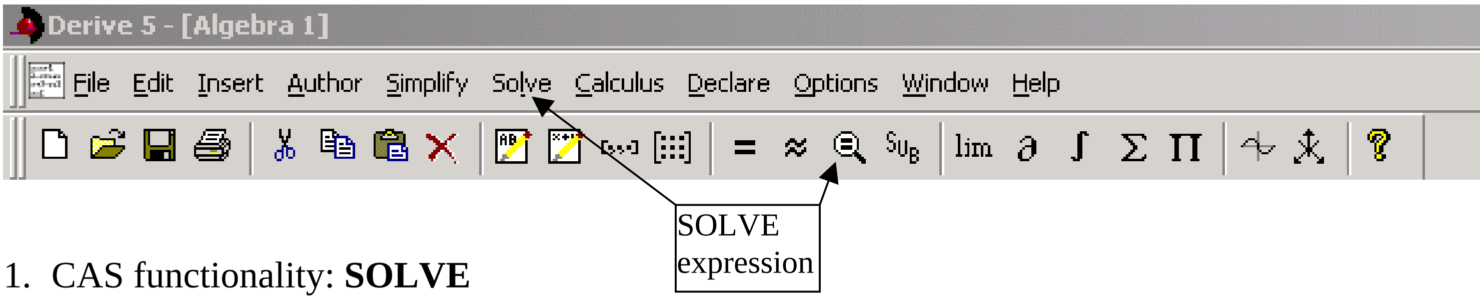
An Introduction to Derive

Part 1: Familiarisation with the software



1. CAS functionality: **SOLVE**

Years 7 to 10 examples

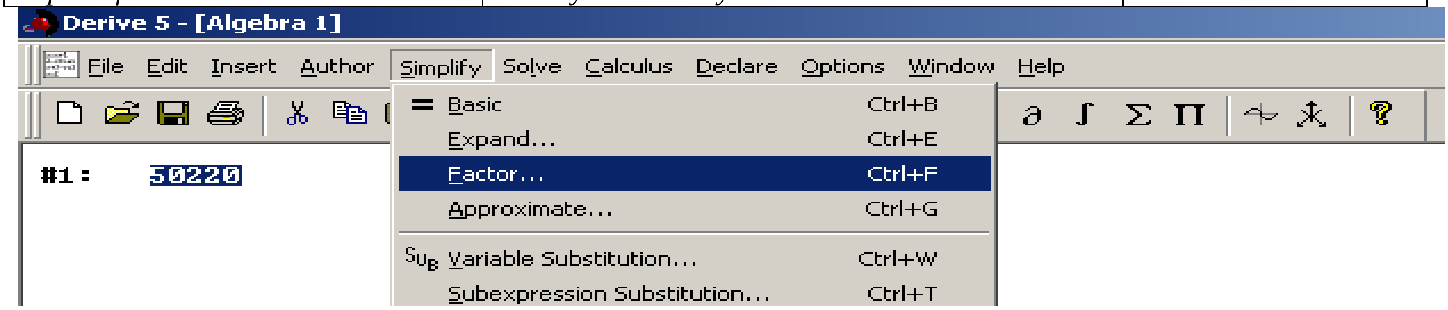
Task	Derive 5 steps	Answer
a. Algebraic solve Solve $10a + 2 = 15a - 1$ for a	Enter $\boldsymbol{10a + 2 = 15a - 1}$ Solve menu: select 'Expression' Dialog box should show Variable: a, algebraic & real solution Click the "Solve" button ***[or type* solve(*10a + 2 = 15a –1,a*)]**	
b. Rearranging a literal equation Rearrange $\boldsymbol{E = mc^2}$ to make $\boldsymbol{m}$ the subject	Enter $\boldsymbol{e = m \times c\text{^}2}$ Solve menu: select 'Expression' Dialog box: Variable: m, algebraic [or *Type* **solve(*e = m × c*^2,*m*)]**	
c. Rearranging a literal equation Rearrange $\boldsymbol{E = mc^2}$ to make $\boldsymbol{c}$ the subject	Edit the entry line to **solve(*e = m × c*^2,*c*)**	
d. Solve the simultaneous equations $y = 4x - 5$ and $2x + 3y = 8$	Solve Menu: select 'System' Dialog box: 2 equations OK Dialog box: Enter equations: $y = 4x - 5$ & $2x + 3y = 8$. SOLVE	

2. CAS functionality: **SOLVE**

Years 11 & 12 examples

Task	***Derive 5* steps**	Answer
a. Algebraic solve Solve $x^2 + 7x - 3 = -2x - 5$ for x	Enter $x^2 + 7x - 3 = -2x - 5$ Solve menu: select 'Expression' Dialog box should show Variable: x, algebraic & real solution. *Click "Solve"*	

	[or Type **solve(***x^2 + 7x – 3 = -2x – 5,real, x)]*	
b. Solving literal equations Find the general solution of the quadratic expression *ax^2 + bx + c = 0* for the variable *x.*	Enter ***ax^2 + bx + c =0*** Solve menu: select 'Expression' Dialog box: Variable: x, algebraic, real [or *Type* **solve(*ax^2+ bx + c =0,x)]***	
c. Find the general solution of the quadratic function *ax^2 + bx + c = 0* wrt parameter *b.*	Edit the entry line to **solve(*ax^2 + bx + c =0,b)***	
d. Solve the simultaneous equations *x^2 + y^2 = 9* and *y^2 = x+3* *NOTE: Derive 5 accepts equations in implicit form.*	Solve Menu: select 'System' Dialog box: 2 equations OK Dialog box: Enter equations: *x^2 + y^2 = 9* and *y^2 = x+3.Click* " SOLVE"	



3. CAS functionality: **FACTOR**

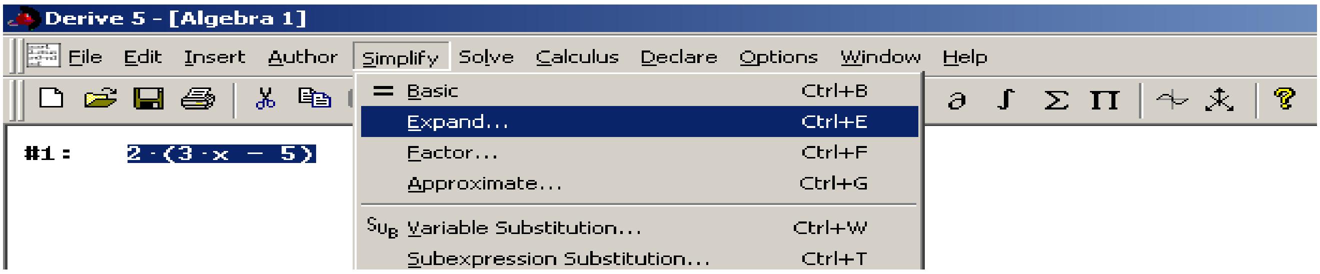
Years 7 to 10 examples

Task	***Derive 5* steps**	**Answer**
a. Prime factors Find the prime factors of 50220	Enter *50220* Simplify menu: select 'Factor' Dialog box: click FACTOR	
b. Lowest Common Denominator Express 1/a + 1/b with a common denominator	Enter ***1/a – 1/b*** Simplify menu: select 'Factor' Dialog box: click FACTOR button	
c. Common factors Factorise 8ab + 12ac	Edit the enty line to ***factor(8ab + 12ac)***	
d. Patterns with quadratic factors Factorise $a^2 – b^2$ $4c^2 – 9d^2$ $b^2 – a^2$	Enter ***a^2 - b^2*** Simplify menu: select 'Factor' Dialog box: click FACTOR button Edit entry line to ***factor(4c^2 – 9d^2)*** Edit entry line to ***factor(b^2 – a^2) etc***	

4. CAS functionality: **FACTOR** (Years 11 & 12 examples)

Task	***Derive 5* steps**	**Answer**
a. Factorising polynomials over Q Find the factors of *x^4 - 9 over the Rational field*	Enter *2x^4 – 3x^3 –3x – 2* Simplify menu: select 'Factor' Dialog box: select 'Rational' solution	
b. Factorising polynomials over R Find the factors of *x^4 - 9 over the Real field*	Enter *2x^4 – 3x^3 –3x – 2* Simplify menu: select 'Factor' Dialog box: select 'Radical' solution	
c. Factorising polynomials over C Find the factors of *x^4 - 9 over the Complex field*	Highlight *2x^4 – 3x^3 –3x – 2* Simplify menu: select 'Factor' Dialog box: select 'Complex' solution	
d. Common Denominator Express $x+2+\frac{2}{x+1}$ with a common denominator	Enter ***x+2+2/(x+1))*** Simplify menu: select 'Factor' (rational) Dialog box: click FACTOR	

e. Partial fractions Express the partial fractions $\frac{3}{x+1}+\frac{5}{x-2}$ as a single expression	Enter ***3/(x+1)+5/(x-2)*** Simplify menu: select ‘Factor’ (rational) Dialog box: click FACTOR	



5. CAS functionality: **EXPAND**

Years 7 to 10 examples

Task	***Derive*** **5 steps**	Answer
a. Binomial expansion Expand ***2(3x – 5)***	Enter ***2(3x – 5)*** Simplify menu: select ‘expand’	
b. Write as the sum of two fractions Express $\frac{2a^2+3b}{5ab^2}$ as the sum of two fractions	***(2a^2+3b)/(5a*b^2)*** Simplify menu: select ‘Expand’	
c. Expanding 3 factors Expand (2x + y)(x – 3y)(x + 2y)	Enter ***(2x + y)(x – 3y)(x + 2y)*** Simplify menu: select ‘Expand’	

6. CAS functionality: **EXPAND**

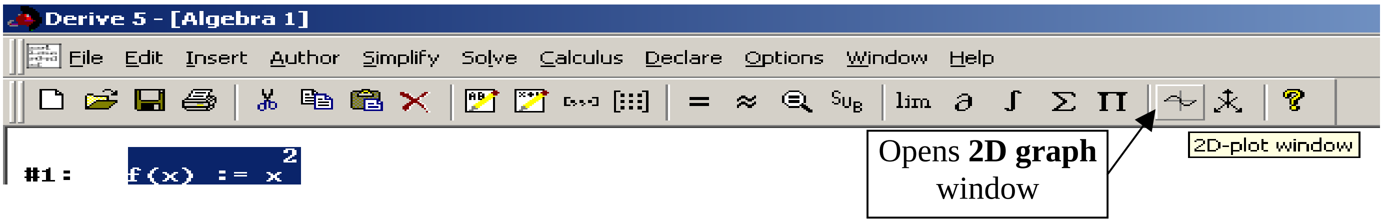
Years 11 & 12 examples

Task	***Derive*** **5 steps**	Answer
a. Binomial expansion Expand $(3x-\frac{5}{x^2})^6$	Enter ***(3x – 5/x^2)^6*** Simplify menu: select ‘expand’ (Rational)	
b. Polynomial division Express $\frac{x^2+5x+6}{x+1}$ as the sum of a quotient and remainder.	Enter ***(x^2+5x+6)/(x+1)*** Simplify menu: select ‘Expand’ (Rational)	
c. Expanding trig expressions Expand ***sin(2x)***	“Declare” Menu. Select “Simplification settings” Dialog box: Trigonometry: Expand. OK *Type* ***expand(sin(2x)).***	

7. CAS functionality: **DEFINE**

Task	***Derive*** **5 steps**	Answer
a. Define ***f***(x) = x^2 Evaluate ***f(-5)*** Simplify	*Type* ***f(x):=x^2.*** ***OR,*** ‘Declare’ menu: ‘Function definition’. In dialog box: function name & argument: *f(x)* Function definition: *x^2* *Then* ***f(-5)*** or type ***“f(-5)=”*** [Enter] ***Then f(x + h)*** or type ***“f(x + h)=”*** [Enter]	
b. Define $g(x)$ = sin(x)	*Type g(x):=sin(x)* ENTER. Then –2g(x), Then	

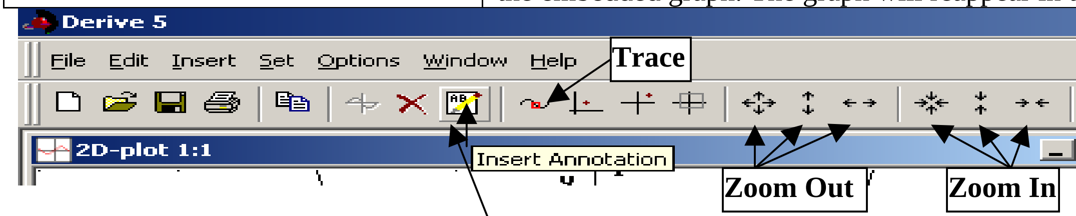
Evaluate *-2g(x)* Evaluate $g(x+\frac{\pi}{2})$	g(x+ π /2). To graph: "Window" menu: Tile vertically. Highlight the expression. Click the graph icon in the Graph window. Done.	



8. CAS functionality: **GRAPH**

a) Basic graphing

Task	***Derive 5* steps**
a. Define ***f(x)*** = x^2 Graph ***f(x)*** and **embed** the graph in the Algebra sheet**.**	*Type **f(x):=x^2*** to define the function. **To graph**: With the expression highlighted, Click "2-D graph" icon. This opens the graph window. Now click the 2D plot icon in the graph window menu bar. **To see the Algebra window and graph side-by-side**, go to the "Window" menu. Select: "Tile vertically". **To embed graph**: Click anywhere in the graph window, to select it. From the "File" menu select "Embed". **See how it works**. Close the graph window (click [×]). Double click on the embedded graph. The graph will reappear in the 2D graph window.

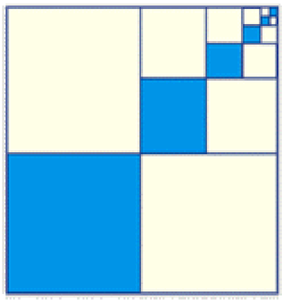


b) More on 2D graphs

Task	***Derive 5* steps**
b. On the same set of axes, graph i) ***f(x)*** ii) ***2f(x)***) iii) ***f(x + 1)*** iv) ***f(x – 2)*** v) ***f(x) – 2***	In the algebra window, enter the expression (eg ***2f(x)***) and click With the expressions highlighted, select the graph window and click the 2D graph icon. **Annotate the graphs.** Click the "Insert Annotation" button and annotate the graphs. **Zoom** in and out on the graphs.

9. CAS functionality: **CALCULUS menu - a Year 7 – 10 applications.**

Task	***Derive 5* steps**	**Answer**
a. Find the partial sums of the areas shades. What is the total area shaded?	**Finding partial sums** Each term is of the form $\frac{1}{4^n}$. **Entering** $\sum_{n=1}^{a}\frac{1}{4^n}$: Enter 1/4^n. From the "Calculus" menu and select "Sum", **or** click the [∑] toolbar button. Set the lower limit at 1 and upper limit at *a*. Click simplify. **Obtaining a table of partial sums for a =1 to a = 12** With $\sum_{n=1}^{a}\frac{1}{4^n}$ highlighted, go to the "Calculus" menu and select "Table". Set "starting value" at 1, end value at 12. Click "Simplify". The partial sums will be given as exact fractions. Click the [≈]	

	toolbar button to obtain the table with approximate decimal values. **Converging to what value?** Right-mouse click on $\sum_{n=1}^{a} \frac{1}{4^n}$. Select "Copy", the paste into the entry line and replace *a* with ∞ (from bottom toolbar) then .	

Part 2: Some classroom activities

Activity 1

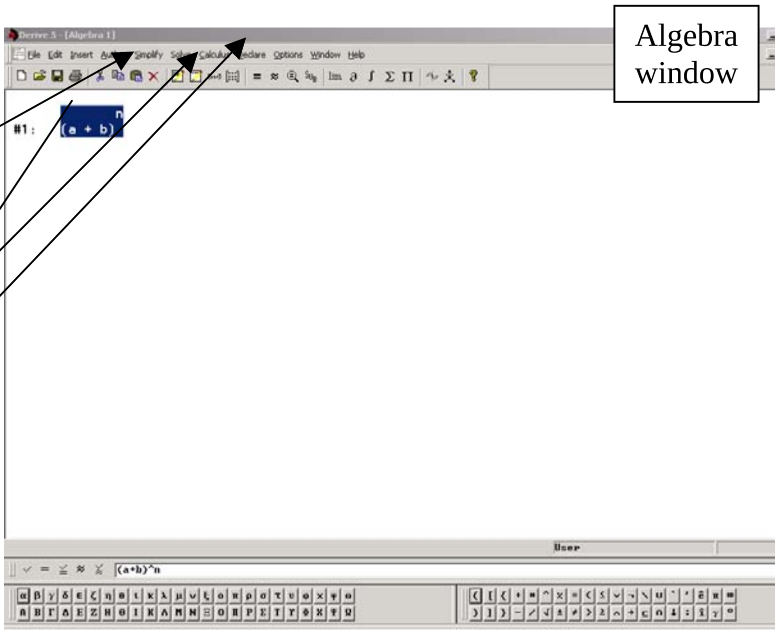
Exploring Patterns: Binomial coefficients & Pascal's Triangle

Aim: To investigate the expansion of $\boldsymbol{(a+b)^n}$, where *n* is a positive integer and n∈[0,10].

1. Open an *DERIVE* worksheet
2. Type **(a+b)^n**, in the "entry line".
3. Press ENTER. The expression will appear in the "Algebra w
4. With expression #1 highlighted, select "expand" from the *Simplify* menu.
5. In the dialog box, select: *variables* a & *b* and "*Rational*" factors. Click OK.

To obtain a table showing the expansion of $(a + b)^0$ to $(a + b)^{10}$:

6. With line #2 highlighted, select "Table" from the u.
7. In the dialog box, *le = n, starting value = 0, ending value = 10* and *step size* = *1*. Click OK.
8. Click the [=] *simplify* toolbar button.



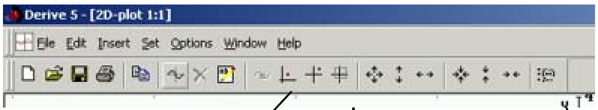
Questions.

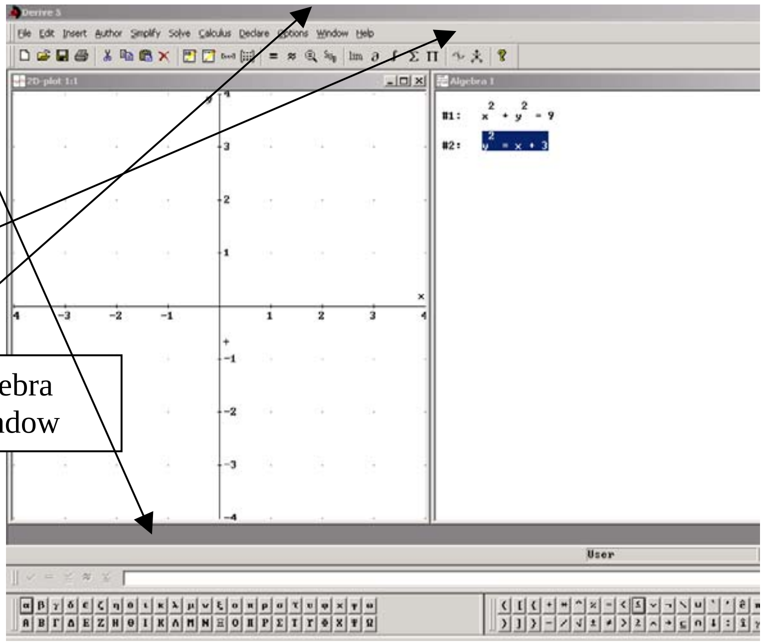
- In the expansions of $\boldsymbol{(a+b)^n}$**,** what patterns do you observe in the:
  a. powers of *a* and *b*
  b. coefficients (e.g. $a^3 + 3a^2b + 3ab^2 + b^3$ has coefficients 1, 3, 3, 1)
- From the patterns that you have observed, write the expansion of $(a+b)^{11}$. Use DERIVE to check your answer.

Activity 2

Graphing and solving in implicit form

Aim: To investigate the graphs and points of intersection of $x^2 + y^2 = 9$ and $y^2 = x + 3$





appear in the "Algebra window".

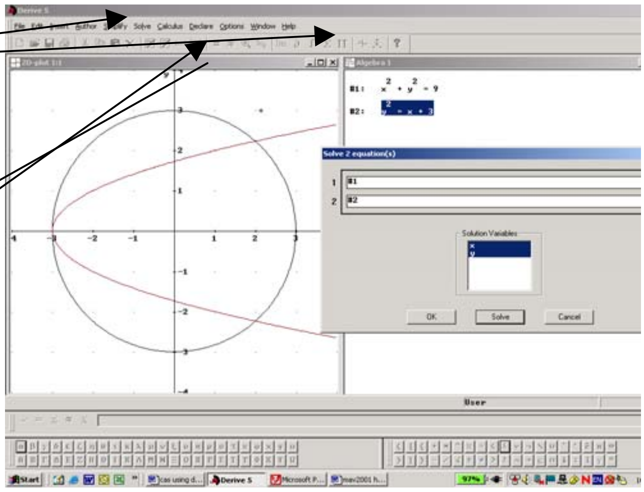
4. Type ***y^2=x+3***, in the "entry line".
5. Press ENTER.

To obtain a 2-D plot window beside the algebra window:

6. Click the *2-D plot* toolbar button.
7. Select *Tile vertically* from the *Window* menu.
8. Highlight ex… click on t… 2-D plot win… *plot expression* toolbar button.

Solving the system of equations $x^2 + y^2 = 9$ and $y^2 = x + 3$

9. Select the *Algebra window* by clicking anywhere in the *Algebra window* area.
10. Go to the *Solve* menu and select "*System*" (to solve a *system* of equations).
11. In the dialog box, select "*2*" equations. OK.
12. In the next dialog box enter #1and #2. OK.
13. To obtain an exact solution set, click [=]
14. To obtain rational approximations, click [≈]
15. The number of significant figures in the rational approximation, can be changed through the "Declare" menu: Declare>Output



Activity 3
Differentiation from first principles

Aim: To investigate derivatives of for the family of polynomial power functions $f(x) = ax^n$, from first principles, for $n \in [0, 10]$

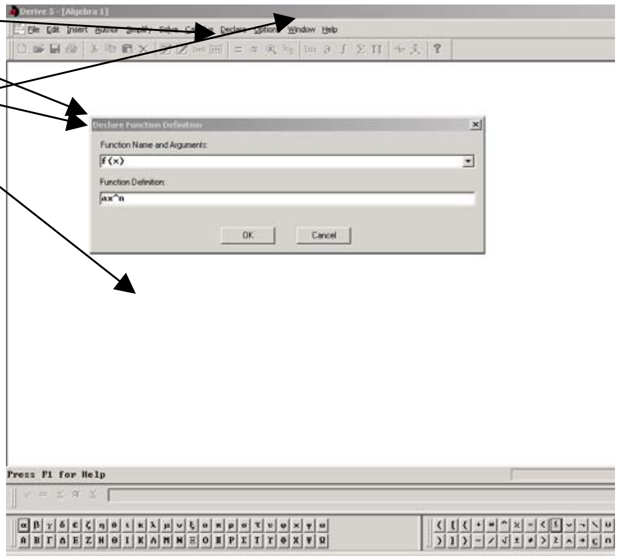
1. Define f(x) as ax^n. Go to the Declare menu.
2. In the dialog box: *function name* … is *f(x)*
3. *Function definition* is *ax^n*.

Now enter $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ as follows:

4. Type: (f(x+h)-f(x))/h in the *entry line*. Press Enter.
5. Click the lim toolbar button. (Or select *limit* from the *Calculus* menu).
6. In the dialog box select: *variable* is *h*, *limit point* is 0 and *approach from* is *both* left and right. OK.

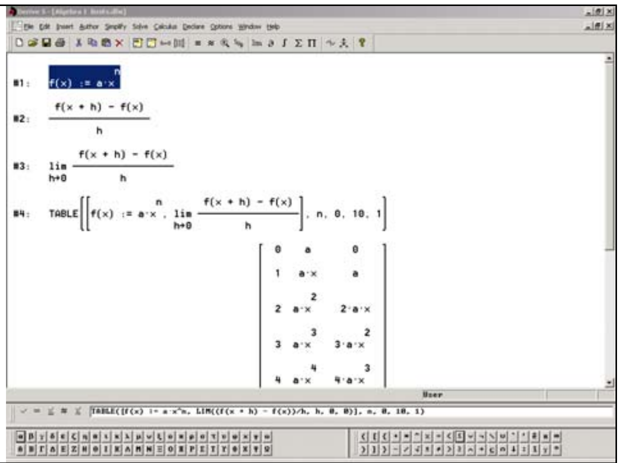
Now obtain a table of the derivatives for n =0 to n= 10

7. Select *Table* from the *Calculus* menu.
9. In the dialog box, select: *variable = n*, *starting value = 0*, *ending value = 10* and *step size = 1.* Click OK.
10. Click the [=] *simplify* toolbar button.



Questions.

- For *f(x)* = ax^n, what patterns do you observe in $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$?
- From the patterns that you have observed, write the derivative of :
  - a. $f(x) = ax^{11}$
  - b. $f(x) = 5x^{12}$
  - c. $f(x) = 5$



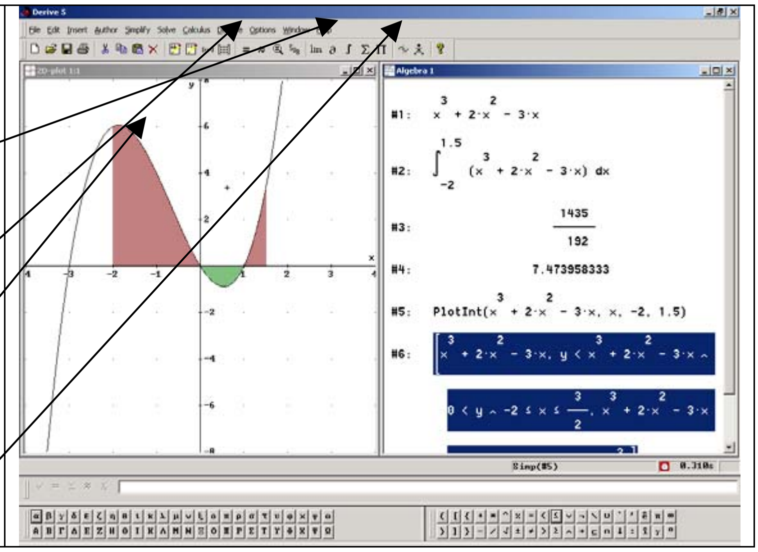
Activity 4
Integral Calculus

Aim: To investigate the integral $\int_{-2}^{1.5} (x^3 + 2x^2 - 3x)dx$ and the area bounded by the graph of *f(x)* =and the x-axis, for $-2 \le x \le 1.5$.

1. Open a new *DERIVE* worksheet

Evaluate $\int_{-2}^{1.5}(x^3+2x^2-3x)dx$ as follows:

2. Type *x*^3 + *2x*^2 – 3*x*, then ENTER.
3. Click the integral [$\int$] toolbar button
4. In the dialog box: *variable* is *x*, *Integral* is *Definite*, *upper limit* is 1.5 and *lower limit* is –2. Click OK.
5. Click [=]. Then click [≈] for approx.



To obtain a 2-D plot window, showing the region bounded by the graph of $f(x)=x^3+2x^2-3x$ and the x-axis, for $-2\le x\le 1.5$, beside the algebra window:

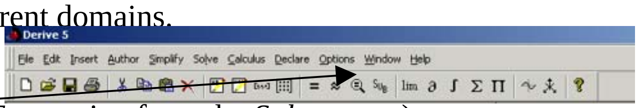
6. Click the 2-*D plot* toolbar button.
7. Select *Tile vertically* from the *Window* menu.
8. Type ***Plotint(#1,x,-2,1.5)*** in the *entry line*. Press Enter. Click [=] toolbar button.
9. With expression #6 highlighted, select the *2-D plot window* (by clicking in the window).
10. Click the *plot expression* toolbar button.

Questions.

- The value of $\int_{-2}^{1.5}(x^3+2x^2-3x)dx\approx 7.47$. Why is the area bounded by the graph of $f(x)=x^3+2x^2-3x$ and the x-axis, for $-2\le x\le 1.5$, not equal to 7.47?
- In fact, area is equal to 8.64 (to 2 decimal places). Write an expression to calculate the shaded area. Use DERIVE to find the exact area.

Activity 5
Solving Trigonometric Equations

Aim: To investigate solution to $2\sin(\theta-\frac{\pi}{6})=1,$ for different domains.



1. Enter, in the entry line. Press enter.
2. Click the *Solve Expression* toolbar button (or select *Expression* from the *Solve* menu)
3. In the dialog box: *variable* is θ, *solution method* is *algebraic*, *solution domain* is *real*. OK. [=].

Questions

- Use the answer provided by DERIVE to solve π over the following:

a. $-\pi\le\theta\le\pi$ b. $0\le\theta\le 2\pi$ c. $-2\pi\le\theta\le 0$ d.