According to the Standards:

Instructional programs from preK-grade 12 should enable students to:

- Recognize and use connections among mathematical ideas
- Make and investigate mathematical conjectures

In grades 9-12 students should

1. Students should develop an increased capacity to link mathematical ideas and a deeper understanding of how more than one approach to the same problem can lead to equivalent results.

Calculus Scope and Sequence: Applications of Derivatives **Keywords:** Rolle, Rolle's theorem **Description:** This activity will illustrate Rolle's Theorem

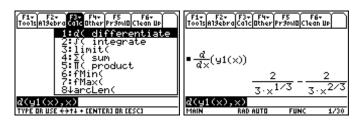
Rolle's Theorem: If f, is a function continuous over a closed interval [a,b], differentiable on the open interval (a,b) and f(a)=f(b)=0, Then there is at least one value of c, in (a,b) such that f'(x)=0

Determine whether or not the hypothesis of Rolle's Theorem holds on the following function: $f(x) = x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$ for [0,8]

1. First input the function into Y= and observe it's graph over the interval (it's always a good idea to set the window a bit larger than the interval to observe behavior around the endpoints)

$\begin{array}{c} \hline f_{1+} & f_{2+} & f_{3} & f_{4} & f_{5} \\ \hline Troots 200m [Edit] & f_{11} & f_{5} & f_{6} & f_{5} \\ +PLOTS & & & & \\ & & & & \\ & & & & \\ & & & & $	F1+Y F2+ Tools200m xmin=-2. xmax=10. xscl=1. ymax=5. yscl=1. xres=2	Tools Zoom Trace Regraph Math Draw Pen C
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- 2. While the function is continuous it would also be good to look at the derivative:
- Go to the Homescreen (Press HOME)
- Go to F3-Calc-#1
- Calculate the derivative function



- 3. This can be entered into the Y= menu as follows
- Go to Y=, choose an empty slot (y2 in this case)
- Press 2^{nd} -Ans then ENTER

That will paste the exact result into the proper slot

• Look at the graph

F1+ F2+13 85 85 - 55 ToolsZoom: 35 / 85 - 55 (855 - 5) -PLOTS	F1+ F2+ F3 F4 F5+ F6+ 5" ToolsZoomEdit / A11Style(3003) +PLDTS	F1+ F2+ F3 F4 F5+ F6+ F7+8:: Too1sZoomTraceReGraphMathDrawPen:-C
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<u> </u>	<u>45=</u> y3(x)= Main rad auto func	MAIN RAD AUTO FUNC

It would appear that the derivative might be undefined at x = 0 let's look at the table for confirmation:

- Go to TableSet and set the values in a small neighborhood of 0
- Look at the TABLE

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Since Rolle's Theorem says it must be differentiable on the open interval we have satisfied the entry points. So, now let's see if the rest of the condition applies:

- Calculate y1(0)
- Calculate y1(8)
- If they are both = 0, then find a value where the derivative (here in $y_2(x)$) is = 0

All this can be done on the Homescreen:

F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9miDClean Up	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl3ebraCalcOtherPr3mIOClean UP
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$\frac{2}{3 \cdot x^{1/3}} - \frac{2}{3 \cdot x^{2/3}}$	
■y1(0) 0	• y1(8) 0
y1(0)	y1(8)
MAIN RAD AUTO FUNC 2/30	MAIN RAD AUTO FUNC 3/30

- To Solve we go to F2-Algebra #1 and enter the following arguments: o (function = 0, variable)
- In this case the derivative function we are using is in $y_2(x)$

F1+ F2+ F3+ F4+ F5 F6+ Tools Al3ebra Calc Other Pr3miDClean Up	F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIDClean Up
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4:zeros(5:approx(6:comDenom(0)	■y1(0) 0 ■y1(8) 0
= 91 7:propFrac(0 ■ y1 8↓nSolve(0	■ solve(y2(x) = 0, x) x = 1 solve(y2(x)=0, x)
TYPE OR USE ++++ CENTER] OR CESC]	MAIN RAD AUTO FUNC 4/30

Therefore, we have confirmed Rolle's theorem for this example.