NUMB3RS Activity: Circling Around
Episode: "Rampage"

Topic: Venn Diagrams, Intersecting Circles, and Intersecting Spheres **Grade Level:** 6 - 12

Objective: Using Venn Diagrams with related circle and sphere problems to narrow searches

Time: 30 minutes

Introduction

In "Rampage," Charlie and Don Eppes discuss Inequality Bounding as a method of trying to locate a suspect. (This method is based on knowing a location where the suspect was earlier and knowing an area of close contact where movement might have taken place.) Though their conversation is interrupted, the ideas lead to a variety of possibilities for narrowing the location of a suspect, or alternatively narrowing a set of data to determine more about a suspect. One aspect of sorting data and narrowing possibilities leads to Venn diagrams and the notion of using circles or spheres to help locate a position. This aspect is explored in this activity.

A Venn diagram, named for its developer, John Venn (1834–1923), uses a set of circles to organize data. To consider a Venn diagram as a tool for narrowing down a list of suspects, consider the following information about a group of citizens.

In a group of 30 citizens, there are 16 males, 16 Caucasians, and 10 individuals over 6 feet tall. Furthermore, there are 6 Caucasian males, 3 males over 6 feet tall, and 5 Caucasians more than 6 feet tall. If there is exactly one Caucasian male over 6 feet tall, how can the rest of the data be sorted?

In the **Discuss with Students,** you are encouraged to use a Venn diagram to analyze the data. Students should be allowed to discuss how the numbers of individuals must overlap to have only a total of 30 citizens involved. (This portion of the activity should be accessible to middle school students.)

The use of Venn diagrams can help in the narrowing and organizing of information. The idea behind the use of circles in the diagrams to narrow information can also be generalized through geometric ideas to help locate points or areas based on given information. For example, suppose two dogs are secured to posts with chains that are 3 feet and 8 feet long. What is the largest overlap of areas of roaming of the two dogs? The smallest?

The ideas of circles overlapping in two dimensions can be expanded to three dimensions using spheres. The Federal Communications Commission (FCC) has required wireless providers and makers of cell phones to use "Enhanced 911 Services" that allow emergency services to find a person by tracking a cell phone to within a 125-meter radius. Virtually all providers use the detection of radio frequencies sent from the phone to emergency service antennas and the normal phone service antennas. Through a process called triangulation, the emergency services personnel can "narrow" down the actual location anytime the cell phone is turned on. Charlie and Don can use this tracking device to help locate a suspect by considering the intersection of three spheres that have the antennas as centers.

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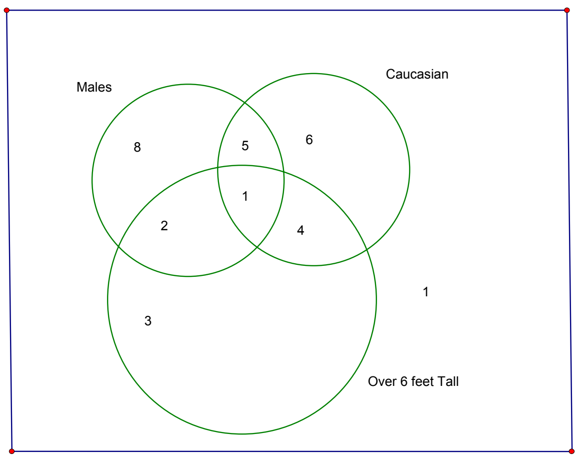
Discuss with Students

1. Analyze the data in the **Introduction** to determine how many non-Caucasian females in the group are **not** over 6 feet tall.
2. Suppose two dogs are secured to posts with chains that are 3 feet and 8 feet long. What is the largest intersection of areas of roaming of the two dogs? The smallest? Draw pictures to support your answers.
3. If there were three overlapping circles, what are the minimum and maximum areas of the overlap of all three circles?
4. What are the possible points of intersection of two spheres? Three spheres?

Answers to Discuss with Students:
1. *A natural way to organize the information is to use a Venn diagram as seen below. There is 1 non-Caucasian female who is not over 6 feet tall.*



2. *Drawings may vary. The greatest area of overlap is seen when both dogs are chained to the same post and is the area of the circle produced by a radius of 3 feet. The least is 0 square feet, when the circles formed with radii the lengths of the chains touch in a single point or when the posts are more than 11 feet apart.* **3.** *The greatest possible area is the area of the smallest of the three circles [Think about the circles nested inside one another but tangent at one point.]; the circles could have exactly one point in common and have no common overlap, thus an area of 0.* **4.** *Two spheres could have no points in common [not touch at all], all points in common [have the same center and the same radius], or one point in common [be tangent]. Three spheres could have no points of intersection, a single point in common if they are nested and tangent to the same plane, two points in common [think of two spheres intersecting in a circle and the third intersecting that circle in two points, or a circle of points [think of two of the spheres being identical and intersecting the third in a circle.*

Student Page Answers: 1a. *(a) represents California adult females (women); (b) represents non-California adult females; (c) represents non-California girls; (d) represents California girls; other groups include non-CA males and Californians* **1b.** *There are 17 non-California women with 29 women all together so there are 12 California women. With 26 California females, there are 26-12 or 14 California girls. With 29 girls altogether, there are 29-14 or 15 non-California girls. Now we have 17 non-California males, 17+15 or 32 non-California females, 26 California females as seen above and 44-26 or 18 California males. Then add the non-Californians, 17 +32, and the Californians, 26+18, to get 93 as the total.* **2a.** *The equations are* $x^2 + y^2 = 225; (x-25)^2 + y^2 = 400$
2b. *(9, 12) and (9, –12)* **2c.** *106.26 degrees; 73.74 degrees*
2d. $\frac{73.74}{360}\pi(20^2) - \left(\frac{1}{2}(24)(16)\right) + \left(\frac{1}{2}(24)(16)\right) + \frac{106.26}{360}\pi(15^2) - \left(\frac{1}{2}(24)(8)\right)$, *or approximately 166.04 square units*
3a. *(6, 8, 10)* **3b.** *Answers will vary but the antennae could be located at the centers of the spheres; the antennae would be the lengths of the radii from the cell phone, and the cell phone user would be at the point of intersection of the three spheres.*

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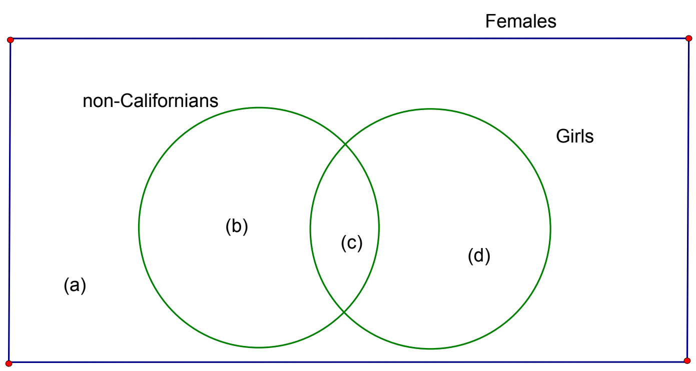
Name ____ Date ______________

NUMB3RS Activity: Circling Around

In "Rampage," Charlie and Don Eppes discuss Inequality Bounding as a method of trying to locate a suspect. This method is based on knowing an earlier location of the suspect and knowing an area of close contact where movement might have taken place. Charlie has defined data points of a stalker and his victim, and his idea is that by employing their last known locations, they can focus their search efforts to a specific location. A Venn diagram is used to visualize this process. A Venn diagram, named for its developer John Venn (1834–1923), uses a set of circles to determine how to **organize** data.

1. In a group of potential suspects, there were both Californians and non-Californians about which bits of information are known. The group contained 26 California females, 17 non-California women (adult females), 17 non-California males, 29 girls (non-adult females), 44 Californians, 29 women, and 24 non-California adults.

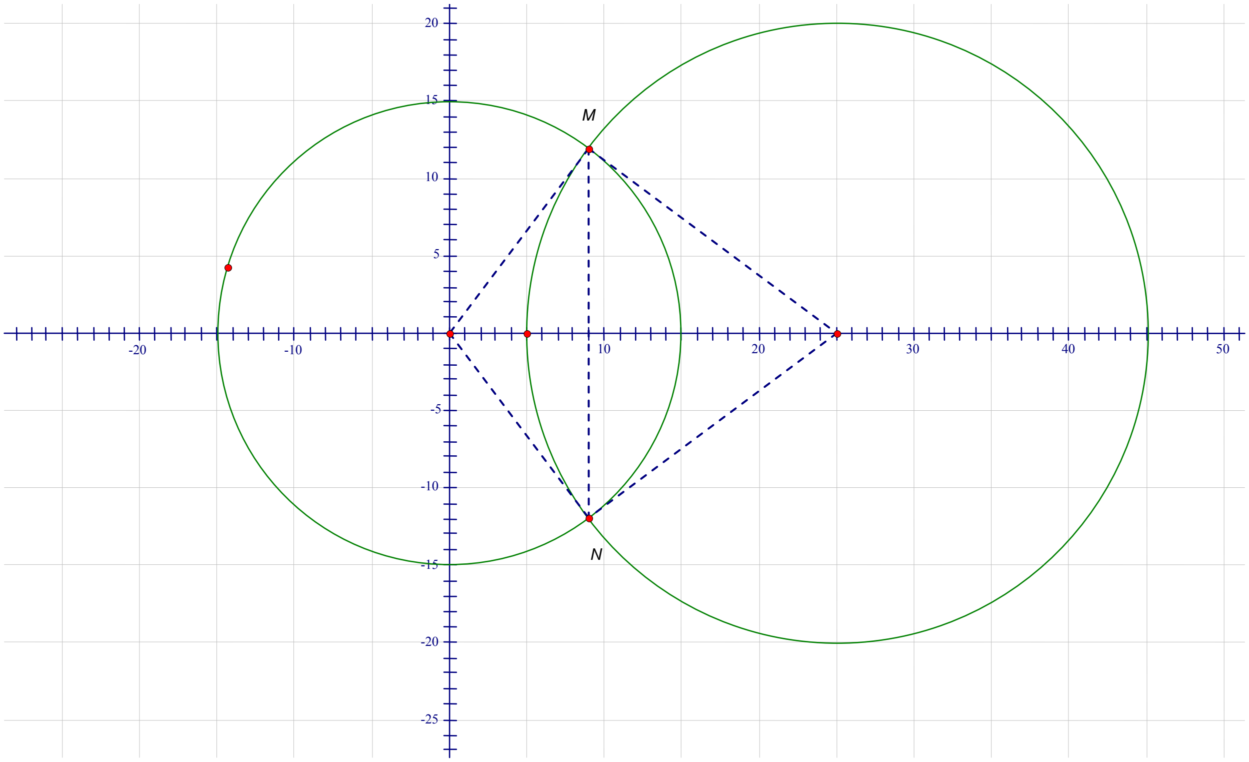


[This problem is based on one found in Billstein, et al. (1984). See reference in resources.]

a. Identify types of people in each region. (Hint: Consider one circle that contains only non-Californians, one that contains only girls, and a rectangle representing the females as shown. Use this information to then draw conclusions about the number of males in the group. Identify the characteristics of each region of the diagram, and groups that are not included in the diagram.)

b. How many total people are in the group?

2. Two suspects in a criminal case have circles of influence in Los Angeles with radii of approximately 20 blocks and 15 blocks respectively as shown.



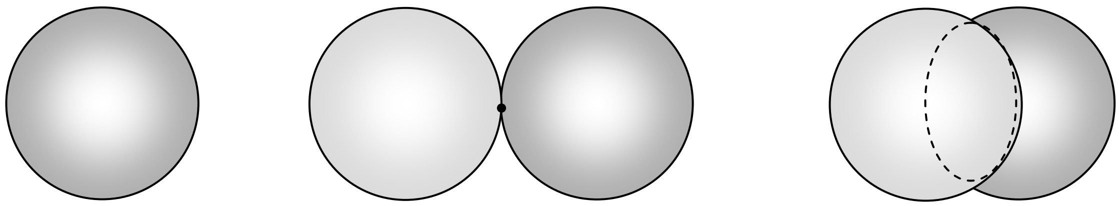
a. Write the equations of the two circles.

b. Find the coordinates of the points of intersection (*M* and *N*) of the two circles. Segments connecting *M* and *N* with the center of each circle are shown in the diagram.

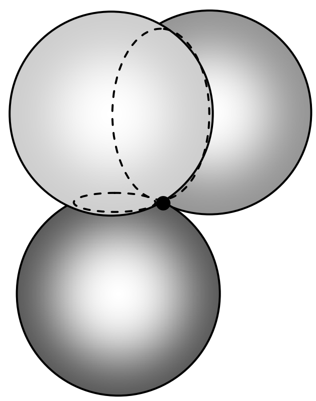
c. Determine the measures of the central angles formed in part b. (Hint: Think about the right triangles formed by the segments shown in the diagram.)

d. A **segment** of a circle is determined by a chord and an arc of the circle. The intersection of the two circles depicted above could be separated into two segments, one in each respective circle. The area of a segment can be found by subtracting the area of a triangle from the area of a sector. Find the area of the intersection of the two circles. (Hint: To find the area of a sector, use the proportion $\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Central angle}}{360^\circ}$)

When two spheres intersect, they could be the same sphere and have all points in common, they could “kiss” (touch in a single point), or they could intersect in a circle as shown below.



If we know that only a single point is located on both of the spheres, then the spheres kiss. However, if three spheres intersect in a single point, two of the spheres kiss and the other sphere intersects each of these spheres in a circle. The point where the three spheres intersect is where both of these circles intersect, as shown below.



By putting the spheres on a three-dimensional grid and writing equations of the spheres in the form, $(x-h)^2+(y-k)^2+(z-j)^2=r^2$ we can write the equations of spheres with centers at (*h, k, j*) and radius *r* and find a single point of intersection if it exists.

3. Consider three spheres with radii of $\sqrt{200}$, $\sqrt{50}$, and 10 inches respectively and that intersect in one point. Suppose the spheres have centers located respectively on a three-dimensional coordinate system at the points with coordinates (0, 0, 0), (3, 4, 5), and (6, 8, 0).

a. Determine the coordinates of the point of intersection of the spheres.

b. If you have a cell phone and are located at the point of intersection of the three spheres and the centers of the spheres represent antennae, describe how your location can be found.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Extension 1
Use the Web site below (or other similar Web sites) to develop a brief essay on the legal issues of tracking via cell phones.
http://www.legalaffairs.org/issues/July-August-2003/feature_koerner_julaug03.msp

Extension 2
Use the Web site below to consider how private investigators could track cars.
http://www.pimall.com/nais/news/n.satellite.html

Extension 3
The Web site below has a lesson and activity about using the Global Positioning System (GPS). This can be used with middle school students.
https://www.ion.org/satdiv/education/lesson8.pdf

Other Resources
Billstein, R., S. Libeskind, and J. Lott. *A Problem Solving Approach to Mathematics for Elementary Teachers*. Menlo Park, CA: The Benjamin/Cummings Publishing Company, 1984.

There is a *NUMB3RS* activity which uses the TI-Navigator™ system to form a two-dimensional model of how GPS systems work. This activity accompanies the “Convergence” episode, and can be downloaded for free from the Web site below.
http://www.cbs.com/primetime/numb3rs/ti/activities/TINavigator_Trilateration_Convergence.pdf

The Web site below contains a coordinate activity using TI-Navigator.
http://education.ti.com/educationportal/activityexchange/activity_detail.do?activityid=4186&cid=us

The Web site below contains a TI-Navigator activity that uses triangulation to determine a location.
http://education.ti.com/educationportal/activityexchange/activity_detail.do?activityid=5541

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