NUMB3RS Activity: Where's the Source?
Episode: "Undercurrents"

Topic: Vector fields   **Grade Level:** 9 - 12
Objective: Use vector fields to make predictions
**Time:** 25 minutes

Introduction

In "Undercurrents," Don's team is asked to determine the point of origin for the body of a dead woman that washes up on the Los Angeles shore. Charlie attempts to trace where the woman's body might have entered the water by using oceanic fluid dynamics and current flow equations. He analyzes the current flow equations by dropping a number of small buoys (known in fluid dynamics as "tracers") into the water and mapping how they approach the shore. Through this process, he is able to determine the vector field (collection of vectors) that tells him where the body most likely originated.

This activity focuses on vector fields; not the calculus of calculating them, but using them to make predictions. The activity on the Extensions page uses the Texas Instruments CBL™ 2 device and a flow rate sensor to extend this activity into a class project to calculate the flow rate of water in a stream.

Discuss with Students

A **vector** is a quantity that has both a magnitude and a direction, and can be represented by a directed line segment. The length of the directed line segment describes the magnitude of the vector and its orientation describes its direction. In this case, the speed of an object in motion is the magnitude of the vector, and the direction is described in terms of north, south, east, and west.

For example, the vector drawn at the right represents 30 mi/hr due west. The length of the vector represents 30 mi/hr, and the arrow points due west. If we wanted to draw a vector that represents 60 mi/hr due west, it would be twice as long and face in the same direction.

30 mi/hr West

60 mi/hr West
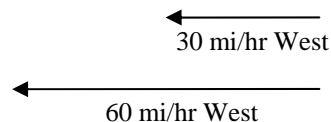
A **vector field** is a collection of vectors. Imagine a draining bathtub containing soap bubbles. The soap bubbles move in a spiral path at different speeds. If we calculate the vectors for each point in the tub, it would make a vector field like this:
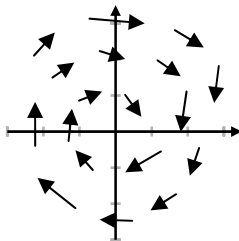
The soap bubbles follow the arrows. As you can see, the path of a bubble will take it right down the drain at the origin.

Student Page Answers:
1. *Approximately (3, 1.5)* **2.** *(10, −10)* **3.** *(−10, −10)* **4.** *Four drains: (10, 10), (−10, 10), (−10, −10), (10, −10)* **5.** *south-central South Dakota* **6.** *south-central South Dakota or Ontario, Canada [Note: for #5 and #6, the balloons could have started anywhere along the air currents.]*

Name: _____   Date: _____

NUMB3RS Activity: Where's the Source?

In "Undercurrents," Don's team is asked to determine the point of origin for the body of a dead woman that washes up on the Los Angeles shore. Charlie attempts to trace where the woman's body might have entered the water by using oceanic fluid dynamics and current flow equations. He analyzes the current flow equations by dropping a number of small buoys (known in fluid dynamics as "tracers") into the water and mapping how they approach the shore. Through this process, he is able to determine the vector field (collection of vectors) that tells him where the body most likely originated.

A **vector** is a quantity that has both a magnitude and a direction, and can be represented by a directed line segment. The length of the directed line segment describes the magnitude of the vector and its orientation describes its direction. In this case, the speed is the magnitude of the vector. For example, the figure at the right is a vector that represents 65 mi/hr due east.
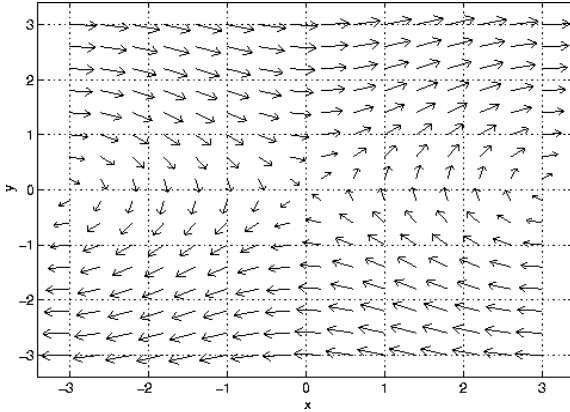
65 mi/hr East
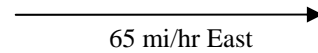
Vectors are useful because they illustrate how an object will behave with many different forces acting on it at the same time. For example, suppose a boat crossing the ocean is acted upon by three different vectors (wind, current, and engine propulsion.) To see how these three vectors affect the path of the boat, they are drawn using the "head to tail" method. This means to draw the wind vector where the engine vector ends and the current vector where the wind vector ends. The actual path the boat will take (the resultant vector) is found by drawing a vector from the start of the first vector to the end of the last vector.

wind   current

engine

Path of the boat
(Resultant vector)
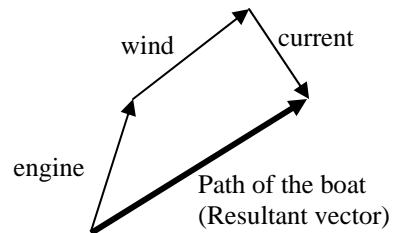
Suppose one of Charlie's tracer buoys is placed in the water with a wind vector of 20 mi/hr north and a current vector of 15 mi/hr east. The vector the tracer buoy follows is calculated at the right using the "head to tail" method.

Current 15 mi/hr

Wind 20 mi/hr

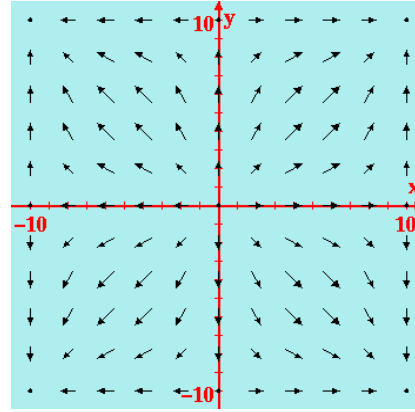Resultant Vector
(path of the buoy)
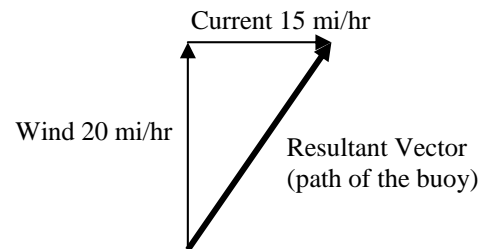
This vector is for just one of Charlie's tracer buoys. When the vectors of *all* the tracer buoys are graphed, the result is called a **vector field**. Vector fields are collections of vectors that describe the possible movements of an object through a substance. Two examples are shown on the next page.

Vector Field A

Source: **http://www.math.oregonstate.edu/ home/programs/undergrad/CalculusQuestStudy Guides/vcalc/vecfield/vecfield.html**

Vector Field B

Source: **http://www.ee.byu.edu/em/ embook/ch3/demo3.13.html**

1. If these vector fields represent the movement of water, where does a soap bubble leave the vector field if it is placed at (3, –1) in vector field A?

2. What happens to a soap bubble placed in the same location in vector field B?

3. Where would any object placed in the 3rd quadrant of vector field B land?

4. If vector field B represents a tub of water, how many drains do you think there are? Where do you think they are located?

In a real application, vectors are calculated from stationary buoys in the ocean set up by NOAA (National Oceanic and Atmospheric Administration.) These vectors depend upon many factors, like wind, water, current, and temperature.

To calculate his vector fields, Charlie drops many buoys into the water and traces their movements. If the same thing is done in the atmosphere using weather balloons, the picture below might appear, where the path of each balloon is shown as a line, and arrows show its direction:
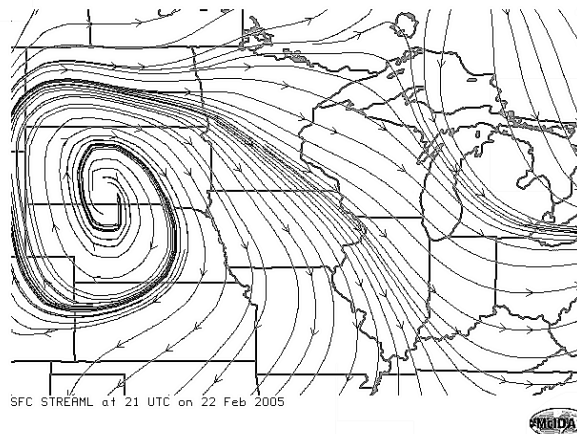
[Source: Adapted from **http://www.ssec.wisc.edu/mcidas/images/mcidas_capabilities/STREAML.GIF**]

5. Suppose a balloon landed in the southeast corner of Wisconsin. Where did it most likely originate?

6. Suppose a balloon landed in the southeast corner of Michigan. Where did it most likely originate?

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Activity: Class Project – Calculating the Stream Flow of a River

<u>For the Student</u>
Stream flow is the volume of water that moves past a specific point along a stream over a given period of time. To calculate a river's stream flow, a cross-sectional area of the stream or river is measured, and the velocity of the flow is measured using a flow rate sensor. The stream flow can then be calculated by multiplying the cross-sectional area by the flow velocity.

Select a nearby stream that is easily accessible and can be safely crossed without any sand bars or objects in its path. Mark a straight path across the river at equal increments (at least six). Record the depth at each location along with the distance from the riverbank. Once this is done, measure the water flow at each of these locations using the Flow Rate sensor. The flow rate sensor can directly be connected to the TI-84 Plus with the EasyLink cable and the EasyData App installed (otherwise it can be connected using the Texas Instruments CBL^{TM} 2). Be sure to have the flow rate sensor's blades turned into the current, and be sure to hold the sensor halfway between the surface and the riverbed at each location while collecting the data.

Measurement locations

River Bank   River Bank

River Bed
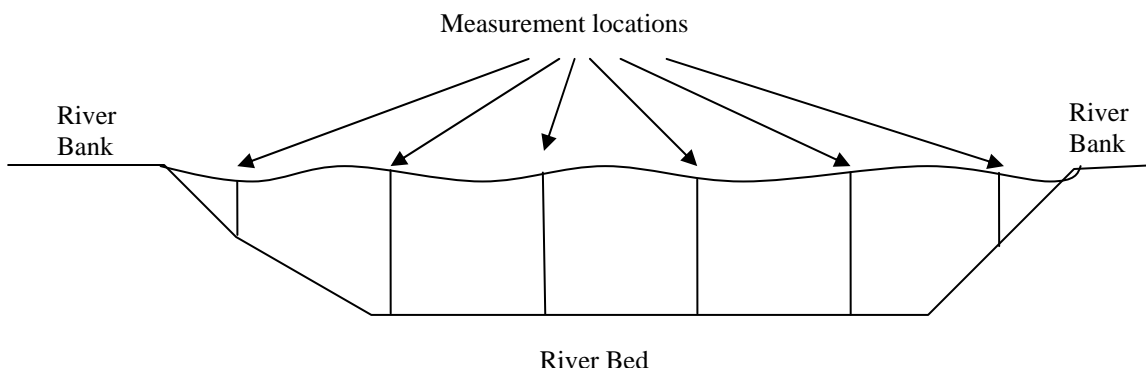
Once the data collection is complete, average each flow rate and compute the cross-sectional area by dividing the cross section into shapes for which you can find the area (trapezoids and rectangles). The flow rate for the river at this location is the product of the velocity and the cross-sectional area. Repeat this experiment several times over the next few weeks or months and resample your measurements. How does the flow rate change with respect to season and weather?

Additional Resources

- NOAA deploys hundreds of drifter buoys each year. Make your own vector field by accessing the data at **http://www.aoml.noaa.gov/phod/dac/gdp.html**.

- You can view many animated vector fields and observe their behavior at: **http://www.falstad.com/vector**

- This article describes how Italian scientists in 2001 used vector fields to solve a crime very similar to the one in "Undercurrents": **http://www.ve.ismar.cnr.it/~carniel/publications/Carniel%20p143-152.pdf**

Related Topic

- For another approach to measuring with vectors, see the *NUMB3RS* Activity "Narrowing the Search," which can be downloaded for free from the Web site below. **http://www.cbs.com/primetime/numb3rs/ti/activities/ Act3_NarrowingTheSearch_Undercurrents_final.pdf**

- For an activity that uses the TI-Navigator™ system to explore vector fields, see the *NUMB3RS* Activity "Riding the Waves," which can be downloaded for free from the Web site below. **http://www.cbs.com/primetime/numb3rs/ti/activities/ Act4_RidingTheWaves_Undercurrents_final.pdf**