NUMB3RS Activity: Up to Speed
Episode: “Traffic”

Topic: Elementary traffic flow models **Grade Level:** 10 - 12
Objective: Use a spreadsheet to model the acceleration, speed, and distance of a moving vehicle, to gain a fundamental appreciation of modeling in traffic flow theory
Materials: TI-83 Plus/TI-84 Plus graphing calculator, with the CellSheet™ App
Time: 15 - 25 minutes

Introduction

In the episode “Traffic,” Charlie believes that there may be a pattern to what seems like random attacks on highway drivers. He then launches into a discussion of how difficult it is to model traffic flow. He mentions the use of partial differential equations to calculate the optimum number of lanes and on- and off-ramps, as well as the synchronization of traffic signals.

Traffic flow theory began in the 1930s using probability theory. Since then it has become the very complex study of modeling the interactions of vehicles, drivers, and the highway infrastructure itself, including control devices, signs, and markings. With the use of computers, highway designs have become very sophisticated, requiring highly complex mathematics. In this activity, students will study a very basic traffic flow model that predicts the location of a single vehicle in a line of traffic.

Discuss with Students

Any type of mathematical modeling has to accomplish three things: make simplifying assumptions in order to make the model manageable, identify the key variables and their relationships in the model, and use the appropriate mathematics of physical laws that control the behavior of the model. This process requires that the model be described using language that can be translated into mathematical relationships. In the case of more complex models, the mathematics is then translated into computer code.

To model the position of a vehicle, use the following:

1. *Speed* is the change in distance per unit of time. *Acceleration* (constant in our case) is the change in speed per unit of time.
2. *Distance* traveled (with constant acceleration over a time interval) is the product of the time interval and the average of the speeds at the start and end of the interval.
3. With constant acceleration, change in speed is the product of acceleration and time.
4. The vehicle accelerates at a constant rate until its speed reaches the speed limit, then the speed becomes constant at that limit.

Student Page Answers:
1. $S_{i+1} = S_i + AT$. **2.** $S_{avg} = 0.5(S_i + S_{i+1})$ **3.** *18 +0.5(12 + 16)(1) = 32 m* **4.** $D_{i+l} = D_i + 0.5(S_i + S_{i+1})T$
5. *The vehicle stops accelerating when its speed reaches the speed limit. Note: the formula for distance is C2+0.5(B2+B3).* **6.** *72 mph* **7.** *64 meters*

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Name: ______________________________ Date: ______________

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The model uses time T, acceleration A, speed S, and distance D as its variables. We will use S_0, T_0, and D_0 as starting points, and assume that A is constant. The subscript i is used to specify a point in time (e.g., S_i) and $i + 1$ is used to specify the next point in time (e.g., S_{i+1}).

1. If a vehicle has speed S_i and acceleration A, write an expression for S_{i+1}. _________

The basic model that relates speed, distance and time is $D = RT$, where R is a constant rate of speed. For a vehicle with constant acceleration, the rate R is the average of the speeds at the start and end of the time interval.

2. Write an expression for the average of S_i and S_{i+1}. __________

3. Suppose a vehicle has so far traveled a distance of 18 m, has a current speed of 12 m/s, and acceleration of 4 m/s^2. How far will it have traveled after one additional second? __________

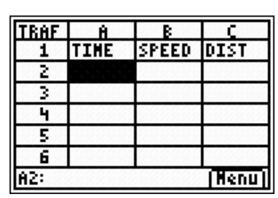
4. If a vehicle has traveled D_i m so far, write an expression for D_{i+1}. __________

5. Suppose that the speed limit is L m/s. Explain the meaning of the following equations:

$S_{i+1} = S_i + A,\ S_i < L$

$S_{i+1} = L,\ S_i \geq L$ ____

On your calculator, press [APPS] and open the CellSheet App. (If you like, you can use the menu to name this spreadsheet "TRAF" as shown; this is not necessary unless you want to save this file.) To put in the column headings, type `"TIME`, `"SPEED`, and `"DIST` in the appropriate cells.



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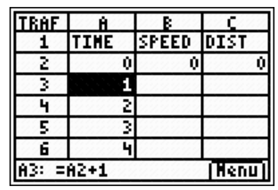
In row 2, enter the initial values of 0 for time, speed, and distance. For our model, we use time steps of 1, so each successive value can easily be calculated using the previous one.

To enter the formulas in row 3, move to cell A3 and press [STO▸] to place an equal sign in the edit line, then enter the formula.

In our model, $T_{i+1} = T_i + 1$; $S_{i+1} = S_i + A$ (up to the speed limit, otherwise it is L); and $D_{i+1} = D_i + 0.5(S_i + S_{i+1})$.

Because completing the spreadsheet can be a little tricky, especially for novices, here is how to fill in the TIME column (for 25 time steps). The same process is used for the other columns.

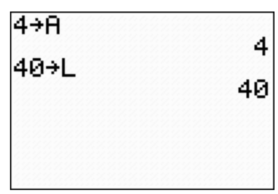
- Move to cell A3.
- Press [STO▸] to make an equal sign, so you can enter the formula.
- Since the value in A2 represents T_0 and the value in A3 will be T_1, enter the formula as "A2+1" (as shown).
- To copy the formula down the column to represent 25 time steps, do the following:
  a. Press [ZOOM] to highlight the cell with the formula
  b. Press [Y=] to select **Range** from the menu
  c. Move the cursor down to row 27 (remember, we used row 1 for the heading and row 2 for T_0)
  d. Press [TRACE] to select **Paste** from the menu



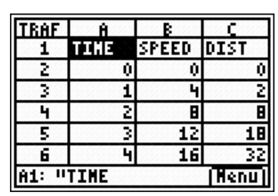
This is a good time to note that when you copy a formula, the calculator automatically adjusts the row numbers so that each cell is calculated from the one directly above it.

Recall that the formula for speed is rather tricky. Enter it as though you were graphing a piecewise function: (B2+A)(B2<L)+(L)(B2≥L). (Note: to enter the inequality symbols, press [2nd] [TEST]. Enter the formula for distance following the format for time.

In order to run your model, quit CellSheet for the moment ([2nd] [QUIT]). On the home screen, store values for A and L (for example, let A = 4 and let L = 32).



Restart the CellSheet App, select **Menu**, **1:File...**, and **6:Recalc**(ulate) to update all of the values. Experiment with different sets of values.



6. To the nearest mile per hour, how fast is 32 meters per second?

7. A car going 32 m/s brakes hard, decelerating at 8 m/s^2. How far does the car travel from the time it starts decelerating until it comes to complete stop? (Hint: The answer will be the same as the distance covered by a car that accelerates at 8m/s^2 from standing still to 32 m/s.)

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Obviously, the model in this activity is too simple to be very useful. After experimenting with the model, the next step is to introduce more vehicles and "relax" some of the assumptions, as discussed in the activity. High school level mathematics can only begin to scratch the surface of the very complex yet well-studied field of traffic flow theory. On real highways, vehicles travel at varying speeds, enter and exit the highway with certain probabilities, change lanes (while passing each other or not), and even break down and stop. The causes of traffic jams as a function of the speed and density of traffic are of extreme interest to mathematicians, and many models have been developed to study the phenomenon.

For the Student

If you are experienced with CellSheet, or a spreadsheet program for a computer, improve this model so that the initial values for acceleration, and speed limit are on the spreadsheet itself. Add other enhancements that show the graphs of the speed and/or distance for the vehicle.

You could also develop a similar model to show the stopping distance of a vehicle when the brakes are applied. A Driver's Education instructor should be able to provide you with the formulas used for computing stopping distances.

Related Topic

In season 2 of *NUMB3RS* ("In Plain Sight" – Episode 208), Charlie used *flock theory* to model behaviors in a crime ring. Flock theory studies the interactions of objects like flocks of birds or schools of fish. It is closely related to traffic flow theory; in fact, many of the same models are used. The major difference is that flocks and schools generally operate freely in three dimensions, making it even more complicated. To learn more about flock theory, download the activity "Follow the Flock" by going to **http://education.ti.com/exchange** and searching for "6206."

Additional Resources

The CellSheet App comes preloaded on the TI-83 Plus Silver Edition and TI-84 Plus Silver Edition graphing calculators. For the TI-83 Plus or TI-84 Plus, this App can be downloaded for free from **http://education.ti.com/cellsheet**.

The Turner-Fairbank Highway Research Center has a comprehensive and very readable article regarding the history of modeling traffic flow mathematically. It also gives a good perspective on all the different aspects and their respective variables that have to be considered. For more information, go to:
http://www.tfhrc.gov/pubrds/janfeb99/traffic.htm

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