Problem 1 – Finding Zeros Graphically

Function	Zeros
$f(x) = x^3 - 3x^2 - x + 3$	
$f(x) = x^3 - 3x - 2$	
$f(x) = x^4 + 5x^3 + 3x^2 - 5x - 4$	
$f(x) = x^4 - x^3 - 7x^2 + x + 6$	
$f(x) = x^4 - 3x^3 - 6x^2 + 28x - 24$	
$f(x) = x^5 - 2.6x^4 - 1.11x^3 - 3.74x^2 - 0.73x + 0.3$	

- 1. Make a conjecture about the number of real zeros of a polynomial in relation to the degree of the polynomial.
- 2. A fourth degree polynomial has four zeros: Sometimes Always Never.
- 3. A polynomial can have more zeros than the highest degree of the function. *True False*.
- 4. What is the greatest number of zeros possible for the function $f(x) = x^5 15x^3 + 10x^2 + 60x 72$?
- 5. Determine the number of real zeros for $f(x) = x^5 15x^3 + 10x^2 + 60x 72$.
- 6. What are the real zeros of $f(x) = x^5 15x^3 + 10x^2 + 60x 72$?

Problems 2-6 - Rational Root Theorem

For each polynomial given, use the Rational Root Theorem to list the possible rational zeros. When you have the possible zeros, use the handheld to determine which values are actual zeros of the function.

On each spreadsheet, type the possible rational zeros of f(x) into the first column. The second column will return a zero if it is an actual zero.

Function	Possible Zeros	Actual Zeros
$f(x) = x^3 + 2x^2 - 11x - 12$		
$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$		
$f(x) = 10x^4 - 3x^3 - 29x^2 + 5x + 12$		
$f(x) = x^4 - 2x^3 - x^2 - 2x - 2$		

CAS EXTENSION

If a, b, and c are zeros of a polynomial, then the polynomial can be written in the factored form p(x) = (x - a)(x - b)(x - c).

- 1. What happens to the factored form if there are less zeros than the degree?
- 2. How does the factored form change if a zero is a multiple (double or triple, etc.) root?
- 3. How does the factored form change if there are imaginary zeros?

Use the information from the three questions to write the polynomials in the table above in factored form. On a *Calculator* page, select **MENU > Algebra > Factor**, then type in the equation. Explain why the equation factors as given by the handheld.

Function	Factored Form
$f(x) = x^3 + 2x^2 - 11x - 12$	
$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$	
$f(x) = 10x^4 - 3x^3 - 29x^2 + 5x + 12$	
$f(x) = x^4 - 2x^3 - x^2 - 2x - 2$	