Gateway Arc Length

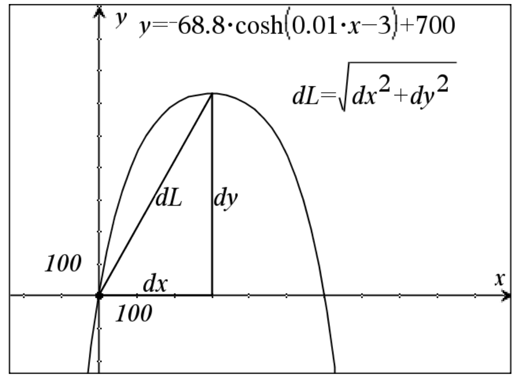
Student Activity

Name ____________________

Class ____________________

Part 1 – Arc Length Introduced

The Gateway to the West is an inverted catenary arch in St. Louis that is *approximately* 630 feet tall and 630 feet wide at its base. A catenary (a hyperbolic cosine function) is the shape that a chain or cable forms when it hangs between two points. For this activity, the shape of the Gateway Arch can be *approximately* modeled by the following equation: $y(x) = -68.8\cosh(0.01x - 3) + 700$.



1. If you were to ride in the elevator tram of the Gateway Arch, at least how far would you travel to get to the top? Explain.

Using the Pythagorean Theorem, we get $dL = \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2}\,dx.$ As dx becomes smaller, so do dy and dL. As dL becomes smaller, the difference in length of dL and the length of the arc from x to $x + dx$ is eventually infinitesimal. So we can integrate both sides to give us the formula for arc length:

$$L = \int_a^b \left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right)dx.$$

2. On the Home screen, use the formula to find the arc length from $x = 0$ to $x = 300$, (*approximately*), for $y(x) = -68.8\cosh(0.01x - 3) + 700$. Write the formula and answer. Is this reasonable (when compared to your answer from Exercise 1)?

For parametric equations, the Pythagorean Theorem would yield $dL = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$ Integrating both sides gives us the arc length formula $L = \int_a^b \left(\sqrt{(x'(t))^2 + (y'(t))^2}\right)dt.$

Graph the parametric equations $x(t) = 2\cos(t)$ and $y(t) = 2\sin(t)$.

3. For the parametric equation $x(t) = 2\cos(t)$ and $y(t) = 2\sin(t)$, use the arc length formula to find the length from $t = 0$ to $t = \frac{\pi}{2}$. Show each step.

Now graph the equation **y1**(x) = $\sqrt{4-x^2}$. When x = 0 to x = 2, this graph should look the same as the previous parametric curve.

4. Use the Home screen to find the arc length of **y1**(x) = $\sqrt{4-x^2}$ from x = 0 to x = 2. Write out the equation and answer. Does this agree with the previous answer? Why or why not?

5. Graph **y2**(x) = x^2 – 9 and approximate the arc length from x = 0 to x = 3. Write the arc length formula and solution for this arc length. Try using **arcLen(y2(*x*),*x*,0,3)** on the Home screen to check your answer.

6. Use the Pythagorean Theorem to approximate the arc length from x = 0 to x = 3 of $y=-x^2+\frac{5}{3}x+4$.

On the Home screen, find the arc length using the formula. Write the formula and solution. Discuss if this is reasonable.

Part 2 – Additional Practice

1. Which of the following integrals gives the length of the graph of $y = \sin^{-1}\frac{x}{2}$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

a. $\int_a^b \sqrt{\frac{x^2+8}{x^2+4}}dx$ **b.** $\int_a^b \sqrt{\frac{x^2+6}{x^2+4}}dx$ **c.** $\int_a^b \sqrt{\frac{x^2-2}{x^2-4}}dx$

d. $\int_a^b \sqrt{\frac{x^2-5}{x^2-4}}dx$ **e.** $\int_a^b \sqrt{\frac{2x^2+3}{x^2+1}}dx$

2. The length of the curve determined by the parametric equations $x = \sin t$ and $y = t$ from $t = 0$ to $t = \pi$ is

a. $\int_0^\pi \sqrt{\cos^2 t+1}dt$ **b.** $\int_0^\pi \sqrt{\sin^2 t+1}dt$ **c.** $\int_0^\pi \sqrt{\cos t+1}dt$

d. $\int_0^\pi \sqrt{\sin t+1}dt$ **e.** $\int_0^\pi \sqrt{1-\cos t}\,dt$

3. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

a. $\int_a^b \sqrt{x^2+\tan^2 x}\,dx$ **b.** $\int_a^b \sqrt{x+\tan x}\,dx$ **c.** $\int_a^b \sqrt{1+\sec^2 x}\,dx$

d. $\int_a^b \sqrt{1+\tan^2 x}\,dx$ **e.** $\int_a^b \sqrt{1+\sec^4 x}\,dx$