Modelling Compound Interest using Recursion

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Teacher Notes/Answer Sheet

7 8 9 10 **11 12**

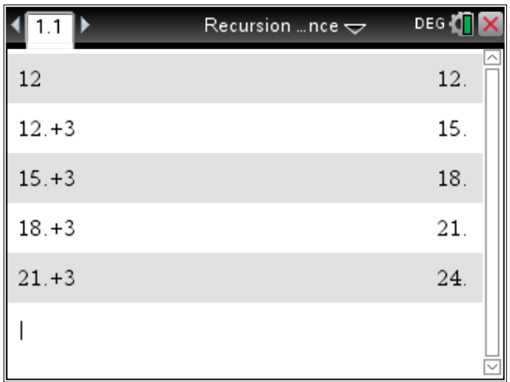
TI-Nspire | Investigation | Teacher | 60 min

Introduction

Recursive techniques are very useful for finance problems that use a repetitive process based on previous answers (iterations).i.e. compounding interest or reducing loans at set rates.

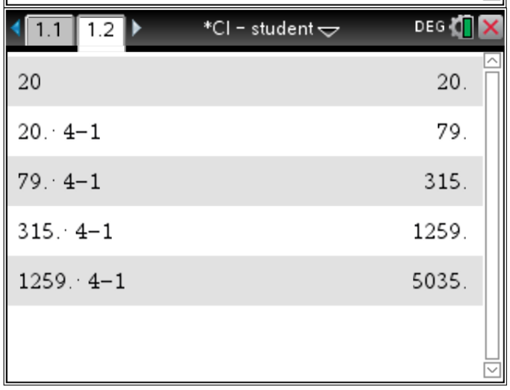
Introductory tasks

A simple example of recursion is to add 3 to an existing number, say 12. On a calculator, pressing enter repeatedly will add 3 to the answer. Each new calculation is called an iteration.



The "+3" is the repeated process to get the value at the next time period.

This is sometimes written as V_0=12, V_{n+1}=V_n+3, i.e. the initial value V_0 =12, the next value (V_{n+1})= previous value (V_n) +3.



Using recursive techniques, the V_n is the value of the investment (answer at each line) at the end of each compounding period. The initial value entered will be P (the principal). In the simple example above it would be the 12.

Instead of "+3" you would enter "$\times\left(1+\frac{r/c}{100}\right)$" where r/c is the annual interest rate divided by the compounding periods per year.

Repeat for initial value 20, with constant "×4-1". i.e. V_0=20, V_{n+1}=V_n×4-1

Example 1:

Determine the amount of money accumulated after 4 years if $10 000 is invested at an interest rate of 6.25% per annum, compounded <u>annually</u>.

Note that period 4 (i.e. after 4 compounding periods) you are referring to the 5th term as term 1 (period 0) is the initial value.

For simplicity, the value inside the brackets $\left(1+\frac{6.25/1}{100}\right)$, is often computed first. i.e. 1.0625

Author: R.Brown

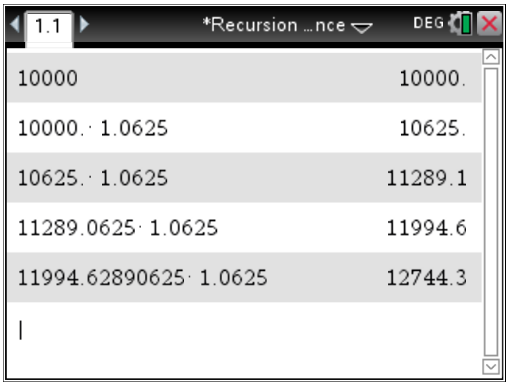
Note: you may wish to discuss why the term $\left(1+\frac{6.25/1}{100}\right)$ is used. i.e. in the compound interest formula the component $P*\left(1+\frac{r/c}{100}\right)$ is equivalent to $P*1+P*\left(\frac{r/c}{100}\right)$ which is the last value, $P*1$, plus the new interest value, $P*\left(\frac{r/c}{100}\right)$.

This problem can be also written as V_0=10000, $V_{n+1}= V_n + V_n\times .0625$ or $V_{n+1}= V_n\times 1.0625$

The simplest recursive approach is as follows:

In a **Calculator** page enter the initial value (Principal) and press enter. i.e. 10000

On the 2nd line press ***(1+6.25/100) or 1.0625**, then enter. It will use the previous answer to complete this computation. Continue to press enter to generate the required iterations.



What is the amount of money accumulated after 4 years compounded annually?

After 4 years the accumulated value is $12744.30.

Note that you are looking for the 5th term as the 1st term is the initial value.

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Example 2:

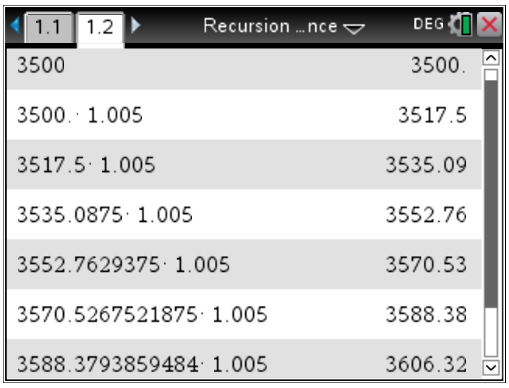
Maria invests \$3500 at an interest rate of 6.0% p.a., compounded monthly. After how many months will her investment first exceed \$3600?

This problem can be also written as $V_0=3500$, $V_{n+1}= V_n + V_n\times .06/12$ or $V_{n+1}= V_n\times 1.005$

In a **Calculator** page enter the initial value (Principal) and press enter. i.e. 3500.

On the 2nd line press ***(1+6.0/12/100) or 1.005**, then enter. It will use the previous answer to complete this computation. Continue to press enter to generate the required iterations.

Note the /12 as the value is compounded monthly.



After how many months will her investment first exceed \$3600?

After 6 months the accumulated value is first greater than \$3600.

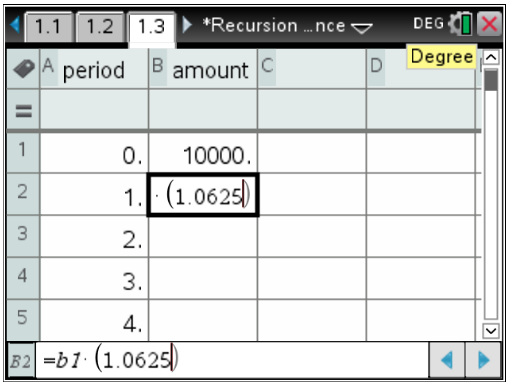
Note that this is the 7th term in the sequence.

Using a spreadsheet to show recursive technique for Example 1 above

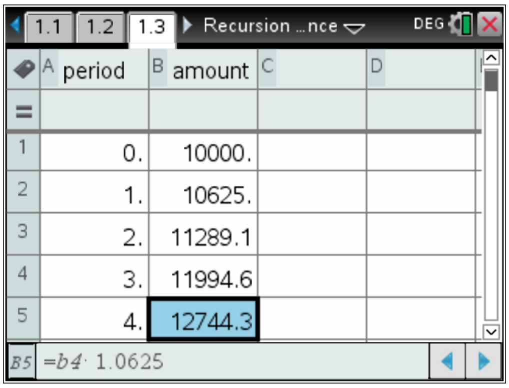
Recursive approach using a spreadsheet.

Enter compounding periods in column A. Start with 0 for the initial start period equalling the initial investment. Enter the initial investment value in b1

In cell b2 enter = b1*(1+6.25/100), enter. Fill down (menu **>Data>Fill**) to obtain the amount after 4 years.



Hint: make sure a multiplication sign is used between b1 and the bracket when entering the formula.



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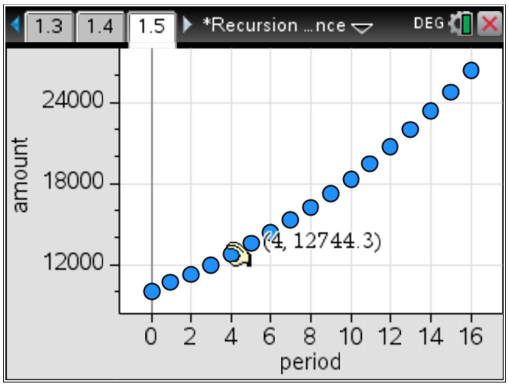
Plotting in a **Data & Statistics** page as a normal bivariate example.

To make the curve more obvious more compounding periods (i.e. 16 years) were used.



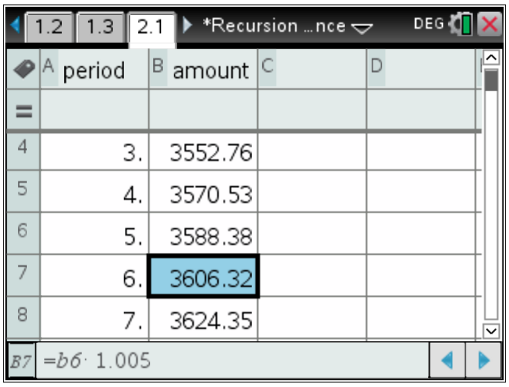
Discuss the shape of the curve.

Answer: The curve is not linear (as would be the case for Simple Interest) but is increasing at an increasing rate.

Does it appear to increasing at an even amount each year or increasing at a higher rate as the periods get larger?

Hint: *emphasise the terms explanatory (IV) and response (DV) variables.*

Create a spreadsheet for Example 2

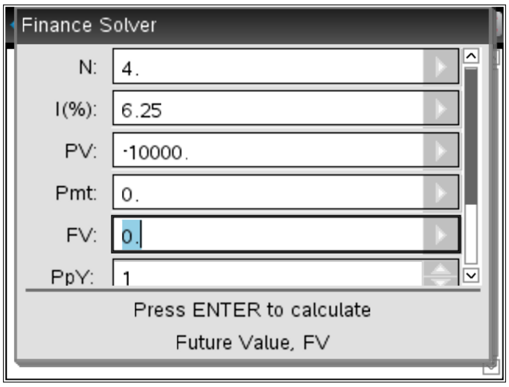


	A period	B amount	C	D
=				
4	3.	3552.76		
5	4.	3570.53		
6	5.	3588.38		
7	6.	3606.32		
8	7.	3624.35		

B7 =b6· 1.005

Using TVM Solver

Access on a **Calculator** page using menu **>Finance>Finance Solver**



N is the number of payments or compounds for the term of the investment.

I is the interest rate (p.a)

PV (Present Value) is entered as negative as it has been loaned to the financial institution.

Ppy and Cpy will be the same and is the number of compounds per year.

Note that the Finance Solver can only be used for compounding/reducing balance examples and not simple interest based problems.

Place cursor on the unknown value (in this case the Future value, FV),and press enter to evaluate.

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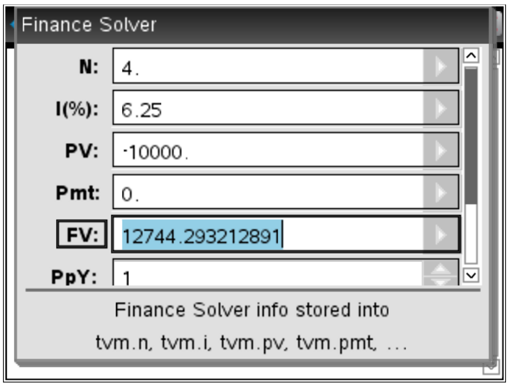
Finance Solver
N: 4.
I(%): 6.25
PV: -10000.
Pmt: 0.
FV: 12744.293212891
PpY: 1
Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...