Name ________________________

Class ________________________

Problem 1 – Parametric Equations and Projectile Motion

Parametric graphs can be great for showing projectile motion. Page 1.3 shows a graph depicting the position of an object based on the parametric equations

$$\begin{cases} x1(t) = v_0 \cdot \cos(\theta) \cdot t \\ y1(t) = y_0 + v_0 \cdot \sin(\theta) \cdot t + \dfrac{g}{2} \cdot t^2 \end{cases}.$$

Explore the graph using the sliders to change the angle, θ, and the initial height, y_0. Click in the MathBox in the top left corner of the page to change the initial velocity, v_0. Use the **Graph Trace** tool to trace along the curve to find the *x* and *y* positions of the object at any time *t*.

1. When $\theta = 45°$, $g = –9.8$ m/s^2, $v_0 = 10$ m/s, and $y_0 = 0$ m, what are the *x* and *y* positions of the object moving along the curve when $t = 0.9$ seconds? Round to 3 decimal places.

Problem 2 – Polar Graphs using Parametric Equations

On page 2.1, review the two pairs of formulas that are important polar and parametric equations.

Page 2.2 shows a graph of $r(\theta) = 4\sin(n\cdot\theta)$ and a tangent line to the curve. Use the slider to change the coefficient, *n*, of θ. Observe the changes in the curve as you change the value of *n*. Drag the point on the curve. Observe the changes in the value of the slope of the tangent line at the bottom right of the graph.

2. Find the value of the derivative of $r(\theta) = 4\sin(3\cdot\theta)$ when $\theta = \dfrac{\pi}{6}$. Show the steps that lead to your solution.

Problem 3 –Parametric Equations in Three Dimensions

On page 3.1, read the scenario about the launch of an egg-carrying container. The conditions of the launch are:

- the initial position, z_0, of the projectile (the egg-carrying container) is 5.6 m
- the initial velocity, v_0, is 15 m/s
- the angle of elevation, θ, is 30°
- a cross breeze is blowing at a rate of –5 m/sec in the *y*-direction

Does the projectile hit the roof?

3. Given the conditions described, what is the **speed** of the projectile when t = 1 second? Round to 3 decimal places.

You have most likely learned that $a_x = \frac{d^2}{dt^2}(x)$ is the magnitude of the acceleration in 2D.

4. Write an expression that would represent the magnitude of the acceleration in 3D.

5. Given the following position equations, what is the magnitude of the acceleration at time t = 1 second?

$$\begin{cases} x(t) = 15\cos(30^\circ)\cdot t \\ y(t) = -5t \\ z(t) = -4.9t^2 + 15\sin(30^\circ)\cdot t + 5.6 \end{cases}$$

Problem 4 – Volumes of Solids of Revolution

On page 4.1, read about the process used to find the volume of a solid rotated about the x-axis.

On page 4.2, use the slider for c on the left side of the page to move the location of the orange disk. Observe the changes of the rectangle on the right side of the page. Select ctrl + tab to move to the right side of the page. Drag the movable point on the x-axis and observe the changes in the location and size of the disc on the left.

6. What is the volume of the solid formed by rotating $f(x) = 0.2x^2$ from $x = 1$ to $x = 3$ around the x-axis?

Extension

Problem 5 – More on Volumes of Solids of Revolution

On page 5.1, review the equations that will rotate the function around the *x*-axis and the *y*-axis.

On page 5.2, read the instructions on how to graph a parametric equation in the Graphs application in 3D. You will also see the formula for the volume of a graph rotated about the *x*-axis.

On page 5.3, enter the expressions for **xp2, yp2** and **zp2**. Be sure that you are graphing in parametric mode.

7. Show the setup of the integration and the solution for $f(x) = x^2$ rotated around the *y*-axis from $x = 0$ to $x = 2$.

Problem 6 – Arc Length in 3D

On page 6.1, read about the formula for arc length.

Scrolling down the page, read about the ways to calculate the arc length of the graph shown on page 6.2. After studying the solution to the arc length of the given curve, how does it compare to the value of $2\pi r$ when $r = 3$?