Investigation 2

Adding and Subtracting Fractions

Knowing how to combine and separate quantities is helpful in understanding the world around you. The mathematical names for combining and separating quantities are *adding* and *subtracting*.

For example, if you own two acres of land and you buy another half-acre lot, you will have $2 + \frac{1}{2}$, or $2\frac{1}{2}$, acres of land. The number sentence that shows this relationship is:

$$2 + \frac{1}{2} = 2\frac{1}{2}$$

The *sum* refers to the $2\frac{1}{2}$ acres of land you own.

If you then sell $\frac{3}{4}$ of an acre of your land, you will own $2\frac{1}{2} - \frac{3}{4}$ acres of land. The number sentence that shows this relationship is:

$$2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$$

The *difference* refers to the $1\frac{3}{4}$ acres of land you will own.

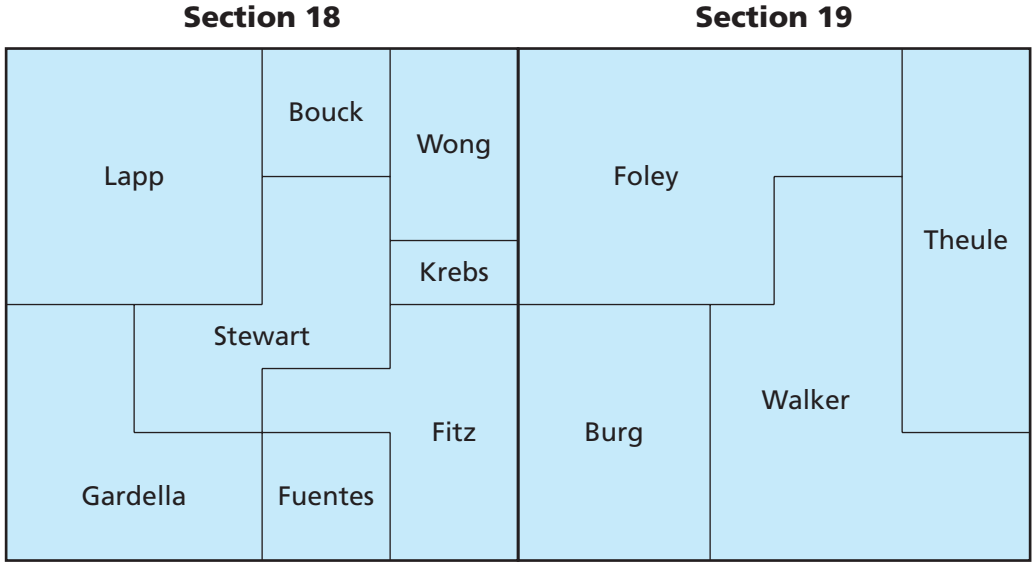
The problems in this investigation require you to add and subtract fractions. As you work, use what you have learned in earlier units and investigations about fractions and finding equivalent fractions. Practice writing number sentences to communicate your strategies for solving the problem.

2.1 Land Sections

TEKS / TAKS **6(2)A** Model addition and subtraction situations involving fractions with pictures and words.
6(2)B Use addition and subtraction to solve problems involving fractions.

When Tupelo Township was founded, the land was divided into sections that could be farmed. Each *section* is a square that is 1 mile long on each side. In other words, each section is 1 square mile of land. There are 640 acres of land in a square-mile section.

The diagram below shows two sections of land that are *adjacent*, or side by side. Each section is divided among several owners. The diagram shows the part of a section each person owns.



Problem 2.1 Writing Addition and Subtraction Sentences

A. What fraction of a section does each person own? Explain.

B. Suppose Fuentes buys Theule's land. What fraction of a section will Fuentes own? Write a number sentence to show your solution.

C. 1. Find a group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land. Write a number sentence to show your solution.

2. Find another group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land.

D. 1. Bouck and Lapp claim that when their land is combined, the total equals Foley's land. Write a number sentence to show whether this is true.

2. Find two other people whose combined land equals another person's land. Write a number sentence to show your answer.

3. Find three people whose combined land equals another person's land. Write a number sentence to show your answer.

E. How many acres of land does each person own? Explain your reasoning.

F. Lapp and Wong went on a land-buying spree and together bought all the lots of Section 18 that they did not already own. First, Lapp bought the land from Gardella, Fuentes, and Fitz. Then Wong bought the rest.

1. When the buying was completed, what fraction of Section 18 did Lapp own?
2. What fraction of Section 18 did Wong own?
3. Who owned more land? How much more land did he or she own?

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2.2 Visiting the Spice Shop

TEKS / TAKS **6(2)A** Model addition and subtraction situations involving fractions with objects and numbers. **6(2)B** Use addition and subtraction to solve problems involving fractions.

All over the world cooks use spices to add flavor to foods. Because recipe ingredients are often measured using fractions, cooking can involve adding and subtracting fractional quantities.

Reyna owns a spice shop in Tupelo Township. Some of her recipes are shown below.

Spice Parisienne

$\frac{2}{5}$ oz ground cloves

$1\frac{1}{5}$ oz ground nutmeg

$1\frac{1}{5}$ oz ground ginger

$1\frac{1}{10}$ oz cinnamon

Garam Masala

$\frac{2}{3}$ oz cinnamon

$6\frac{1}{2}$ oz cardamom

$2\frac{1}{2}$ oz cumin

$\frac{1}{3}$ oz ground cloves

$\frac{2}{3}$ oz coriander

$2\frac{3}{4}$ oz black pepper

Grind all spices together with a mortar and pestle or in a coffee mill.

Betty's Fruitcake Spices

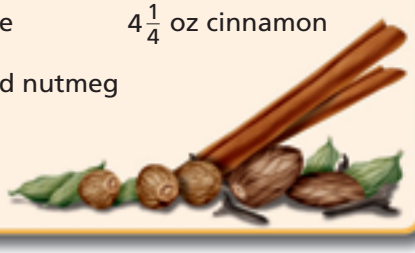
$1\frac{1}{8}$ oz cardamom

$2\frac{1}{2}$ oz allspice

$2\frac{5}{8}$ oz ground nutmeg

$\frac{5}{8}$ oz ground cloves

$4\frac{1}{4}$ oz cinnamon



Problem 2.2 Using Addition and Subtraction

Use number sentences to show your thinking.

A. Latisha buys the spices to make one batch of Spice Parisienne.

1. How many ounces of spice does Latisha buy?

2. a. Suppose she already has the nutmeg at home. How many ounces of spice does she buy?

b. Show a way to determine the answer using subtraction.

B. Ms. Garza buys spices to make one batch of Garam Masala.

1. How many ounces of spice does Ms. Garza buy?

2. a. Suppose she already has enough cinnamon and coriander at home. How many ounces of spice does she buy?

b. Show a way to determine the answer using subtraction.

C. Betty buys spices for her famous fruitcake.

1. How many ounces of spice does Betty buy?

2. Betty makes the fruitcake but forgets the nutmeg! How many ounces of spice does she actually use?

3. Tevin is allergic to cinnamon. If Betty removes cinnamon from the recipe for him, how many ounces of spice does she buy?

D. Use what you have learned to find the value for N that makes each sentence correct.

1. $1\frac{2}{3} + 2\frac{7}{9} = N$

2. $\frac{2}{5} + \frac{1}{4} = N$

3. $2\frac{3}{4} - 1\frac{1}{3} = N$

4. $3\frac{1}{6} - 1\frac{3}{4} = N$

5. $N + \frac{3}{4} = 1\frac{1}{2}$

6. $2\frac{2}{3} - N = 1\frac{1}{4}$

E. Describe a good strategy for adding and subtracting mixed numbers.

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2.3 Just the Facts

TEKS / TAKS **6(12)B** Communicate mathematical ideas using numerical and algebraic mathematical models. **6(13)A** Make conjectures from patterns.

In Problem 2.2, you wrote an addition or subtraction sentence to show a calculation you did. For each addition sentence you write, there are three related number sentences that show the same information.

addition sentence: $2 + 3 = 5$
related number sentences: $3 + 2 = 5$
$5 - 2 = 3$
$5 - 3 = 2$

These four number sentences form a **fact family.**

You can also create fact families with fractions. For example, $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ has these three related number sentences:

$$\frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$
$$\frac{7}{8} - \frac{3}{4} = \frac{1}{8}$$
$$\frac{7}{8} - \frac{1}{8} = \frac{3}{4}$$

You can write this entire fact family using eighths by changing $\frac{3}{4}$ to $\frac{6}{8}$. It looks like this:

$$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$$
$$\frac{1}{8} + \frac{6}{8} = \frac{7}{8}$$
$$\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$$
$$\frac{7}{8} - \frac{1}{8} = \frac{6}{8}$$

Problem 2.3 Fact Families

A. For each number sentence, write its complete fact family.

1. $\frac{2}{3} + \frac{5}{9} = \frac{11}{9}$

2. $\frac{5}{10} - \frac{2}{5} = \frac{1}{10}$

B. For each mathematical sentence, find the value of N. Then write each complete fact family.

1. $3\frac{3}{5} + 1\frac{2}{3} = N$

2. $3\frac{1}{6} - 1\frac{2}{3} = N$

3. $\frac{3}{4} + N = \frac{17}{12}$

4. $N - \frac{1}{2} = \frac{3}{8}$

C. After writing several fact families, Rochelle claims that subtraction undoes addition. Do you agree or disagree? Explain your reasoning.

D. In the mathematical sentence below, find values for M and N that make the sum exactly 3. Write your answer as a sum that equals 3.

$$\frac{5}{8} + \frac{1}{4} + \frac{2}{3} + M + N = 3$$

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2.4 Designing Algorithms for Addition and Subtraction

TEKS / TAKS **6(2)B** Use addition and subtraction to solve problems involving fractions. **6(13)A** Make conjectures from sets of examples. **6(13)B** Validate conclusions using mathematical relationships.

To become skilled in solving problems that involve addition and subtraction of fractions, you need a plan for carrying out computations. In mathematics, a plan, or a series of steps, for doing a computation is called an **algorithm** (AL guh rith um). For an algorithm to be useful, each step should be clear and precise.

In this problem, you develop algorithms for adding and subtracting fractions. You may develop more than one for each computation. You should understand and feel comfortable with at least one algorithm for adding fractions and at least one algorithm for subtracting fractions.

Problem 2.4 Designing Algorithms for Addition and Subtraction

A. 1. Find the sums in each group.

Group 1	Group 2	Group 3
$2\frac{2}{9} + \frac{4}{9}$	$\frac{4}{9} + \frac{1}{3}$	$\frac{1}{8} + \frac{2}{3}$
$\frac{5}{8} + \frac{1}{8}$	$2\frac{1}{2} + \frac{5}{12}$	$\frac{2}{9} + 3\frac{1}{4}$
$\frac{3}{5} + \frac{9}{5}$	$\frac{7}{8} + \frac{1}{2}$	$3\frac{4}{5} + 3\frac{3}{4}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that will work for adding *any* two fractions including mixed numbers. Test your algorithm on the problems in the table. If necessary, change your algorithm until you think it will work all the time.

B. 1. Find the differences in each group.

Group 1	Group 2	Group 3
$3\frac{5}{6} - \frac{1}{6}$	$1\frac{3}{4} - \frac{1}{8}$	$3\frac{5}{6} - 1\frac{1}{4}$
$\frac{11}{7} - \frac{1}{7}$	$2\frac{7}{16} - 2\frac{1}{4}$	$\frac{1}{4} - \frac{1}{5}$
$1\frac{2}{3} - \frac{1}{3}$	$6\frac{7}{8} - 3\frac{3}{4}$	$4\frac{3}{5} - \frac{1}{3}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that will work for subtracting *any* two fractions, including mixed numbers. Test your algorithm on the problems in the table.

5. Describe how the subtraction problems below are different from the problems in the subtraction table in part (1).

Group 1	Group 2	Group 3
$1\frac{1}{3} - \frac{2}{3}$	$6\frac{3}{4} - 3\frac{7}{8}$	$3\frac{1}{4} - 1\frac{5}{6}$

6. If needed, change your algorithm until you think it will work all the time.

C. Use your algorithms for addition and subtraction to find each sum or difference.

1. $8 - 2\frac{2}{3}$ **2.** $8\frac{2}{3} - 2$ **3.** $2\frac{7}{16} + \frac{4}{9}$ **4.** $1\frac{4}{5} + 1\frac{5}{6} + 1\frac{3}{4}$

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