Math Objectives

- Students will identify the three defining characteristics of a normal curve related to shape, center, spread, and area.
- Students will recognize that normal curves form a family whose members share these same characteristics.
- Students will use appropriate tools strategically (CCSS Mathematical Practices).
- Students will model with mathematics (CCSS Mathematical Practices).

Vocabulary

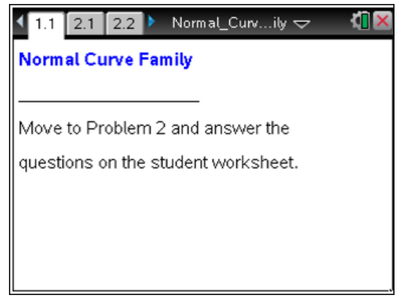
- normal curve
- mean
- standard deviation
- point of inflection
- density functions

About the Lesson

- This lesson involves investigating the relationship of the equation of a normal curve to its graph.
- As a result, students will:
  - Identify the axis of symmetry as the line *x* =, the mean of the distribution represented by the normal curve, and the standard deviation as the distance from that line to the point of inflection.
  - They will use a slider to change the values of two parameters, μ and σ, to investigate their effects on the normal curve, noting in particular that μ represents the location of the mean and that σ represents the distance from the mean to the curve at the point of inflection.
  - Estimate the area under a normal curve graphed on a coordinate grid and investigate how this area changes as the mean and standard deviation are changed.

TI-Nspire™ Navigator™ System

- Send the .tns file to students.
- Use Screen Capture or Quick Poll to examine the values of *x* that make the expression or equation true.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Normal_Curve_Family_Student.pdf
- Normal_Curve_Family_Student.doc

TI-Nspire document

- Normal_Curve_Family.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: In this activity, students will not move the two points on the curve manually. To change the value of μ or σ, students should move the cursor to the up or down arrow and press when the arrow becomes shaded.

Teacher Tip: The initial discussion refers only to the parameters μ and σ and not to the characteristics of a distribution. That comes into the investigation at Question 7, where the curve is connected to the mean and standard deviation of a distribution. A description of the familiar situation of graphing a line and knowing the effects of the parameters *m* and *b* might help students see more clearly what is happening in the current activity.

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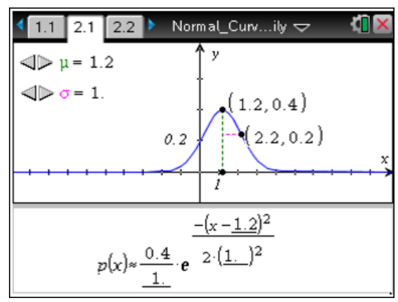

1. The distributions of many real-world variables can be closely approximated by a normal distribution. The equation of a normal curve is approximately $p(x) \approx \frac{0.4}{\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, where μ is the mean and σ the standard deviation.

   a. Describe the shape, center, and spread of the curve.

   Sample Answers: The curve is mound-shaped and seems to be symmetric to a line through the middle. The center of the curve is at *x* = 1. The spread seems to go from negative infinity to positive infinity, but the curve is bunched up mostly between –1.5 and 3.5 or –2 and 4. The maximum point, or height, of the curve seems to be about 0.4 at *x* = 1.

Teacher Tip: The actual equation for the normal curve is $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$. If you show the equation to students be sure they recognize that π and *e* are constants. Students might be asked to determine how the equation explains the behavior of the graph as (*x*) as *x* approaches infinity. They should observe that the number *e* to a negative power is really 1 over *e* to that power, so as *x* increases, the fraction becomes 1 divided by a very large number. That number will get closer and closer to 0, and the product of a number very close to 0 and 0.4 will also get increasingly closer to 0.

b. Find $p(1)$ when $\mu = 1$ and $\sigma = 1$. Explain how this point relates to the graph.

Answer: $p(1) = 0.4$, which represents the height of the curve. The point (1, 0.4) is the maximum point on the curve.

c. Use the arrows to change μ and σ. Describe the changes in the graph of the normal curve.

Answer: The graph of the curve shifts horizontally when μ is changed, moves to the right when μ increases, and moves to the left as μ decreases. When σ is changed, the height of the graph changes.

2. The point at which a graph changes from concave up to concave down is called the **point of inflection**. How far is the point of inflection from the center of the graph? Explain how you know.

Answer: The distance from a point to a line is the perpendicular distance from the point to the line, which is represented by the horizontal segment in the graph. The segment is about 1 unit long. Using the displayed ordered pairs, you can subtract the *x*-coordinates.

Teacher Tip: To help students understand the point of inflection, you might ask them how they would construct the curve using parts of curves they already know. They might suggest using part of a parabola that opens down and part of an exponential curve. The point at which the two would be glued together is the point of inflection. It can also be described as the point at which the curve, like a bowl with sloped edges, switches from "spilling water" to "holding water." Stress, however, that the curves in the normal graph are not really parabolas.

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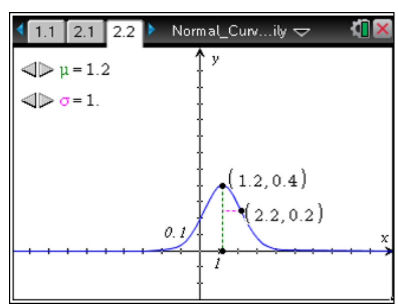

3. a. Two characteristics of this curve are the maximum point (center) and the distance from the center to the point of inflection (measure of spread). Use the arrows to change μ and σ. Describe how the parameters in the equation affect the maximum point and why.

Answer: The height of the maximum point is affected by changing σ. The curve gets "squeezed up" as *s* gets smaller. The width is also affected by changing σ; a smaller σ means the point of inflection is closer to the mean. The horizontal location of the maximum point is affected by changing μ.

b. Predict the center, shape, and spread of the curve if $\boldsymbol{\mu}$ **= 3** and $\boldsymbol{\sigma}$ **= 2**. Verify your prediction using the sliders.

Answer: The center of the curve will be at *x* = 3 on the horizontal axis, the axis of symmetry will be the line *x*=3. The curve will be flatter than when the standard deviation is 1, and the tails will still approach but not touch the *x*-axis. The distance from the axis of symmetry to the point of inflection is 2.

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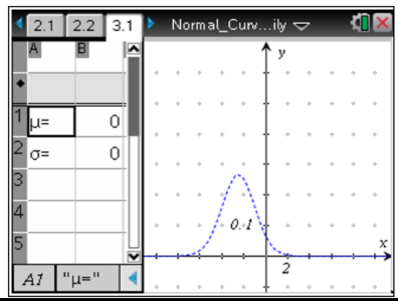

4. Consider the dashed curve.
   a. Predict the values for μ and σ that were used to create the graph. Explain why you think your prediction makes sense.

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 1 at the end of this lesson.

Sample Answers: Students might suggest the values are about μ = –2.5 and σ = 1.5. The value for μ is the *x*-coordinate of the maximum point on the curve, and the distance from the value of μ to the point of inflection is the value of σ.

b. Verify the predictions by typing values into Column B of the spreadsheet.

Answer: μ = –2.5 and σ = 1.5

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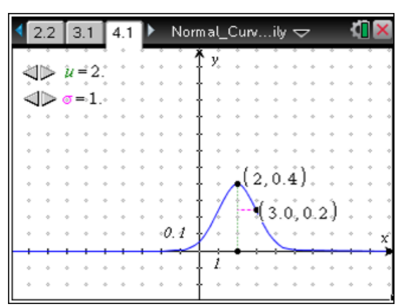

5. a. Describe the axis of symmetry for the curve.

Answer: The axis of symmetry is a line that divides the curve into two congruent parts. In this case, it is the vertical line that goes through *x* = μ.

b. What happens to the axis of symmetry as μ and σ change?

Answer: The axis of symmetry is not affected at all by a change in σ. It is determined by μ; the line of symmetry is the vertical line $x = \mu$.

6. a. The length of the segment connecting the point of inflection and the axis of symmetry represents the standard deviation. Describe the changes in the graph as the standard deviation increases.

Answer: As the standard deviation increases, the graph becomes flatter and flatter, with the *y*-coordinate of the peak decreasing proportionately with increases in the value of σ.

b. Compare a normal curve with a mean of –2 and a standard deviation of 1 to a normal curve with a mean of 1 and a standard deviation of 1.

Answer: The curves are congruent. The curve with a mean of 1 could be translated 3 units to the left, and it would match the curve with a mean of –2 exactly.

7. a. Calculate the area of one grid box, and then count boxes to approximate the area between the curve and the horizontal axis when $\mu = 0.6$ and $\sigma = 1.8$.

Answer: The area of one grid box is 0.05 by 1 or 0.05 square units. By counting and estimating, the number of rectangles under the curve is about 19, so the total area under the curve is about 0.95 plus the small parts under the tails as the *x*-values continue to increase or decrease, for a total area of about 1.

> **Teacher Tip:** Students might need to be reminded of the extra area that is not clearly visible and that accumulates as x goes to plus and minus infinity. A smaller grid might be a way to help them get a better estimate, which can be done by going to **MENU > Window/Zoom > Window Settings** and resetting the *x*-scale to a smaller value, e.g., 0.5 or 0.25.

b. Change the value of μ. Predict the total area between the curve and the horizontal axis. Verify by counting the boxes.

Answer: The value of μ will not change the area because the shifted curve will be congruent to the original, so areas bounded by the curve and the horizontal axis for both curves will be equal.

c. Reset μ to 0, and change the value of σ to 0.5. Use the grid boxes to approximate the area between the curve and the horizontal axis.

Answer: Again, each rectangle formed by the grid has an area of 0.05, and there are about 19 rectangles plus the extra as the curve continues along the *x*-axis, so the total area is about 1 square unit.

d. Change σ to a new value. Predict the area between the curve and the horizontal axis. Verify by counting the boxes.

Answer: The area continues to be about 1 square unit.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

8. A normal curve has three defining characteristics. What are these characteristics and how can you recognize them in a graph?

Answer: (i) The *x*-coordinate of the maximum point is the mean of the distribution. The vertical line through the mean is the axis of symmetry for the curve.

(ii) The standard deviation of the distribution is the distance from the vertical line through the mean to the point of inflection.

(iii) The total area under any normal curve is always 1 square unit. You can divide the area under the curve into rectangles or squares and approximate the area.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- A normal curve has three defining characteristics:
  - *x* equals the mean, which defines an axis of symmetry and the maximum value of the function.
  - The standard deviation occurs at the point of inflection.
  - The area under the curve is 1.
- Normal curves form a family of curves whose members share these same characteristics.

Assessment

Identify the following as true or false. Be prepared to explain your reasoning in each case.

a. A normal curve can be short and flat or tall and skinny.

Answer: True; a large value of σ creates a short, flat curve, and a small value of σ creates a tall, skinny curve.

b. You can make two normal curves so that one would fit completely inside the other.

Answer: False; the area under every normal curve is 1 square unit, so it is impossible to have one normal curve completely under another.

c. It is possible to create a normal curve such that the area under the curve is more than 1.

Answer: False; the area under every normal curve is 1 square unit.

d. All normal curves are symmetric.

Answer: True; the curves are symmetric around the line x = the mean.

e. There is some value of x such that $(x, 0)$ is on the normal curve.

Answer: False; if this were true, the curve would touch the *x*-axis, but that is impossible given the equation is about 0.4 over e to a power, which is never 0.

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Note 1

Question 4 part a, *Quick Poll*

Use Quick Poll to gather students' predictions for the values of μ and σ. Tip: Poll the variables one at a time by having them enter just the number. This will minimize the variance in the responses.
Pick a value from those submitted and discuss with students why they think it will or will not match the curve graphed.

Note 2

Question 8, *Quick Poll*

To assess students understanding of the area under a normal curve, send one or several open response Quick Polls. For example, "What is the area under a normal curve with μ = 5 and σ = 2?" Students should always response with "1" or "approximately 1".