Parabola Geometry

Student Activity

7 8 9 10 11 12

TI-Nspire™ Investigation Student 50 min

Teacher Notes:

The purpose of this activity is to help students understand that a parabola is more than just a 'U-shaped curve'; it has specific properties. The first activity 'modelled' an envelope using a parabola. In this activity students use a mixture of geometry (triangle congruency) and algebra (Pythagoras) to show that the envelope is a parabola and that the parabola has a quadratic equation.

Definition of a parabola:

> "A set of points equidistant from a line and point where the point is not on the line."

The definition of a parabola helps identify an important characteristic utilised by parabolic reflectors, a concept that is explored in the applications section of this activity bundle.

The QR code video links in this activity provides some discussion about the difference between 'modelling' and 'actual', in much the same way as this activity proves that the model used to describe the envelope is the actual curve. The video also provides some necessary background information around triangle congruency and detailed support towards proving the envelope is a parabola and quadratic equation.

The student worksheet can be downloaded from the Texas Instruments Australia website.

http://education.ti.com/australia > **Teachers** > **Australian Curriculum Nspired** > **Year 10** > Algebra

Australian Curriculum Standards

AC9M9A03

Find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane.

AC9M10A04

Use mathematical modelling to solve applied problems … choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings.

AC9M10SP01

Apply deductive reasoning to proofs involving shapes in the plane and use theorems to solve spatial problems.

Lesson Notes:

Students should have previously worked with triangle similarity and congruence. The activity serves as an opportunity for students to experience some spaced retrieval with a purpose, applying these skills to solve a problem.

Students should be encouraged to save their TI-Nspire file in their "Parabolas" folder.

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Calculator Instructions: Defining the Curve

In the previous activity (Paper Folding), a parabola was used to *model* the shape of the envelope created by the folds. There is a significant difference between modelling and determining the actual, theoretical outcome.

In this activity, you will see if the envelope was just a model, or if it is indeed a parabola, and what that equation might look like. The video tutorial linked to the QR code will help you on your journey.

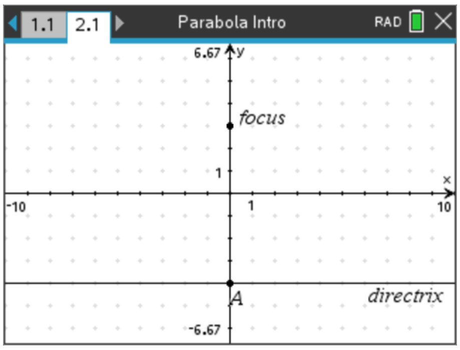
https://youtu.be/Qg5RHJ0uUYY

Open your previous activity: "Parabola Intro" and insert a new Problem.

Press:

doc▾ > **Insert** > **Problem**

Insert a Graphs application, horizontal line (perpendicular to y axis) and point on the Y axis (focus); the same set up as the previous activity.

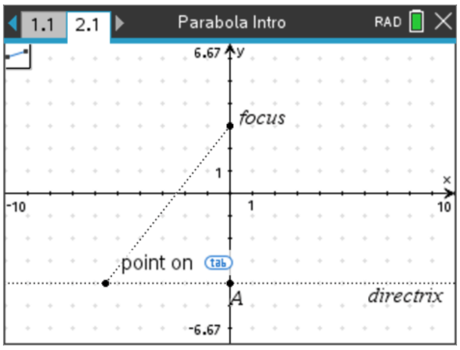


Place the point (focus) and line (directrix) on a axis hash-marks. When objects are placed on a hash mark, their incremental / decremental values follow the scale on the axis.

Draw a line segment connecting the focus to the directrix.

menu > **Geometry** > **Points & Lines** > **Segment**

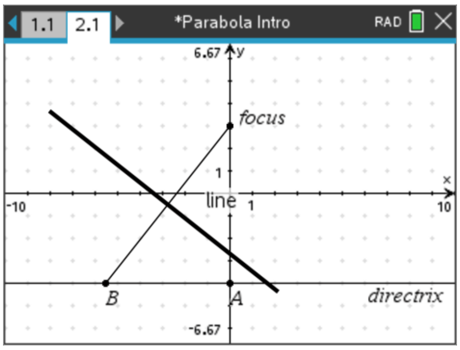
Make sure the point is on the directrix. If you have the grid switched on, make sure the point is on the line and not stuck to the grid.



Construct a perpendicular bisector to this new line segment.

menu > **Geometry** > **Construct** > **Perpendicular Bisector**

So far, this construction is effectively the same as the previous one, but with the inclusion of the line segment joining the focus and the directrix.



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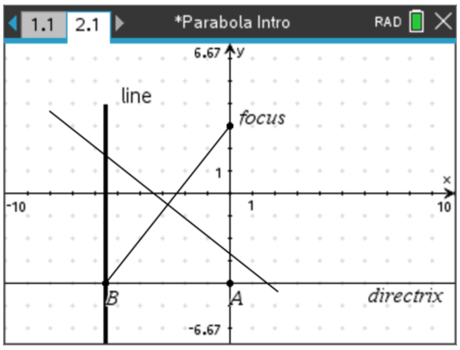
Now construct a perpendicular line, perpendicular to the directrix and passing through point B (in the diagram).



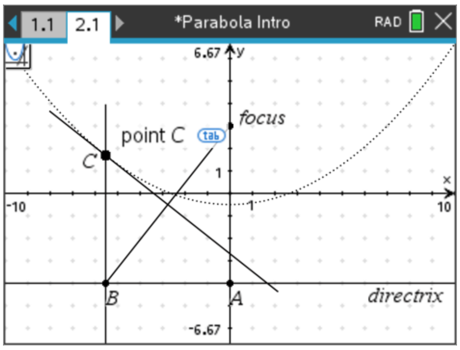
menu > **Geometry** > **Construct** > **Perpendicular**

This new part of the construction provides for a new version of the locus by studying the point where this perpendicular line intersects the perpendicular bisector.

menu > **Geometry** > **Points & Lines** > **Intersection Point**

Select the two lines, the point of intersection will automatically identify the location.

The new locus relates to the movement of point B on the directrix and the point of intersection, (Point C on the diagram).



menu > **Geometry** > **Construct** > **Locus**

Select Point B followed by point C.

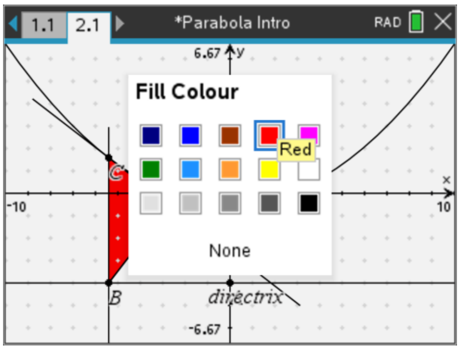
Release the locus tool and drag point B along the directrix.

Question: 1.

How does the locus for Point C relate to the previous envelope from the paper folding and digital representation?

Answer: The curve is the same shape as the envelope. The more folds on the envelope the better the envelope follows the parabola (locus). The perpendicular bisector is 'tangent' to the curve.

A triangle is formed by the line segment joining the focus to point B, the line BC and the perpendicular bisector. The shape tool in the Geometry menu can be used to construct and fill this triangle.



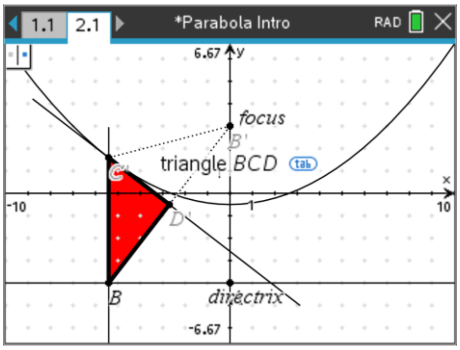
menu > **Geometry** > **Shapes** > **Triangle**

To construct the triangle, select the three vertices. Press esc to release the triangle tool when finished.

With the triangle complete, move the mouse over the triangle and press:

ctrl + menu > **Colour** > **Fill Colour** (Select a colour)

With the triangle now clearly in view, press:



menu > **Geometry** > **Transformations** > **Reflection**

The aim is to reflect the triangle in the line: CD (perpendicular bisector).

Make this triangle a different colour.

Move point B along the directrix and observe the original triangle and its reflection.

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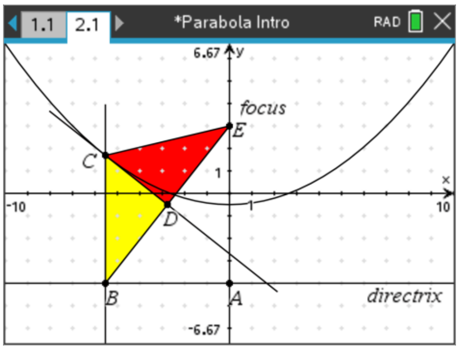
Question: 2.



In the image shown opposite two triangles are shown.
ΔBCD is formed by the perpendicular bisector (CD), perpendicular (BC) and line segment (BD). The second triangle has been drawn: ΔCDE where E is the focus. How are these triangles related and what does this relationship say about BC and CE?

Answer: ΔBCD ≡ ΔCDE.

Proof: BD = DE (CD Bisects BE)
CD is common
∠BDC = ∠EDC (Perpendicular)
∴ ΔBCD ≡ ΔCDE (SAS)

Since ΔBCD ≡ ΔCDE, it follows that BC = CE.

Question: 3.

A **parabola** is defined as: "the set of all points in the plane equidistant from a line and a point not on the line. Is the locus (curve) in your construction, a parabola? (Justify your answer)

Answer: Based on the previous question, the curve (locus) is the set of points equidistant from the directrix (line) and point (focus), therefore the curve is a parabola.

For Questions 4 to 6, let point C be represented by the coordinates (x, y).

Question: 4.

Move the directrix so that it represents the line y = -4. Move the focus so that it is located at the point (0, 4).
If the point on the y axis (focus) won't move, it could be that the triangle vertex is 'hiding' the focus. In this case, with the hand over the point (the word TAB appears), then press the tab key and grab the point.

a) Determine an expression for the distance from point C to the directrix.
Answer: Distance: y – ⁻4 = y + 4.

b) Use Pythagoras's theorem to determine an expression for the distance from point C to the focus.
Answer: $d = \sqrt{(x-0)^2 + (y-4)^2}$

c) Given the distances in (a) and (b) are the same, write an expression in the form "y = " for the location of point C. Graph the rule and compare it to the locus.
Answer:
$$y + 4 = \sqrt{x^2 + y^2 - 8y + 16}$$
$$y^2 + 8y + 16 = x^2 + y^2 - 8y + 16$$
$y = \frac{x^2}{16}$ The graph matches the locus … of course the graph is not dynamic like the locus

Question: 5.

Move your directrix and focus to new locations and repeat Question 4, determining the corresponding equation for the parabola.

Answer: Answers will vary, depending on the location of the focus and directrix.

General solution: Directrix y = b with focus (0, a) will have equation: $y = \frac{x^2}{2(a-b)} + \frac{a+b}{2}$

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