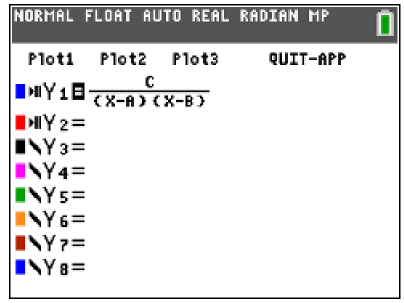
Exploring Vertical Asymptotes

TEACHER NOTES

TI-84 PLUS CE FAMILY



Math Objectives

- Students will determine the domain of rational functions.
- Students will use algebraic concepts to determine the vertical asymptotes of a rational function.
- Students will determine the removable discontinuities of a rational function.
- Students will describe the graph of a rational function given the equation.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- rational function
- factors
- domain
- zeros
- removable discontinuity

About the Lesson

- This lesson involves observing how changing the values in a rational function affects the continuity of the graph of the function.
- As a result, students will:
- Manipulate the factors of the numerator and denominator to observe the effects of changes in the factors.
- Explain how the values in a rational function determine the vertical asymptotes.
- Identify the conditions that must be met for a rational function to have a removable discontinuity.

Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Exploring Vertical Asymptotes_84CE_Student.pdf

Exploring Vertical Asymptotes_84CE_Student.doc

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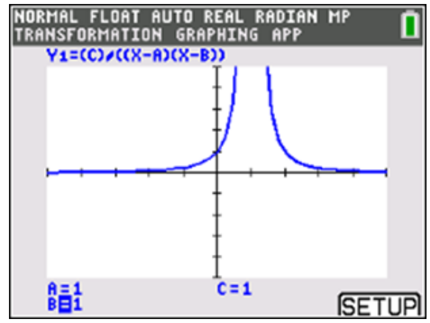
Given the equation of a rational function, will you always be able to determine the domain? In this activity, you will explore vertical asymptotes and removable discontinuities using the **Transformation Graphing App** on the handheld.



Problem 1

To turn on the **Transformation Graphing** app, press **apps**, **:Transfrm**, and press any key. Press $\boldsymbol{y} =$ and in Y_1, type in the equation $Y_1 = \frac{C}{(X-A)(X-B)}$.

1. Use the up/down arrows to change between the values of *A, B, and C*. Use the left/right arrows to change each individual value. Change the value of *A.* Describe how the graph changes.

   Sample answers: One of the vertical asymptotes moves. The other vertical asymptote stays the same. When *A* = *B*, there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.

2. Change the value of *B*. Describe how the graph changes.

   Sample answers: The other vertical asymptote moves, while the first vertical asymptote stays the same. When *A* = *B*, there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.

3. What do the values of *A* and *B* represent in the function?

   Answer: *A* and *B* are the zeros of the denominator or the values of x at which the function is undefined.

4. What are the equations of the vertical asymptotes?

   Answer: *x* = *A* and *x* = *B*. When *A* = *B*, there is only one asymptote, *x* = *A* = *B*.

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5. State the domain of the function in terms of *A*, *B*, and *C*.

Answer: $(-\infty, A) \cup (A, B) \cup (B, \infty)$ when $A < B$, $(-\infty, B) \cup (B, A) \cup (A, \infty)$ when $B < A$, or $(-\infty, A) \cup (A, \infty)$ when $A = B$

6. Change the value of *C*. How does changing *C* affect the domain?

Answer: Changing *C* only dilates the curves bounded by the asymptotes. It does not move the asymptotes. Therefore, the domain is not affected.

7. Describe how you could find the vertical asymptotes for any rational function with a constant numerator.

Answer: Factor the denominator. Solve the denominator to find the zeros of the denominator.

> **Teacher Tip:** Removable discontinuity will be addressed in the next section. Students could discuss the need for the "with a constant in the numerator" qualifier.

Problem 2

8. For problem 2, type the following equation into Y_1, $Y_1 = \frac{(X-A)(x-B)}{(X-C)}$. Using the arrows, set *A* = 2 and *B* = –1, and then change the value of *C*. For which values of *C* are there no asymptotes? Explain why there are no asymptotes for these values of *C*.

Answer: When *C* = *B* = –1 and *C* = *A* = 2, there is no asymptote. The graph looks like a line with a "hole" at *x* = *C* = *A* or *x* = *C* = *B*.

When *A* = *C*, the factor (*x* – *A*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph *y* = *x* – *B*. When *B* = *C*, the factor (*x* – *B*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph *y* = *x* – *A*.

9. The "hole" in the graph is called a removable discontinuity. Explain why the hole exists and how you might remove it by modifying the function definition.

Sample answer: The domain of the function is $(-\infty, C) \cup (C, \infty)$. The value of *C* will always be undefined regardless of the values of *A* and *B*. When the value of *C* does not equal either *A* or

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B, there is an asymptote at *x* = *C*. However, when *C* equals either *A* or *B*, there is only a hole. If that hole was removed, the graph would be continuous. Suppose that there is a hole at x = C. If the function definition is modified so that f(C) is defined to equal the limit of the function as x approaches C, the discontinuity is removed. For example, in question 10, there is a hole at x = -6. Since the limit of the function as x approaches -6 is -9, modifying the function definition to include f(-6) = -9 will remove the discontinuity.

Extension: This could be a nice place to discuss limit notation with a hole. $\lim_{x \to -6} f(x) = -9$.

10. Answer the following question:
    The function $f(x) = \frac{(x+6)(x-3)}{(x+6)}$ has
    (a) an asymptote at x = -6 (b) a removable discontinuity at x = -6
    (c) an asymptote at x = 6 (d) a removable discontinuity at x = 6
    (e) continuity

    Answer: (b) a removable discontinuity at x = -6 due to the common factors of (x + 6) in the numerator and the denominator.

Problem 3

11. For problem 3, type the following equation into Y_1, $Y_1 = \frac{(X-A)}{(X-B)(X-C)}$. Using the arrows, set *B* = –1 and *C* = *4*, and then change the value of *A*.

    a. Describe how the graph changes as the value of *A* changes.

    Sample answer: The graph bounded by the two asymptotes *x* = –1 and *x* = 4 dilates, but the asymptotes do not move. For certain values of *A*, there is only one asymptote.

    b. What is the domain of the function in terms of *A*, *B*, and *C*?

    Answer: $(-\infty, B) \cup (B, C) \cup (C, \infty)$ when $B < C$.

    Teacher Tip: It is important to note that the domain would be $(-\infty, C) \cup (C, B) \cup (B, \infty)$ when $C < B$ or $(-\infty, B) \cup (C, \infty)$ when $B = C$.

c. For which values of *A* is there only one asymptote? Describe the graph at these values.

Answer: When *A* = *B* = –1 or *A* = *C* = 4, there is only one asymptote. The graph looks like a translation of an inverse variation. There is also a hole (a removable discontinuity).

d. Explain algebraically why the graph looks as it does at these points.

Answer: When *A* = *B*, the factor (*x* – *A*) in the numerator reduces with the factor (*x* – *B*) in the denominator, leaving the graph $y = \frac{1}{X-C}$. When *A* = *C*, the factor (*x* – *A*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph $y = \frac{1}{x-b}$
The domain of the function remains the same regardless of the factors that reduce. When *A* equals either *B* or *C*, there is a removable discontinuity instead of an asymptote.

12. Describe how the domain would change if you changed the values of *B* and *C*.

Answer: The domain is $(-\infty, B) \cup (B, C) \cup (C, \infty)$ when $B < C$, $(-\infty, C) \cup (C, B) \cup (B, \infty)$ when $C < B$, or $(-\infty, B) \cup (C, \infty)$ when $B = C$

Teacher Tip: This is a good time to revisit Question 7. Students could discuss how to find the vertical asymptotes for *any* rational function.

Move to page 3.2.

13. Answer the following question:
The function $f(x) = \frac{(x-3)}{(x+6)(x-3)}$ has

(a) one asymptote at x = 3
(b) a removable discontinuity at x = 3
(c) two asymptotes at x = -6 and x = 3
(d) one asymptote at x = -6
(e) continuity

Answer: (b) and (d) as it has a common factor of (x – 3) in the numerator and the denominator and the factor of (x + 6) only in the denominator.

Problem 4

For problem 4, type the following equation into Y_1, $Y_1 = \frac{(X+1)^A}{(X+1)^B}$. Using the arrows, set *B* = –1 and *C* = *4*, and then change the value of *A*.

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14. Answer the following questions:

Holes were discussed in question 9. While manipulating *A* and *B* on your graph, what would *A* and *B* have to be for *f1(x)* to have a hole?

(a) If $A < B$
(b) If $A = B$
(c) If $A > B$

Answers: (b) and (c), both result in the scenario of having 1 or more common factors in the numerator and the denominator, but once reduced, there remains zero or more factors in the numerator and none in the denominator.

What would *A* and *B* need to be to have a vertical asymptote?

(a) If $A < B$
(b) If $A = B$
(c) If $A > B$

Answer: (a) Even though common factors may be reduced, this scenario leaves one or more factors in the denominator, resulting in a vertical asymptote and not a hole.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- How to determine the vertical asymptotes of a rational function.
- What conditions must be true for a rational function to have a removable discontinuity.

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