

Coin Toss

ID: 9412

Time required

45 minutes

Activity Overview

In this activity, students will run two experiments that simulate pouring out coins from a bag. They will collect the data and graph it, using different methods to find equations to model the data. Then students will find the inverse of the data and corresponding equation to model it.

Topic: Logarithmic Functions

- Express the inverse of a given exponential function as a logarithmic function and graph it.
- Show that the graphs of the exponential and logarithmic functions are reflected images in the line $y = x$.

Teacher Preparation and Notes

- Prior to beginning the activity, students should have some experience with the general form of an exponential function $f(x) = a \cdot b^x$, understand the properties of logarithms, and also be familiar with properties of inverse functions.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “9412” in the keyword search box.

Associated Materials

- CoinToss_Student.doc
- CoinToss.tns
- CoinToss_Soln.tns

Suggested Related Activities

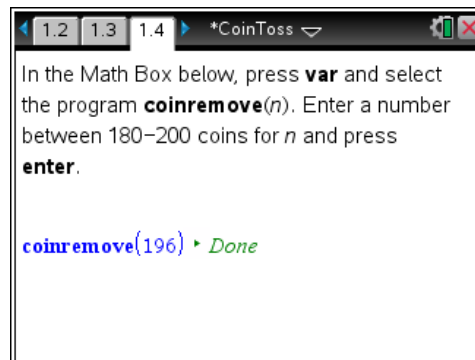
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Exponential Models and Logarithmic Scales (TI-84 Plus family) — 3722
- NUMB3RS – Season 2 – “Bones of Contention” – How to Get a Date (TI-84 Plus family) — 6274
- Exponential Growth (TI-Nspire technology) — 8254

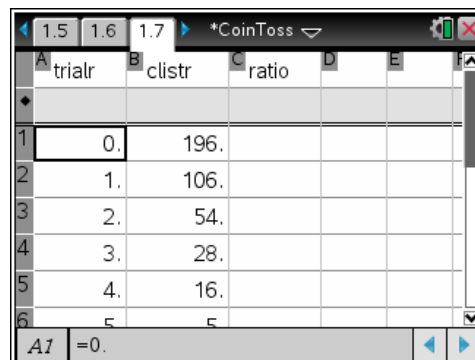
Exponential growth and decay are usually encountered with population and half-life problems. This activity uses two programs that simulate tossing coins out of a bag. The data is gathered and analyzed to develop exponential models and then to find the inverse logarithmic functions.

Problem 1 – Coin Toss Simulation: Exponential Decay

Students are directed to run the program **coinremove(n)** and enter in a number from 180–200. To access the program, students can press the **VAR** key (**var**), highlight **coinremove** and press **enter**. They can also use the alpha keys to type the name.

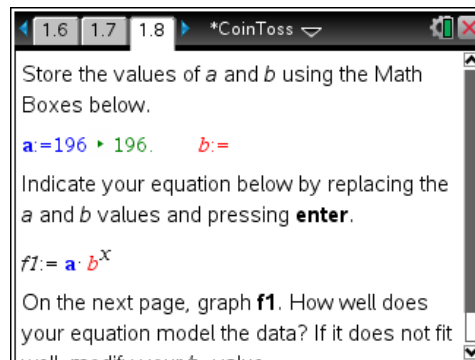


The program will generate two lists of data on page 1.7 with the trial number and the number of coins remaining after the ones that land heads up are removed.

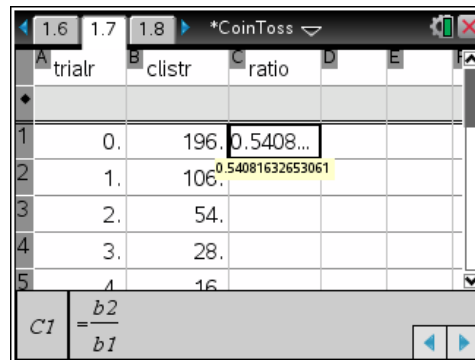


Students will find an equation in the form $y = a \cdot b^x$ to model this data where a is the initial value and b is the multiplier.

Students are to store their starting value as a on page 1.8. To do so, they should type in their value for a in the correct Math Box and press **enter**.



Students will find the ratio of the number of coins by dividing the 2nd term in the number of coins list by the 1st term. In cell C1, students should enter the formula $=b2/b1$.



To calculate the other ratios, have students select the **Fill Down** command from the Data menu. This will put a dashed box around the cell. Tell them to arrow down until the second to last data value and press **enter**.

trialr	clistr	ratio	
2.	34.	0.540816...	
4.	3.	28.	0.5714...
5.	4.	16.	0.3125
6.	5.	5.	0.4
7.	6.	2.	

$C6 = \frac{b7}{b6}$

Once students have their ratios calculated, they will find the average ratio and to store it as their *b*-value on page 1.8. To use the **mean(** command, the students can either type **=mean(** or type **=** and then select **mean(** from the Catalog ().

trialr	clistr	ratio		
1.	0.	196.	0.540816	0.47545
2.	1.	106.	0.509434	
3.	2.	54.	0.518519	
4.	3.	28.	0.571429	
5.	4.	16.	0.3125	
6.	5.	5.	0.4	

$D1 = \text{mean}(\text{ratio})$

Next, students are to enter in their equation into the expression box provided. At this juncture, it would be good to discuss what values they chose for the number of coins and what values they calculated for their average ratio.

Store the values of *a* and *b* using the Input Boxes below.

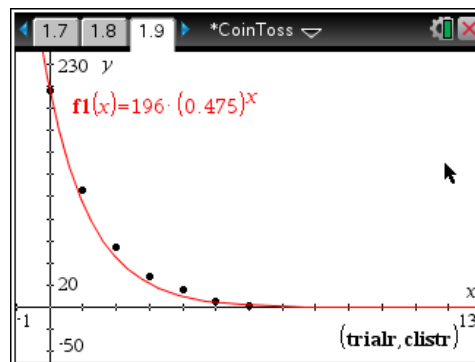
$a=196$ $b=0.47545$

Indicate your equation below by replacing the *a* and *b* values and pressing **enter**.

$f1(x) = 196 \cdot .47545^x$

On the next page, graph **f1**. How well does your equation model the data? If it does not fit well, modify your *b*-value.

On page 1.9, there is a scatter plot of their data. In the entry line, students should arrow up to **f1** and press **enter** to display the graph of their function. They should assess how well their equation models the data. If it does not fit well, students are to modify their *b*-value until they have a good fit.

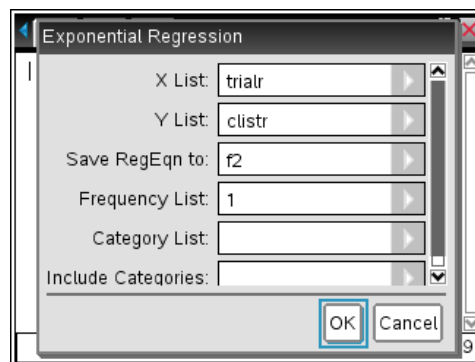


The students now have a series of questions that are good for a class discussion. Sample responses are in parentheses.

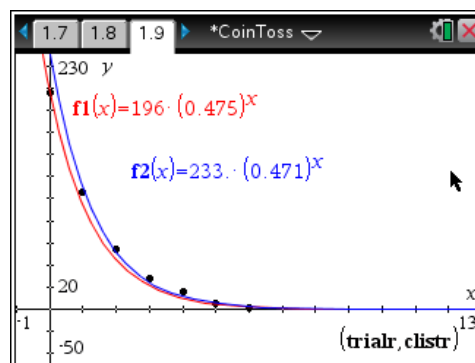
- Is this a growth or decay model? (decay)
- At what percent are the coins being removed? (48%)
- Explain why the b -value is what it is with this experiment. (Since coins have around a 50% chance of coming up heads, that would determine the b -value.)

TI-Nspire™ Navigator™ Opportunity: Quick Poll
See Note 1 at the end of this lesson.

On page 1.12, students will perform an exponential regression. They need to select **Exponential Regression** from the Statistics menu (**MENU > Statistics > Stat Calculations > Exponential Regression**), choose **trialr** for X List, **clistr** for Y List, and save the RegEqn to **f2**.



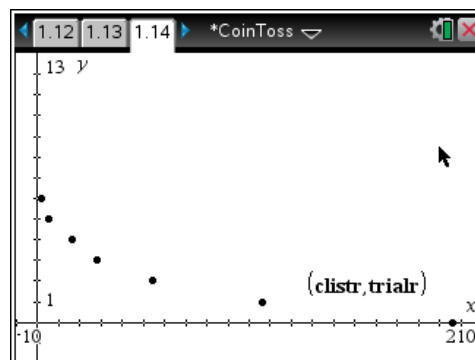
Once this is completed, have them go back to the graph page 1.9, press **ctrl** + **G** to show the graph entry line, up arrow to **f2(x)**, and press **enter** to see the regression equation with their own equation.



Ask: How does the exponential regression equation compare to your equation and how well does it model the data?

Now that the students have had two experiences with exponential modeling, the activity turns to exploring the inverse of exponential functions.

As the students have learned before, the inverse reverses the x - and y -coordinates of points. Moving to page 1.14, students will view a graph of **(clistr, trialr)**. Their objective is to find the inverse function for this data set.



To find the inverse function for an exponential equation, students need to think about exponential functions and their inverses, logarithmic functions.

Exponential functions: $\text{Base}^{\text{Power}(x)} = \text{Answer}(y)$.

Logarithmic Functions: $\log_{\text{Base}} \text{Answer}(y) = \text{Power}(x)$.

To find the inverse function of $y = a \cdot b^x$, first switch the x-and y-values: $x = a \cdot b^y$ and solve for y:

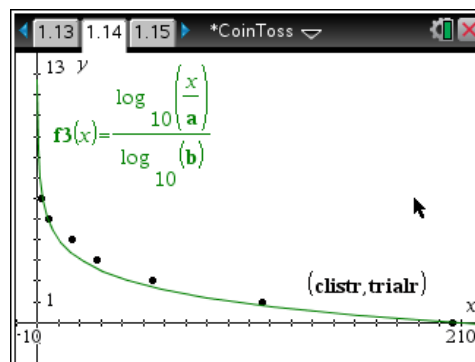
Divide both sides by a: $\frac{x}{a} = b^y$

Apply logs to both sides: $\log \frac{x}{a} = y \log b$

Divide both sides by log b: $y = \frac{\log\left(\frac{x}{a}\right)}{\log b}$

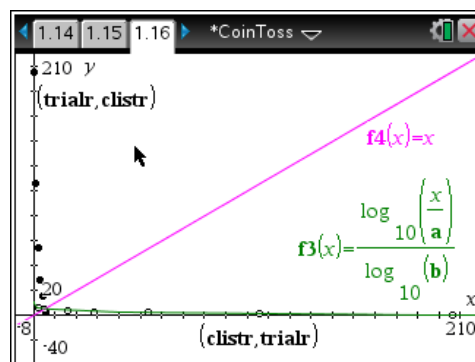
On page 1.14, students should type in the equation that was just derived. To enter a fraction, press **ctrl** + $\frac{\square}{\square}$. To access the log function, press \log_{10} . Students can also use the alpha keys to type **log**. The example uses base 10, but other bases could be used.

Students should assess how well the log function modeled the inverse data.



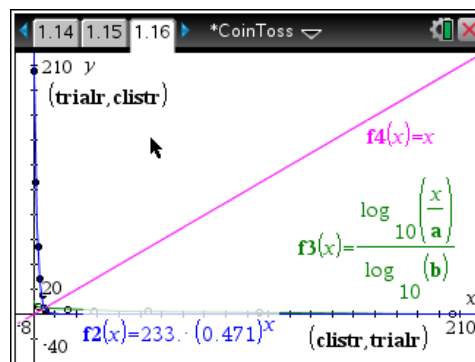
On page 1.16, both the data and its inverse data graphed together. If students have trouble viewing the points, they can change the window settings.

Students should see the two sets of data are mirror images of each other and the reflection line is $y = x$. They are to graph the reflection line with the data.



On page 1.16, graph the exponential regression equation (**f2**) and the reflection line (**f4**).

Then students graph the inverse logarithmic function. To do this, they need to set up the function as they did for the inverse function, but use the variables from the exponential regression equation. To access the regression variables, type **stat.** and a drop menu will appear. Select **a**. Repeat the steps to access the value for **b** and press **enter**.



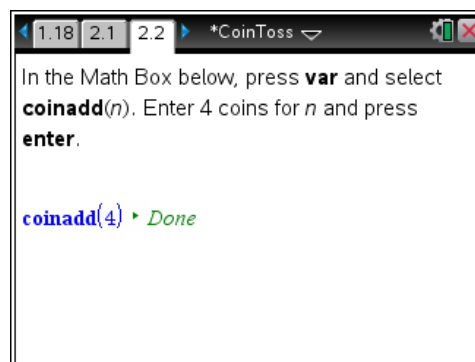
The last page asks the student to specifically state all the aspects of the relationship between an exponential function and its inverse. This is an opportunity to review all the properties of inverses that were discovered throughout this activity.

Problem 2 – Coin Toss Simulation: Exponential Growth

The second part of this activity follows the same format as Problem 1 with the exception that students start with four coins and do the reverse process of adding the number of coins that come up heads.

Students will run the program **coinadd** on page 2.2 entering the value of 4 for the number of coins.

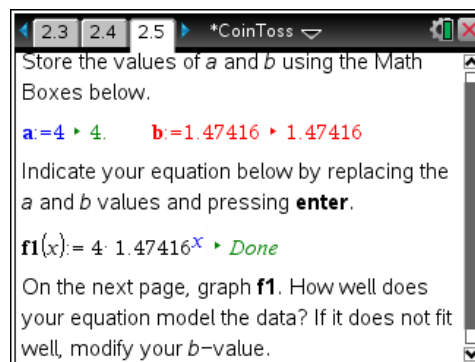
Again, data will be generated on page 2.4. Students should store the starting number as their value of *a* on page 2.5.



Page 2.4 is already set up for the ratio formula in cell C1. Students will need to use the **Fill Down** command as before to complete the calculation of the other ratios.

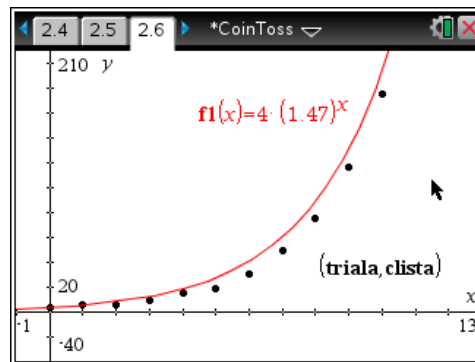
They can then calculate the mean of the ratios, and move to page 2.5 and store this value as *b*.

Note: This *b* value is different than the previous experiment. This is a good opportunity to discuss why.



Students then record their equation on page 2.5 and graph it on page 2.6.

They are also asked how well their equation models the data. This is an opportunity to have students share their data and equations with each other and the class. If the students do not get a good fit, they need to modify their *b* value.



The students now have a series of questions that are good for a class discussion.

- Is this a growth or decay model? (growth)
- At what percent are the coins being added? (150.3%)
- Explain the significance of the *b*-value for this experiment. (Since coins have around a 50% chance of coming up heads, that would determine the *b* value plus 100% because coins are being added.)

On page 2.9, students now find an exponential regression of this data using **triala** for X List, **clista** for Y List, and **f2** for the RegEqn.

Then they can graph their equation on page 2.6. Students should compare the regression equation to their equation.

As before, students will explore the inverse of the data. On page 2.11 is the scatter plot of **(clista, triala)** which is the switching of the x - and y -data values.

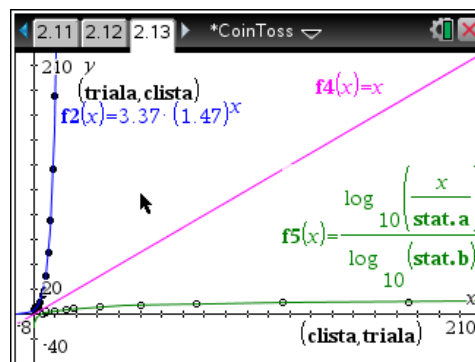
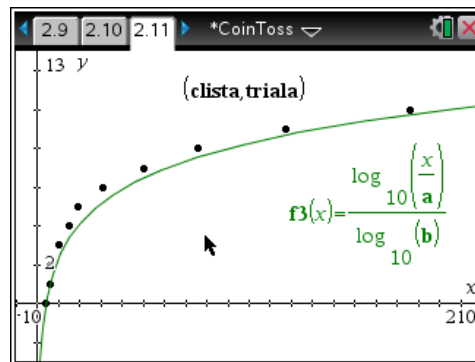
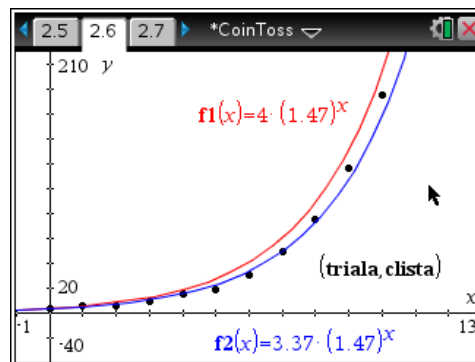
Students should derive the inverse function as they did in Problem 1 and then graph the equation.

This is a good opportunity to discuss with students how well the logarithmic function modeled the inverse data.

Page 2.13 displays both sets of data. Students should graph the reflection line, the exponential regression equation (**f2**), and the logarithmic function.

The activity ends with the final question: *How was the difference between the two experiments represented in their exponential and logarithmic model equations?*

This is an opportunity to review all the key concepts covered in this exercise.



TI-Nspire™ Navigator™ Opportunities

Note 1

Problems 1–2: Quick Poll

Use Quick Poll to verify students understand the concepts of domain and range. The questions provided on the student worksheet may be used for this purpose.