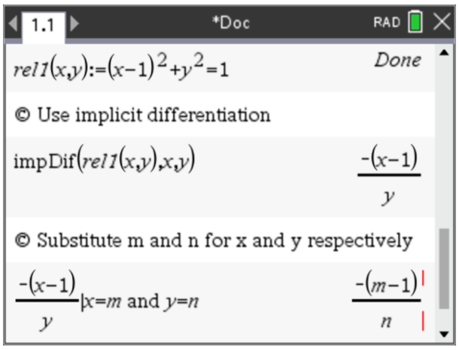
Answer: Several options here.

© Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.

Author: Peter Fox

Option 1: Transpose the circle equation to make y the subject; differentiate then substitute m and n. (Could use CAS)

Option 2: Implicit differentiation (Could use CAS – shown)



Option 3: Geometry – gradient is perpendicular to QP $\therefore\ \dfrac{1-m}{n}$

Question: 4.

Determine the gradient of the line DP in terms of m and n by consideration of the y intercept.

Answer: Line DP passes through (0, 2) & (m, n). Gradient = $\dfrac{n-2}{m}$

Question: 5.

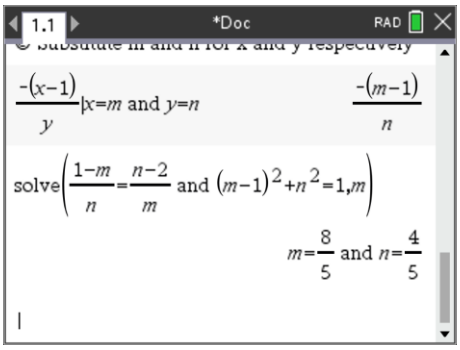
Combine the results from Q4 and Q5 to form a new equation. [Equation 2]

Answer: Equation 2 = $\dfrac{n-2}{m}=\dfrac{1-m}{n}$

Question: 6.

Use simultaneous equations to determine the values of m and n, hence determine the equation to the line DP.

Answer: Could use Solve on CAS (shown) then use translational form of a straight line: $y=-\dfrac{3}{4}\left(x-\dfrac{8}{5}\right)+\dfrac{4}{5}$



Question: 7.

Determine the length of segment DP and discuss the results from a Geometrical perspective.

Answer: DP = 2

Lines AD and DP are both tangent to the circle with point D in common, therefore length AD = DP.

AD = 2 therefore DP = 2. Furthermore:

∆ ADQ = ∆ PDQ. [Congruent – SSS]

∠PQB = 180° – 2 ∠AQD

Let R be on QB such that ∠QRP = 90°. [R is directly below P therefore has same x coordinate]

$$AR=1+\cos(180-2\angle AQD)$$

$$AR=2\sin^2(\angle AQD)$$

$$AR=2\times\left(\frac{2}{\sqrt{5}}\right)^2$$

$$AR=\frac{8}{5}$$

Therefore … P (8/5, 4/5)

Teacher Notes: What other ways can you solve this problem?

© Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.

Author: Peter Fox