NUMB3RS Activity: Ants Go Marching
Episode: “End of Watch”

Topic: Ant colony optimization **Grade Level:** 9 - 12
Objective: to model how an ant colony finds the best path to food
Materials: Calculator with random integer generator
Time: 20 - 25 minutes

Introduction

In “End of Watch,” Charlie claims that the gang member accused of murdering a police officer is probably innocent. He bases this on the likelihood that there was no way for the gang member to find a single safe path between rival gang turfs. He refers to “Collective Behavior Theory” for a mathematical model more precisely known as “Ant Colony Optimization” (ACO) to demonstrate that certain gang territories prevent members from leaving the safety of their own turf. In this activity, students will use directed graphs to simulate an ACO.

Discuss with Students

When searching for food, an ant colony sends out scouts, that wander around randomly. If a trail leads to food, a scout marks the path it took as a good one. Other ants that find such a trail will not wander randomly, but tend to follow the good one instead. They also leave their mark, making the trail even more attractive. Over time, however, the trail marking starts to evaporate, reducing its attractive strength. The more time it takes for an ant to travel down the path and back again, the more time the markings have to evaporate. A shorter path to the food gets marched over faster and the markings remain at a higher level.

Discuss with students the concept of directed graphs. In this activity, a graph is a series of vertices with lines, or edges, connecting them. A directed graph is a graph in which the edges of the graph only go in one direction. The ants will travel from the nest along the edges of the graph randomly searching for food. The ant will only travel in the direction of the arrows on the graph. At each vertex, the ant has either two or three choices for which edge to take next.

For this activity, students should work in pairs. One student should have a calculator set for **RandInt(1,2)**; the other should be set for **RandInt(1,3)**. The ant’s path either leads to food or a dead end. Each path will have a length of 3, 4, or 5 edges. If a path leads to food (a “good” path), then each edge on the path gets a “marking score.” For a path with three edges, each edge gets 5 points; for four edges, each gets 3 points; and for five edges, each gets 1 point. One student should be in charge of recording the edges in each path, as well as the score each edge in the path receives if the path leads to food. Repeat the process until 10 ants have found food.

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Student Page Answers:

1. *There are four: A-D-K, B-H-K, B-I-O, and B-I-P.* **2.** *There are five that start with A: A-D-L-O, A-D-L-P, A-E-H-K, A-E-I-O, and A-E-I-P. Four start with B: B-H-L-O, B-H-L-P, B-J-M-O, and B-J-M-P.* **3.** *2 × 2 × 2 = 8* **4a.** *N and Q are the last edges to the dead end, so an ant will never travel on these edges and find food.* **4b.** *Since edge C leads downward, it is more likely to lead to the dead end than edges A or B.* **5.** *Edges N and Q have to be 0 since they cannot earn any score, as they are the last edges into the dead end. Experimentally, the other low scoring edges will probably include F, G, and J, as they are in the lower part of the graph, supporting the answer to #3. It is interesting to note that edge E may also score low, as it directs the path into the lower part of the graph.* **6.** *Experimentally, edge B scores high because it leads to the most good paths. Edges K and P (and possibly O) score well, as they are the last edge before food. Edge O may not score as high because it is the last edge of more of the good paths that are long. Edges A and H may score well for the same reason as B, but probably not as high. Generally, edges in the top half of the graph have a better chance of being part of a good path.* **7.** *Answers will vary, but the most likely choices are A-D-K and B-H-K.*

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Name: ________________________________ Date: _______________

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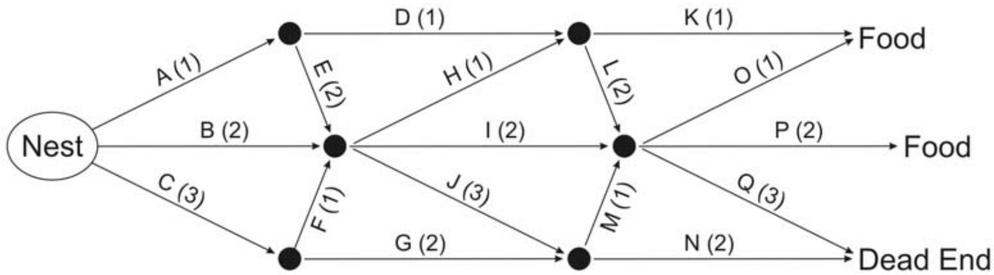
The graph below is called a directed graph. A directed graph is a graph in which the edges only go in one direction. On the graph, an ant leaves the nest and searches randomly for food. At each vertex, the ant has either two or three choices for which edge to take next. These edges are numbered for use with the simulation following #4. If a path leads to food, then it is a "good" path.



1. List and count all good paths that contain exactly three edges..

2. List and count all good paths with four edges that start with edge A. Do the same for those that start with edge B.

There are no good paths with three edges that start with edge C, and five with four edges: C-F-H-K, C-F-I-O, C-F-I-P, C-G-M-O, and C-G-M-P.

3. Good paths with five edges must start out either A-E or C-F; the next steps must be H-L or J-M, and the last step must be O or P. How many good 5-edge paths are there?

Here is a summary of how many times each edge is used in good paths with 3 or 4 edges:

Edge	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Freq.	6	7	5	3	3	3	2	5	6	2	3	4	4	0	7	7	0

When paths with five edges are included, the total number of paths that lead to the ending vertices are: top food, 15; middle food, 11; and dead end, 15, for a total of 41 paths.

4. a. Why do edges N and Q have values of 0?
b. Why is starting edge C used the least of the three initial choices?

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Knowing only how many times each edge gets used does not help to determine what paths ants would follow in the search for food. The following simulation can help. One student should have a calculator set for **RandInt(1,2)**; the other should be set for **RandInt(1,3)**. Use whichever calculator is appropriate at each vertex on the ant's path. The ant's path either leads to food or a dead end. Each path will have a length of 3, 4, or 5 edges. If a path leads to food (a "good" path), then each edge on the path gets a "marking score". For such a path with three edges, each edge gets 5 points; for four edges, each gets 3 points, and for five edges, each gets 1 point. One student should be in charge of recording the edges in each path, as well as the score each edge in the path receives if the path leads to food. Repeat the process until 10 ants have found food.

Edge Scores for Good Paths

Ant #	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1																	
2																	
3																	
4																	
5																	
6																	
7																	
8																	
9																	
10																	
Total																	

When you have finished, combine your results with at least two other pairs of students to get a better picture of the actions of a larger number of ants.

Edge Scores for Good Paths

Edge:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Total:																	

5. Which edges have the five lowest scores? Why?

6. Which edges have the five highest scores? Why?

7. Suppose that after the first group of ants searches for food, the rest of the ants will travel so that at each vertex they follow the edge with the highest score. What path(s) would they be most likely to follow?

This model is a very simple version of a real ACO. It replaces the evaporation of the path markings with higher scores. Also, all of these ants chose their edges randomly. With real ants, as the trail "score" (strength of markings) gets higher, they no longer choose randomly and will tend to follow the marked path, like in Question #7.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

ACO was first introduced by Marco Dorigo in his PhD. thesis in 1992. It was originally called Ant System (AS). Since 1995, Dorigo and others and been working on extended versions of ACO. Possible applications of ACO are of great interest to researchers, because it is now possible to generate a new class of "natural" algorithms based on the behavior of living creatures.

For the Student

One of the first applications of ACO is as a method to solve what is popularly known as a "TSP" (Traveling Salesman Problem). This is a classic problem that uses vertex-edge graphs, just like this activity. The main difference in the graph is that for a TSP, the edges have values for distances. In a TSP, the object is to visit each of the vertices in the graph exactly once, then return to the starting point, traveling the minimum total distance. For large and complicated graphs, there is no known efficient way to determine the best answer. Based on the idea that ants tend to follow edges between vertices that produce smaller totals leads to using ACO as a problem-solving tool. Recently, Mark Sinclair from the Institute of Electrical and Electronics Engineers created a Java applet to demonstrate how ACO is used to solve a TSP:
http://uk.geocities.com/markcsinclair/aco.html

When ants mark a trail, they use a *pheromone*, a chemical that provides signals to other ants. Many species use pheromones, but for an extremely interesting experiment, you can make termites follow a trail drawn with a Bic® pen. It seems that a chemical in the ink is very similar to a pheromone that termites use to mark trails. For a lab activity on this from the University of North Carolina at Chapel Hill School of Education, see:
http://www.learnnc.org/lessons/JackiClark5232002016

Additional Resources

ACO is based on the behavior of real ants. For more information on how real ants locate the best path, see: **http://www.aco-metaheuristic.org/RealAnts.html**

Another "natural" approach to solving a TSP uses a "genetic" algorithm. In essence, a chromosome is "computed" to visit all of the vertices in just one particular order. It is then allowed to "reproduce" with another such chromosome, and the child is checked for its solution to the problem. Children who are successful are then allowed to become parents; subsequent generations continue to solve the problem until no better solution is reached. For a comparison of this method to an ACO method, see:
http://www.codeproject.com/cpp/GeneticandAntAlgorithms.asp

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