Math Objectives

- Students will explore the family of exponential functions of the form $f(x) = c \cdot b^{x+a}$ and be able to describe the effect of each parameter on the graph of $y = f(x)$.
- Students will be able to determine the equation that corresponds to the graph of an exponential function.
- Students will understand that a horizontal translation and a vertical stretch of the graph of an exponential function are essentially the same.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

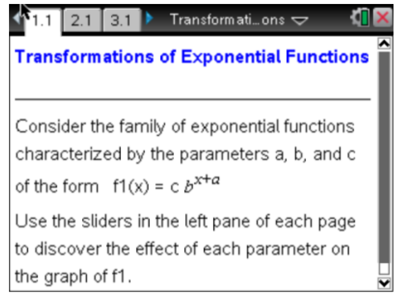
- exponential function
- parameter
- translation
- reflection
- vertical shift

About the Lesson

- This lesson involves the family of exponential functions of the form $f(x) = c \cdot b^{x+a}$.
- As a result students will:
    - Manipulate sliders, and observe the effect on the graph of the corresponding exponential function.
    - Conjecture and draw conclusions about the effect of each parameter on the graph of the exponential function.
    - Compare horizontal translation and vertical stretch and manipulate equations to demonstrate they are the same.
    - Match specific exponential functions with their corresponding graphs.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing [ctrl] [G].

Lesson Files:
Student Activity
Transformations_of_Exponential_Functions_Student.pdf
Transformations_of_Exponential_Functions_Student.doc
TI-Nspire document
Transformations_of_Exponential_Functions.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: To change a slider setting, right-click in the slider box, and select option 1. Consider changing the (start) value, minimum and/or maximum value, and/or the step size in order to help discover or confirm the effect of a specific parameter.

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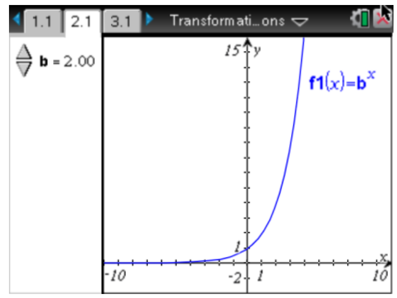

1. The graph of $y = f1(x) = b^x$ is shown in the right panel. Click the arrows in the left panel to change the value of b, and observe the changes in the graph of $f1$.

   a. Explain why for every value of b, the graph of $f1$ passes through the point $(0,1)$.

   Sample Answers: The graph of $y = f1(x) = b^x$ passes through the point $(0,1)$ for all values of $b > 0$ because $f1(0) = b^0 = 1$. The *y*-intercept of the graph of $f1$ is 1.

   b. For $b > 1$, describe the graph of $y = f1(x) = b^x$.

   Sample Answers: The graph is above the x-axis and is always increasing. As x takes on smaller and smaller negative values ($-10,\ -100,\ -1000,\ \ldots$), the values of $f1$ get closer to 0. In more precise mathematical language, we would say as x decreases without bound, b^x approaches 0. As x gets larger and larger (10, 100, 1000, $\ldots$) the values of $f1$ get larger and larger. In more precise mathematical language, as x increases without bound, b^x also increases without bound. As b gets larger, the graph becomes steeper, or increases more rapidly. As b gets closer to 1, the graph becomes less steep approaching the graph of the line $y = 1$.

   c. For $0 < b < 1$, describe the graph of $y = f1(x) = b^x$.

   Sample Answers: The graph is above the x-axis and is always decreasing. As x gets smaller and smaller ($-10,\ -100,\ -1000,\ \ldots$) the values of $f1$ increase without bound. As x gets larger and larger (increases without bound), the values of $f1$ get smaller and approach 0. As b gets closer to 0, the graph becomes steeper. As b gets closer to 1, the graph becomes less steep and approaches the graph of the line $y = 1$.

Teacher Tip: Teachers might need to remind students that a negative exponent inverts the fraction b. The reciprocal of a fraction between 0 and 1 is a number greater than 1.

d. Find the domain and range of function $f1(x)=b^x$.

Answer: The domain is all real numbers, and the range is all positive real numbers: $(0,\infty)$.

e. Does the graph of $y=b^x$ intersect the x-axis? Explain why or why not.

Answer: For $b>1$: as x decreases without bound, the graph of $y=b^x$ approaches the x-axis but never touches it. For $0<b<1$: as x increases without bound, the graph of $y=b^x$ approaches the x-axis but never touches it. The x-axis, the line $y=0$, is a horizontal asymptote to the graph of $y=b^x$.

Tech Tip: The limited resolution on the handheld screen might result in a graph that appears to intersect the x-axis. This presents an opportunity to trace the graph and/or create a table of values to show that values of the function are small, but not equal to zero.

Teacher Tip: The slider for the variable b is set to minimized, style: vertical, and initially set such that it includes the value 1. Most definitions of an exponential function stipulate $b\neq 1$.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
See Note 1 at the end of this lesson.

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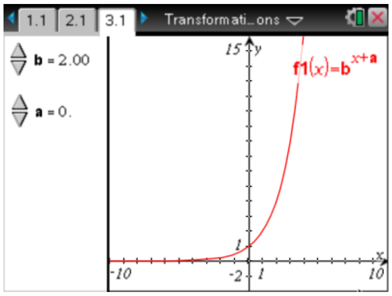

2. The graph of $y=f1(x)=b^{x+a}$ is shown in the right panel. For a specific value of b, click the arrows to change the value of a and observe the changes in the graph of $f1$. Repeat this process for other values of b.
   a. Describe the effect of the parameter a on the graph of $y=b^{x+a}$. Discuss the effects of both positive and negative values of a.

Answer: For $a>0$, the graph of $y=b^x$ is translated horizontally, or moved, left a units. For $a<0$, the graph of $y=b^x$ is translated right a units.

Teacher Tip: This left/right translation occurs for any value of b. Horizontal translations of the graph of an exponential function are difficult to recognize because students often focus on the y-intercept and vertical shifts. Emphasize that the horizontal asymptote ($y = 0$) did not change (move up or down) which would have happened if there were a vertical shift in the graph.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
See Note 1 at the end of this lesson.

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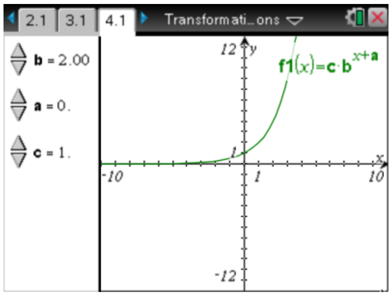

3. The graph of $y = f1(x) = c \cdot b^{x+a}$ is shown in the right panel. For specific values of a and b, click the arrows to change the value of c, and observe the changes in the graph of $f1$. Repeat this process for other values of a and b.
   a. Describe the effect of the parameter c on the graph of $y = c \cdot b^{x+a}$. Discuss the effects of both positive and negative values of c.

   Answer: If $c < 0$, the graph is reflected across the x-axis. For $|c| > 1$, the graph of $y = b^{x+a}$ is stretched vertically. For $|c| < 1$, the graph of $y = b^{x+a}$ is contracted vertically.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
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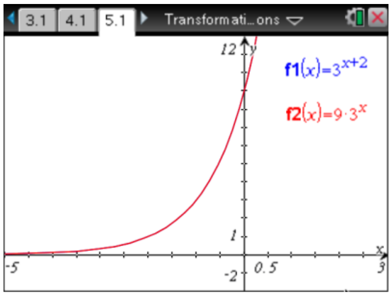


4. Display the graphs of $y = f1(x) = 3^{x+2}$ and $y = f2(x) = 9 \cdot 3^x$.
   a. Describe the similarities between these two graphs. Use the properties of exponents to justify your answer.

   Answer: The graphs of these two exponential functions are the same. $f1(x) = 3^{x+2} = 3^x \cdot 3^2 = 9 \cdot 3^x = f2(x)$.

b. Insert a new problem, and display the graph of $y = f1(x) = 3^{x-2}$. Use the properties of exponents to find a function of the form $f2(x) = c \cdot 3^x$ such that the graphs of $f1$ and $f2$ are the same. Verify your answer.

Answer: $f1(x) = 3^{x-2} = 3^x \cdot 3^{-2} = \left(\frac{1}{9}\right) \cdot 3^x = f2(x)$ The graphs of $f1$ and $f2$ are the same.

c. Use your answers to parts (a) and (b) to explain the relationship between a horizontal translation and a vertical stretch of the graph of an exponential function.

Answer: A horizontal translation and a vertical stretch of the graph of an exponential function are essentially the same. Consider the following expression to show this analytically:

$$f1(x) = b^{x+a} = b^x \cdot b^a = c \cdot b^x = f2(x)$$

This demonstrates that any horizontal translation can also be considered a vertical stretch.

5. Match each equation with its corresponding graph.

(a) $f(x) = 3^{x-4}$

(b) $f(x) = -\left(\frac{1}{3}\right)^x$

(c) $f(x) = (0.7)^{x-4}$

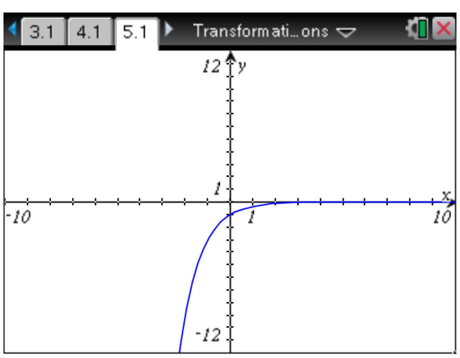
(d) $f(x) = -2\,(0.1)^{x+3}$

(e) $f(x) = e^x$

(f) $f(x) = -\left(\frac{1}{2}\right) \cdot \pi^x$

Note: The function in part (e) is the "natural" exponential function and involves the number $e \approx 2.71828...$

(i)



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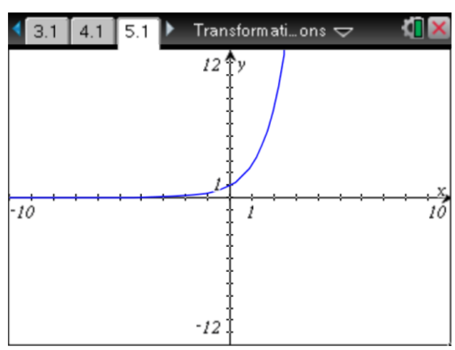

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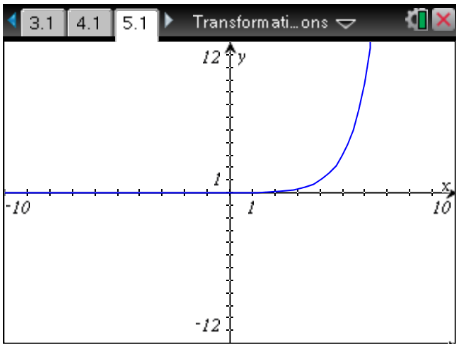

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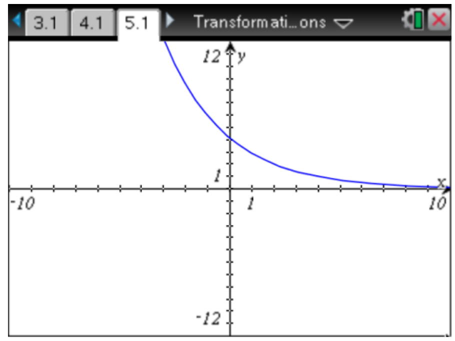

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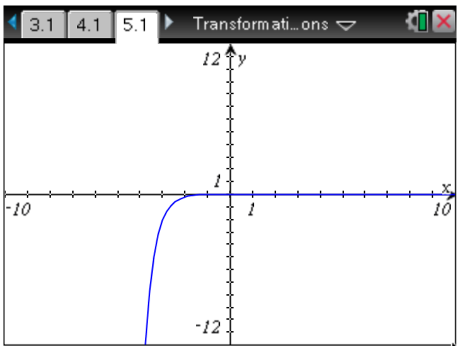

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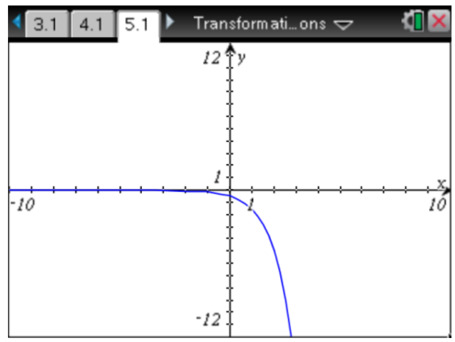

Answers: (a) → (iii) (b) → (i) (c) → (iv) (d) → (v) (e) → (ii) (f) → (vi).

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to:

- Graph and analyze an exponential function of the form $f(x) = c \cdot b^{x+a}$.
- Explain the concepts of reflection and translation.

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Note 1

Name of Feature: Screen Capture and Quick Poll

Use Screen Capture to compare student graphs for various values of each parameter.

A Quick Poll can be given at several points during this lesson. It can be useful to save the results and show a Class Analysis.

Sample multiple choice questions:

For $b > 1$, how many times does the graph of $y = b^x$ cross the x-axis?

(a) 0 ✓

(b) 1

(c) 2

(d) Infinitely many

How does the graph of $y = 2^{x+5}$ compare with the graph of $y = 2^x$.

(a) Translated 5 units to the right.

(b) Translated 5 units to the left. ✓

(c) Shifted five units up.

(d) Shifted 5 units down.

For $c > 1$, how does the graph of $y = c \cdot 3^x$ compare with the graph of $y = -c \cdot 3^x$?

(a) Wider

(b) Stretched

(c) Reflected ✓

(d) Same

