Expanding Binomials
ID: 11138

Activity Overview
In this activity, students will explore the link between Pascal's Triangle and the expansion of binomials in the development of the Binomial Theorem.

Topic: Sequences & Series
- *Combinations*
- *Pascal's Triangle*
- *Binomial Theorem*

Teacher Preparation and Notes
- *To reduce time required for this activity, consider dividing up the tasks among students and sharing results on a board or projection screen.*

- *Consider the following ideas as extensions to this activity:*

  o *Consider exploration of Bernoulli probability experiments as a related concept.*

  o *Explore the additional number patterns studied by Blaise Pascal.*

  o *Relate the given activity to a binomial with two variables, $(ax + by)^n$*

- ***To download the student worksheet, go to education.ti.com/exchange and enter "11138" in the keyword search box.***

Associated Materials
- *ExpandingBinomials_Student.doc*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.
- *Exploring the Binomial Theorem (TI-89 Titanium) — 5376*
- *Using the TI-83/84 to Explore the Binomial Theorem (TI-84 Plus family) — 5768*

Introduce the activity and discuss the pattern observed in Pascal's Triangle on the worksheet. Students will then explore how this pattern relates to combinations by completing the right side of the screen.

At this point, a review of the meaning of the combination notation as well as alternative notation may be discussed as needed.
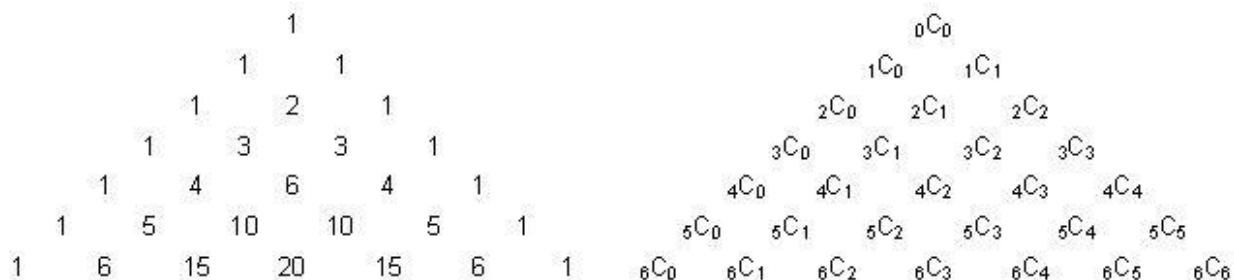

Problem 1 – Exploring $(x + b)^n$

You can show students on the Home screen that any combination will equal the number in the same position in Pascal's Triangle. (i.e., $_2C_1 = 2$). Encourage students to test a few of these combinations to verify that they indeed represent the pattern in Pascal's Triangle.
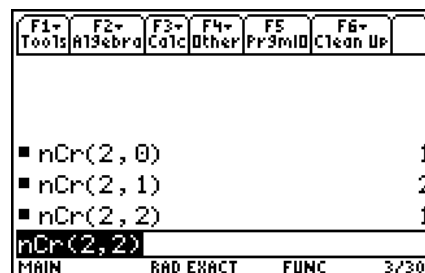
On the worksheet, students are to use the **Expand** command to expand a very basic binomial to see that the resulting pattern is connected to Pascal's Triangle. Remind them that even though the calculator does not show it, the first and last term have a coefficient of 1. Students should record the expansions on their worksheet.
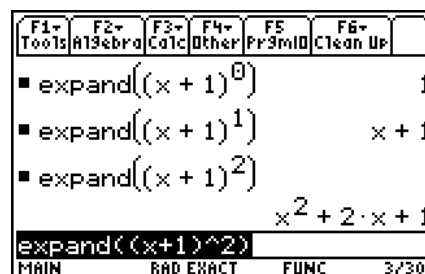
Discuss with students how the exponent of the binomial, before expanded, relates to the row number of Pascal's triangle.

Problem 2 – Exploring $(ax + 1)^n$

Students are to use the Home screen again to expand the binomials given. The binomials include one with specific integers and then one with the coefficient of x replaced with *a*. Explain to students that when a and x have the same exponent, they can be combined, a^2x^2 can be rewritten as $(ax)^2$. They should see that *a* affects the coefficient whenever there is an *x* term.

```
F1▾  F2▾   F3▾  F4▾   F5    F6▾
Tools Algebra Calc Other PrgmIO Clean Up
■ expand((a·x + 1)⁰)                1
■ expand((a·x + 1)¹)         a·x + 1
■ expand((a·x + 1)²)
                    a²·x² + 2·a·x + 1
expand((a*x+1)^2)
MAIN      RAD EXACT     FUNC     3/30
```

After exploring the patterns in the binomials, students are asked to use what they've learned to expand $(ax +1)^4$ by hand with the numbers from Pascal's Triangle and by using combination notation for the binomial coefficients.

Problem 3 – Exploring $(ax + b)^n$

Once students have explored the effects of *a* and *b* separately, they will expand specific and general binomials with *a* and *b* together.

```
F1▾  F2▾   F3▾  F4▾   F5    F6▾
Tools Algebra Calc Other PrgmIO Clean Up
■ expand((3·x + 2)⁰)                1
■ expand((3·x + 2)¹)         3·x + 2
■ expand((3·x + 2)²)
                    9·x² + 12·x + 4
expand((3x+2)^2)
MAIN      RAD EXACT     FUNC     3/30
```

Students will rewrite the expanded binomials from question 11 using combination notation. They can refer back to the first page of the worksheet if necessary. Encourage students to compare their results with a partner. This will help students identify errors and explore the resulting patterns that develop.

Students are to then write the general formula for an expanded binomial $(ax + b)^n$, otherwise known as the Binomial Theorem.

```
F1▾  F2▾   F3▾  F4▾   F5    F6▾
Tools Algebra Calc Other PrgmIO Clean Up
■ expand((a·x + b)⁰)                1
■ expand((a·x + b)¹)         a·x + b
■ expand((a·x + b)²)
                    a²·x² + 2·a·b·x + b²
expand((a*x+b)^2)
MAIN      RAD EXACT     FUNC     3/30
```

The worksheet gives four extra problems that students can complete if there is extra time at the end of class or for homework. They should do it by hand and then check the work with the handheld.

Extra Problems

Students are to use the Binomial Theorem to expand the binomials given on the worksheet.

1. $(6x + 1)^5 = (6x)^5 + 5(6x)^4(1) + 10(6x)^3(1)^2 + 10(6x)^2(1)^3 + 5(6x)(1)^4 + 1^5$

    $= 7{,}776x^5 + 6{,}480x^4 + 2{,}160x^3 + 360x^2 + 30x + 1$

2. $(x + 7)^6 = (x)^6 + 6(x)^5(7)^1 + 15(x)^4(7)^2 + 20(x)^3(7)^3 + 15(x)^2(7)^4 + 6(x)^1(7)^5 + 7^6$

    $= x^6 + 42x^5 + 735x^4 + 6{,}860x^3 + 36{,}015x^2 + 100{,}842x + 117{,}649$

3. $(3x + 5)^4 = (3x)^4 + 4(3x)^3(5) + 6(3x)^2(5)^2 + 4(3x)(5)^3 + 5^4$

    $= 81x^4 + 540x^3 + 1{,}350x^2 + 1{,}500x + 625$

4. $(7x + 4)^8 = (7x)^8 + 8(7x)^7(4)^1 + 28(7x)^6(4)^2 + 56(7x)^5(4)^3 + 70(7x)^4(4)^4 + 56(7x)^3(4)^5 +$
    $28(7x)^2(4)^6 + 8(7x)^1(4)^7 + 4^8$

    $= 5{,}764{,}801x^8 + 26{,}353{,}376x^7 + 52{,}706{,}752x^6 + 60{,}236{,}288x^5 + 43{,}025{,}920x^4 +$
    $19{,}668{,}992x^3 + 5{,}619{,}712x^2 + 917{,}504x + 65{,}536$