NUMB3RS Activity: Parabolic Food Fight
Episode: “Hot Shot”

Topic: Parabolas and curve fitting **Grade Level:** 9 - 12

Objective: Solve simultaneous equations; fit a curve to data

Time: 30 minutes

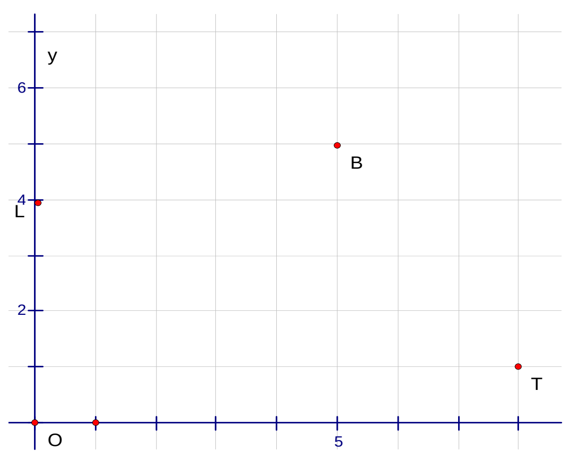
Materials: TI-83/84 Plus Calculator

Introduction

In “Hot Shot,” Larry is in his office catapulting grapes with a spoon to practice for the Physics Department food fight. This catapulting action is similar to work that was done in the Middle Ages, when Galileo Galilei found that the path of a projectile can be modeled by a parabola.

As Larry practices launching a grape with a spoon, things that he might consider are how hard he hits the spoon to launch the grape, where his target is, any obstacles between him and the target that he would like to miss, and so on. Some of these issues are considered below, and an activity to simulate a grape toss is found in the Extensions.

Though not recommended as actual practice for a food fight, this activity allows students to approach the problem mathematically and to review algebraic solutions of equations as well as practice curve fitting to determine a parabolic path of a projectile. In the graph shown below, point *L* represents the thrower (Larry), point *B* is a point that the path passes through (slightly above an obstacle that the path must miss), and point *T* is the target that Larry tries to hit. The toss is modeled on a graph without measurement units but in reality, the units could be feet.



In the discussion with students, you may want to emphasize that the form of a parabola that they will use is $y = ax^2 + bx + c$, where a is nonzero. To find a quadratic equation of this form using the known coordinates of points, *L*, *B*, and *T*, we can substitute the respective values of x and y in the equation resulting in equations with three unknowns, a, b, and c. You may want to review how to solve systems of equations algebraically.

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Later in the activity, we solve the equations by setting up a matrix equation, $AX = B$, where A is the 3 × 3 coefficient matrix, X is a 3 × 1 variable matrix, and B is the constant matrix. The solution is found by multiplying both sides of the equation *on the left* by A^{-1}, the inverse of matrix A (if it exists). Students are asked to compare answers with the different methods.

Next, students experiment with finding a parabola through any three points (if they are not collinear) by finding random values for the coordinates of the points. Keystrokes are included for the calculator portions, but unless students have previously studied the mathematics behind the activities, you might consider omitting the calculator parts. Additionally, it is noted that some students may choose to solve systems of equations using Gauss-Jordan elimination, but we have not included that method here.

Students may review a related activity for the plotting of parabolas using parametric equations in the *NUMB3RS* Activity **Where is the Bullet?** This activity can be downloaded for free from **http://www.cbs.com/primetime/numb3rs/ti/activities/Act4_Whereisthebullet_Convergence_final.pdf**.

Discuss with Students

1. Explain why you think a parabola will (or will not) pass through any three points.

2. How is a quadratic equation related to a parabola?

3. Would Larry have to worry about an obstacle if the obstacle were lower than his target?

4. How far do you think that you could toss a grape with a spoon?

5. What are some techniques that Larry can use to modify the trajectory of his grape?

Discuss with Students Answers:

1. *A parabola will pass through any three points as long at they are non-collinear (not all on the same line). Any three non-collinear points can be located with an equation of degree two.* **2.** *A quadratic equation is a polynomial function of the form* $y = ax^2 + bx + c$, *where a is nonzero. The name "quadratic" comes from the Latin "quadratus," meaning "square," because quadratics arise in the calculation of areas of squares. In the case where x and y are real numbers, the graph is parabolic.* **3.** *If Larry is tossing his object through a parabola that is turned downward, the answer is no. If he were throwing underhanded and "throwing up" at his target (the parabola is turned up), he might have to worry about the obstacle.* **4.** *Answers will vary, but many will be surprised at the distance a grape can travel when catapulted by a spoon.* **5.** *Answers may vary; adjust how hard he hits the spoon; adjust the angle of the spoon on the table; change the spoon size, etc.*

Student Page Answers:

1a. *L(0, 4); B(5, 5); T(8, 1)* **1b.** $c = 4$; $25a + 5b + c = 5$; $64a + 8b + c = 1$;

1c. $a = \frac{-23}{120}$; $b = \frac{139}{120}$; $c = 4$; **1d.** $y = \frac{-23}{120}x^2 + \frac{139}{120}x + 4$ **2.** *a = –0.1916666667; b = 1.15833333; c = 4; the decimals are approximations of the fractions obtained in Problem 1.* **3.** *If there is no solution, the points could be collinear. No parabola will go through three collinear points. This means that matrix A is non-invertible.* **4.** *The solutions are basically the same; there are decimal approximations for the fractions.*

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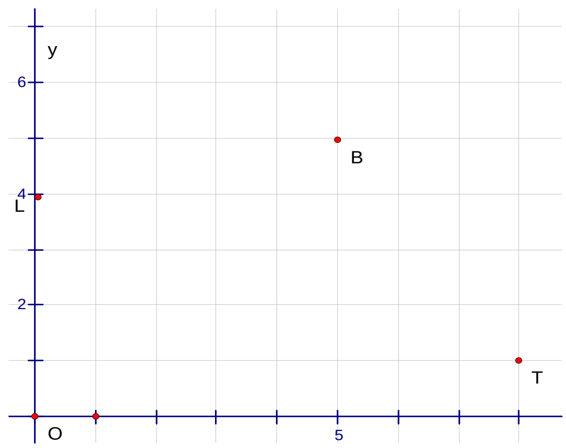
Name ____ Date _________________

NUMB3RS Activity: Parabolic Food Fight

In "Hot Shot," Larry is in his office catapulting grapes with a spoon to practice for the Physics Department food fight. This catapulting action is similar to work that was done in the Middle Ages, when Galileo Galilei found that the path of a projectile is parabolic in nature. These parabolic paths can be modeled with quadratic functions.

As an exercise (and not recommended for actual practice for a food fight), this activity reviews algebraic solutions of equations as well as practices curve fitting to determine a parabolic path of a projectile. In the graph shown below, the point marked *L* represents the thrower (Larry), the point marked *B* is a point that the path passes thorough and is slightly above an obstacle that the path must miss, and the point *T* is the target that Larry tries to hit.



1. **a.** Identify the coordinates of points marked *L*, *B*, and *T*.

b. One way to write the equation of a parabola (as a quadratic function) is $y = ax^2 + bx + c$, where *a* is nonzero. Substitute the values of *x* and *y* from part **a.** into the equation to determine three equations in *a*, *b*, and *c*.

c. Using substitution, solve the system of three equations from part **b.** to find the values of *a*, *b*, and *c*.

d. What is the quadratic equation that determines the path of the object being thrown?

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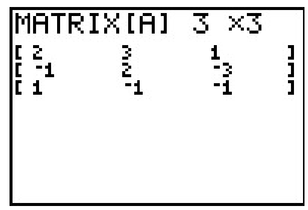
Another method of finding the solution of three equations in three unknowns is to use the TI-84 Plus calculator to solve the matrix equation $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. In the example below, the system of equations on the left could be represented by the matrix equation on the right.

$$\begin{aligned} 2a + 3b + c &= 7 \\ -1a + 2b - 3c &= 4 \\ a - b - c &= 5 \end{aligned} \qquad \underset{A}{\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & -3 \\ 1 & -1 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{B}{\begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix}}$$

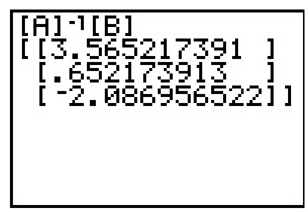
The matrix equation can be solved using the TI-83/84 Plus using the following steps. (You may need to first clear some existing matrices from your calculator's memory.)

* Press [2nd] [MATRIX].
* Go to the **Edit** menu and select Matrix *A*.
* Define Matrix *A* as a 3 × 3 matrix, and enter the coefficients of *a*, *b*, and *c* above.



* Follow the same directions to create a 3 × 1 matrix for *B*, using the constants shown on the right-hand side of the equations.

* To solve the equation, find $A^{-1}B$ (where A^{-1} is the inverse of matrix *A*). Press [2nd] [MATRIX] [1] to select matrix *A*, and press [x^{-1}]. Then press [2nd] [MATRIX] [2] to select matrix *B*. Press [ENTER] to obtain the values seen at the right, which are approximations of the solution to the system of equations.



2. Use your calculator to solve the system of equations from Question 1. How do the calculator's answers compare to those obtained by hand?

3. Define new matrices *A* and *B* as in Question 2, but use the command **randint(1,10)** for each of the entries. (To find **randInt(** press [MATH] [◄] [5].) Solve your new system of equations. What does it mean about the points if there are no solutions? What does this mean about matrix *A*?

4. Use your calculator to determine a quadratic regression equation for the points from Question 1. Enter the *x*-coordinates of the points in list L_1 of your calculator, and enter the *y*-coordinates of the points in list L_2. To find the quadratic regression equation, use the command **QuadReg** $\mathbf{L_1,L_2}$. (To find **QuadReg**, press [STAT], go to the **CALC** menu, and select **5:QuadReg**.) How does this equation compare to the solutions you obtained in Questions 1 and 2?

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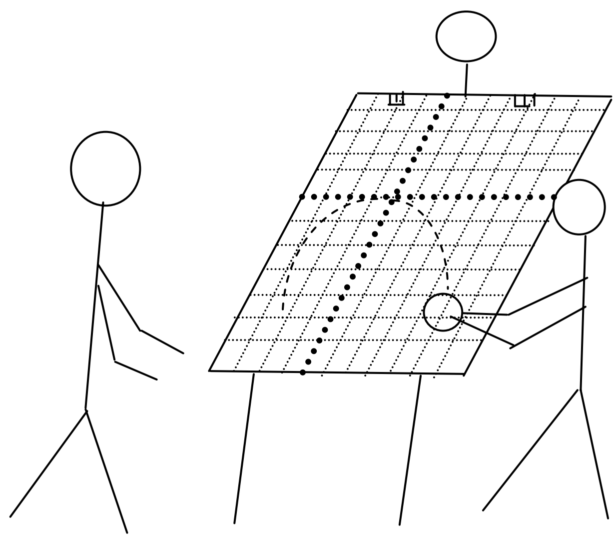
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

<u>For the Student</u>

A grape toss can be modeled with an experiment that requires the following materials: a tennis ball, a container of water soluble finger paint, a flip-chart with a grid imprinted on it, and rubber gloves (to protect the hands from the paint). Note that this is not a catapult, but it does model the path of the grape.

This activity will work best when performed with several people aiding in the work. A setup like the following will work with one person holding the flip-chart (making sure that the bottom of it is supported on a table and that it tilts backwards), one person to catch the ball, and one person to roll the ball. Dip the ball in the paint. The person with the ball rolls it up the flip chart. The ball will leave a paint trail in a parabolic path, similar to the dotted path in the illustration below.



With the graph lines showing on the flipchart, the *x*- and *y*-axes can be chosen. Coordinates of points can be estimated, and the equation of the path of the tennis ball can be found. You may want to move the axes to different places or even turn the graph upside down to get different equations of parabolas. Experiment with the model and try several pieces of graph paper to see at what angle you roll the ball to get the greatest height (and stay on the paper), and how you might roll the ball to get the most “width across” regardless of height. You might even model Larry’s problem by sketching in an obstacle that you want to miss and a target that you want to hit.

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Additional Resources

- The Web site below contains an applet for drawing and moving a parabola. Try this applet to see how moving the vertex of the parabola changes the equation.
  http://members.shaw.ca/ron.blond/TLE/QR.PARABOLA.APPLET/index.html

- Explore the Transformation Graphing App on the TI-83/84 Plus. This App can be downloaded for free from the Web site below.
  http://education.ti.com/transgraph

- For more information about parabolas and other conic sections, visit **http://www.2dcurves.com/conicsection/conicsectionp.html**.

- Technically, the trajectory of such things as grapes, water, etc, is an ellipse, with the far focus of the ellipse located at the center of the earth. Because the foci of the ellipse are so far apart, the ellipse can be modeled extremely well by a parabola. For more information about the elliptical paths of objects, visit the Web site below.
  http://www.lasalle.edu/~smithsc/Astronomy/Orbits/orbits.html

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