Independence Is The Word

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Name ________________________

Class ________________________

Problem 1 – Simulating Independent Events

- Describe two different events that are independent.

- Describe two different events that are not independent.

The probability of obtaining a tail with a coin toss is ½. If a coin is tossed twice, what is the probability that both are tails? Heads? Or one of each? You will investigate this problem using a simulation.

If a coin is tossed twice, what do you think is the probability of tossing no tails? One tail? Two tails?

Let 0 represent a head and 1 represent a tail. Use the spreadsheet on page 1.4.

Step 1: Simulate 100 trials of the first coin toss. In the grey cell of Column A, enter **=randInt(0,1,100)**.

Step 2: Simulate 100 trials of the second coin toss. Enter the same formula for Column B.

Step 3: Calculate the number of tails for each trial. In the grey cell of Column C, enter **=a +b**.

- Survey the results. What is the number of tails that occurs most often? Least often?

Step 4: Graph the results of the two tosses. With your cursor in Column C, choose **MENU > Data > Quick Graph**. Change the dot plot that appears to a bar graph by selecting **MENU > Plot Properties > Force Categorical X**, and then selecting **MENU > Plot Type > Bar Chart**.

Step 5: Move the cursor over each bar in the Bar Chart to see the number count for each outcome. Calculate each of the experimental probabilities for your data and enter it in the table.

Step 6: Combine your data as a group and calculate the experimental probabilities. Then, calculate the class experimental probabilities. Enter all probabilities in the table.

	No Tails	One Tail	Two Tails
Individual Results			
Group Results			
Class Results			

Conclusions:

- Did your results match your predictions? Why or why not?
- Why do you think the probability of getting one tail is higher than the probability of getting no tails or two tails?
- What is the sample space, set of all possible outcomes, for tossing a coin twice?
- Using the sample space, calculate the three theoretical probabilities for tossing a coin twice.

$$\text{Theoretical Probability} = \frac{\text{number of outcomes for event}}{\text{total number of outcomes}}$$

No tails: ________________

One tail: ________________

Two tails: _______________

- As you combined your results with the class, how did the experimental probabilities compare to the theoretical probabilities?
- Knowing that the probability of flipping one coin once and it landing on tails is ½, how can the theoretical probabilities above be computed (added, subtracted, multiplied, or divided) without finding the sample space?
- Explain why the computation for the probability of one tail is different from the others.

Complete the following sentence.

- If two events *A* and *B* are independent, then *P*(*A* and *B*) = ______________________.

Problem 2 – One Independent and One Dependent

Use page 2.1 to answer the following questions.

- What is the probability of choosing a red ball with one draw?

- Use the **Hide/Show** tool (ctrl + menu **> Hide/Show**) to hide the red ball on page 2.1. If a red ball has been chosen and not replaced, what is the probability of choosing a white ball on the second draw?

The probability of choosing a white ball after a red ball was already drawn and not replaced is an example of **conditional probability**. In this example, the events are not independent because knowing that the first one took place affects the probability of the second event.

- The probability of choosing the red ball on the first draw and a white ball on the second draw is $\frac{1}{14}$. How is the probability of each of these events used to compute this probability?

- What is the probability of choosing a red ball and a white ball in two draws (the order does not matter)? Hint: Think of this as two events, (1) choosing red then white and (2) choosing white then red.

Problem 3 – Conditional Probability

When $P(A) > 0$, the conditional probability of B given A is

$$P(B\,|\,A) = \frac{P(A \text{ and } B)}{P(A)}$$

Use the spreadsheet on page 3.2 to answer the following questions.

- What is the probability that a student chosen at random is a male?
- What is the probability that a student chosen at random is of age 15 – 17?
- What is the probability that a student chosen at random is male and is of age 15 – 17?
- Are the events male and 15 – 17 independent events? Prove or disprove using the rules of probability.
- Given that the student is male, what is the probability that the student is 15 – 17? Confirm your answer using the rule above.

Homework

1. A family decides that they would like to have 3 children.

a. What is the probability that a child is a girl?

b. Using a *Lists & Spreadsheet* page, simulate the birth of 3 children. Describe your simulation.

c. What is the experimental probability of having 2 boys and 1 girl?

d. What is the sample space for 3 children?

e. What is the theoretical probability of having 2 boys and 1 girl?

f. What is the theoretical probability of having all girls or all boys?

2. A roulette wheel has 38 slots, numbered 00 and 0 to 36. The slots 0 and 00 are green, 18 of the other are red, and 18 are black. The dealer spins the wheel and at the same time rolls a small ball along the wheel in the opposite direction. The wheel is balanced so that the ball is equally likely to land in any slot as the wheel comes to a stop.

a. What is the probability that the ball will land in any one spot?

b. What is the probability that the ball will land in a red spot?

c. If the wheel is spun twice, what is the probability that the ball will land in a red spot both times?

d. What is the probability that the ball will land in a spot with a 0 or 00?

e. If the wheel is spun twice times, what is the probability that it will **NOT** land on a 0 or 00?

3. Blue eyes are a recessive gene. This means that for a child to have blue eyes, they must inherit the blue eye gene from both parents. Assume that the probability of inheriting the blue-eyed gene is ½. If both parents carry one gene for blue eyes and one gene for brown eyes, what is the probability that they will have a blue-eyed child? If they have two children, what is the probability that both children will be blue-eyed? What is the probability that neither child will be blue-eyed?

4. A standard deck of cards has 4 suits—hearts, diamonds, clubs, and spades. There are 13 cards in each suit—A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Cards a drawn one a time without replacement.

a. What is the probability that a heart will be chosen?

b. What is the probability that an Ace will be chosen?

c. What is the probability that an Ace of hearts will be chosen?

d. What is the probability that the second card drawn will be a heart given that the first card drawn was a heart?

e. What is the probability that the first two cards drawn will be hearts?

f. What is the probability that a hand of five cards will all be hearts?

g. What is the probability that a hand of five cards will contain 4 of a kind?