What's Your Mileage?

Overview

Students use linear equations to model and solve real-world problems. Students also see the correlation between the graph of an equation and its calculated slope by plotting graphs by hand and then calculating slopes with the calculator and comparing.

Math Concepts

- slope
- fractions
- graphing
- multiple representations of numbers

Materials

- TI-30XS MultiView™

Activity

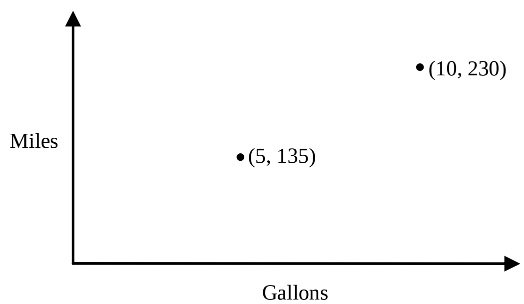
Begin by discussing slope as a constant rate of change in real-world situations.

In addition to the invoice, or sticker, price, many new cars have information posted about their miles per gallon for both city driving and highway driving. This is important information to have before purchasing a car, because it reflects how far drivers can go on a tank of gas and how much money driving the car will cost.

For example, a car manufacturer claims a specific car gets an average of 19 miles per gallon in the city and 27 miles per gallon on the highway. Assume a driver begins a trip with 10 gallons of fuel. If the manufacturer's claims are accurate and the driver first travels 135 miles on the highway, how far will the driver then be able to travel in the city?

Start with a smaller part of the problem, and graph the data you have. Note: The graph will be *blank* when you use it with the students. The coordinates are provided below for teacher guidance only.

Let's plot the data we have and then interpret the graph. We know that after 135 miles, we've used 5 gallons and that there are 10 gallons total.



Use the blank graph provided at the end of the activity on a projector. Ask the students to help plot the individual data points. A rough sketch of the graph is provided above for teacher guidance.

Note: This activity assumes that the car gets 27 mpg constantly on the highway and 19 mpg constantly in the city. Discuss with the students that this is not exactly realistic, given that a car's fuel consumption depends upon acceleration, driving speed, weather conditions, use of air conditioning, etc. However, we assume constant fuel consumption rates here for the sake of discussion.

When all 10 gallons are gone, how many total miles will have been driven if the manufacturer's claims are accurate?

Why is the graph steeper for the first 5 gallons than the second 5 gallons?

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This leads to a good discussion about slope.

Slope is defined as a constant rate of change. For instance, in our example, we are interpreting slope as a rate of change relating miles and gallons of fuel. The first constant rate of change was 27 miles per gallon, which means that for every 1 gallon, the car can go 27 miles, assuming constant conditions. Then, the second constant rate of change was 19 miles per gallon, meaning for every 1 gallon, the car can go 19 miles, again assuming constant conditions.

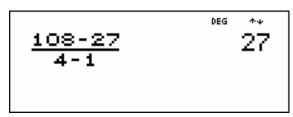
In this example, we were given the rates of change from the beginning. However, that will not always be the case. Let's select two points from our graph and see how to compute slope if it's not given to us from the beginning.

Have the students select two points from the graph created in class. Two points have been chosen here, but this formula will work regardless of the points chosen.

I'll select (1, 27) and (4, 108). We're looking for number of miles per gallon. The formula for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$, *where* (x_1, y_1) *and* (x_2, y_2) *are the two points. Let's use the calculator.*

Follow these steps:

1. Press $\frac{n}{d}$ 108 − 27.
2. Press ⊖ 4 − 1 ⊳.
3. Press enter.
4. The calculator should display:



In this example, we see that the slope is 27, which agrees with the information given.

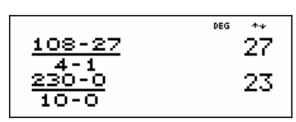
Now, go back to the plotted graph, and calculate the average number of miles per gallon for the entire trip. Draw the straight line from the origin to (10, 230) to illustrate that we're looking at the entire trip.

This driver went 230 miles on 10 gallons of gas. The average fuel consumption can be calculated using (0, 0) and (10, 230) as our two points. We can edit our previous calculator entry to find the average.

This car got an average of 23 miles per gallon.

Follow these steps:

1. Press ⊕ ⊕ enter to copy the previous expression.
2. Then press ⊲ repeatedly to highlight and change.
3. Press enter.
4. The calculator should display:



Again, discuss the meaning. The car did not necessarily travel 23 miles for each gallon of fuel, but on average, taking into account the highway mileage and city mileage given by the manufacturer, 23 miles per gallon was the average fuel consumption.

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Show other examples of finding the slope without having a graph to reference.

If a line passes through the points (0, –4) and (5, 7), what is its slope?

First, to reinforce the idea that slope is a rate of change, plot the two points on a coordinate grid and discuss the line through both points.

Then, again use the recall features of the TI-30XS MultiView to edit the previous key sequence.

Also, it is important to show students that either point could be (x_1, y_1) or (x_2, y_2).

The rate of change, or slope, will be the same regardless of which point you use for (x_1, y_1) or (x_2, y_2).

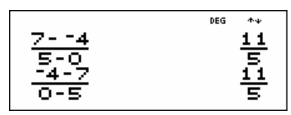
Follow these steps:

1. Press ▲ ▲ enter to copy the previous expression.
2. Then press ◄ repeatedly to highlight and change.
3. Press enter.
4. The calculator should display:

Again, use the calculator to illustrate this.

Follow these steps:

1. Press ▲ ▲ enter to copy the previous expression.
2. Then press ◄ repeatedly to highlight and change.
3. Press enter.
4. The calculator should display:



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Name ______________________

Date ______________________

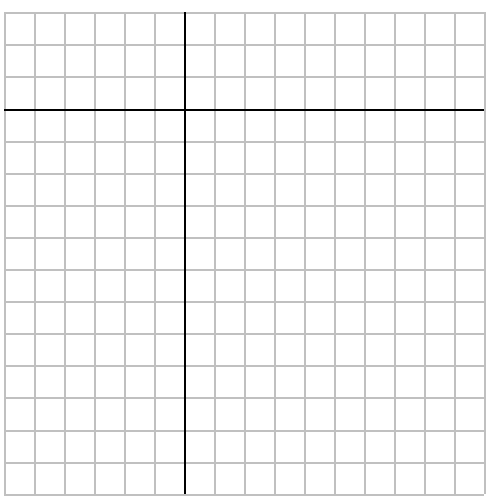
1. Find the slope of the line passing through (–1, 3) and (5, 6).

2. Find the slope of the line passing through (3, –2) and (–1, 5).

3. A 2007 truck has a suggested retail price of $31,540. The 2005 model had a suggested retail price of $30,290. Calculate the slope of the line that would contain these data points. What is the meaning of slope in this situation?

4. A compact car gets an average of 37 miles per gallon on the highway, and 24 miles per gallon in the city. Under constant conditions on the highway, on average, how far could this car go with 8 gallons of gas?

5. Graph the line passing through the points (–2, –9) and (6, –7).



6. Estimate or mentally calculate the slope by looking at your graph for problem 5.

7. Now calculate the slope of the graphed line in problem 5 using the slope formula.

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Name ______________________

Date ______________________

8. A man skydiving jumps from an airplane at 10:30 a.m. from an altitude of 7,500 feet. He is at an altitude of 3,000 feet at 10:34 a.m. Assuming the rate of the fall is constant, find the average rate of change from 10:30 to 10:34 a.m. Use the slope formula to find this rate. What is the meaning of slope in this situation?

9. Graph problem 8. If the skydiver continues to fall at a constant rate, at approximately what time will he hit the ground?

10. Is the slope in problem 8 positive or negative? Explain what that means in this situation.

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Answer Key

1. Find the slope of the line passing through (–1, 3) and (5, 6).

   $\frac{1}{2}$

2. Find the slope of the line passing through (3, –2) and (–1, 5).

   $\frac{7}{-4}$

3. A 2007 truck has a suggested retail price of \$31,540. The 2005 model had a suggested retail price of \$30,290. Calculate the slope of the line that would contain these data points. What is the meaning of slope in this situation?

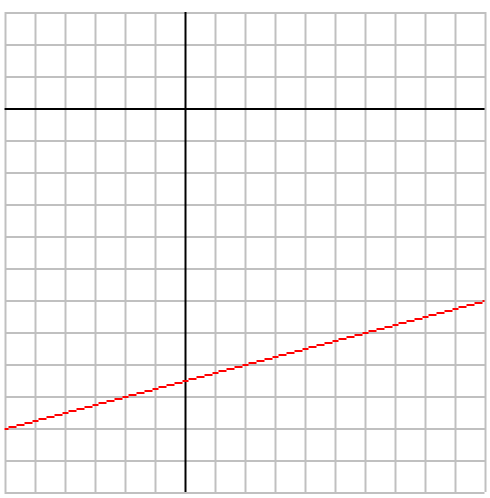
   $\frac{625}{1}$

   The price of the truck increased, on average, at a constant rate of \$625 per yr.

4. A compact car gets an average of 37 miles per gallon on the highway and 24 miles per gallon in the city. Under constant conditions on the highway, on average, how far could this car go with 8 gallons of gas?

   296 miles

5. Graph the line passing through the points (–2, –9) and (6, –7).



6. Estimate or mentally calculate the slope by looking at your graph for problem 5.

   Answers will vary.

7. Now, calculate the slope of the graphed line in problem 5 using the slope formula.

   $\frac{2}{8}$ or $\frac{1}{4}$

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8. A man skydiving jumps from an airplane at 10:30 a.m. from an altitude of 7,500 feet. He is at an altitude of 3,000 feet at 10:34 a.m. Assuming the rate of the fall is constant, find the average rate of change from 10:30 to 10:34 a.m. Use the slope formula to find this rate. What is the meaning of slope in this situation?

   $\frac{-4{,}500 \text{ ft}}{4 \text{ sec}}$ or $\frac{-1{,}125 \text{ ft}}{1 \text{ sec}}$

   The slope describes the skydiver's rate of descent in feet per second.

10. Is the slope in problem 8 positive or negative? Explain what that means in this situation.

    The slope is negative because the skydiver is *falling* at a rate of 1,125 feet per second.

9. Graph problem 8. If the skydiver continues to fall at a constant rate, at approximately what time will he hit the ground?

   Approximately 10:37 a.m. (Somewhere between 10:36 and 10:37 a.m. would be an acceptable answer.)

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Transparency