EXPLORATIONS

Activity 7

Follow the Fence, Then the Hypotenuse

Objective

- To find the length of the hypotenuse of right triangles

Materials

- TI-73
- Student Activity page (p. 80)

In this activity you will

- Discover the method for finding the lengths of non-vertical or non-horizontal segments of right triangles.
- Use the method to find the lengths of segments.

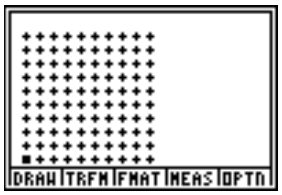
Introduction

Jay and Joyce are ranchers. They must periodically check the fence of their rectangular pasture to make sure that there are no broken parts where the horses or cows can get out onto the busy highway. To do this, they inspect the perimeter of the fence on their Missouri Foxtrotter horses. Jay examines two sides of the rectangle while Joyce covers the other two sides. They start at the same vertex point of the rectangular pasture. Jay rides due north along the three-mile stretch of fence and then turns due east and travels along a four-mile fence segment. Joyce travels due east on the four-mile fence and then turns due north along the three-mile fence segment. They meet at the vertex of the rectangle opposite the starting point. Since all four sides have been followed and examined, they together take the shortcut or hypotenuse back to the starting point. How far do they travel together?

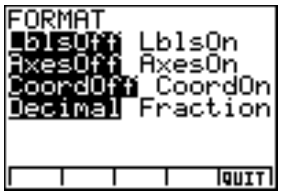
Investigation

It is easy to find the lengths of segments along the verticals or horizontals of right triangles. However, sometimes the segments are not horizontal or vertical. In this investigation, you will discover how to find the lengths of such segments.

1. From the main Geoboard menu, select **4:10x10**.



2. To format the geoboard, select **FMAT** and make sure that the following settings are selected:

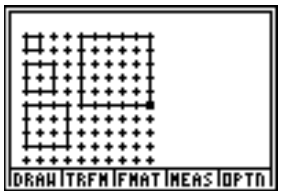


   LblsOff (Labels are off)
   AxesOff (Axes are off)
   CoordOff (Coordinates are off)
   Decimal (Measurement is in decimal form)

   Select **QUIT** to exit the FORMAT menu.

3. Using only vertical and horizontal sides, draw squares with areas of 1, 4, 9, and 25 square units.

   Your geoboard should look like the screen at the right.



4. Complete this table.

Area of square	Length of square
1	
4	
9	
25	

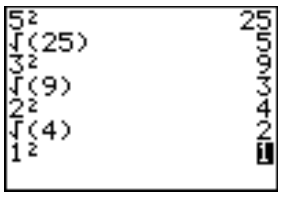
5. To find the area of a square, multiply the length of one side by itself. This is called squaring a number. The formula is written as follows:

   Area (A) = Side (S) x Side (S)

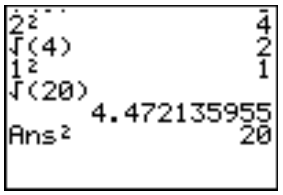
   or

   $A = S^2$

6. The inverse of squaring a number is to take its square root. Quit the Geoboard application. You should be on the home screen of the TI-73. [CLEAR] the home screen and type the expressions at the right. (Notice that the [√] is a [2nd] function above the [x^2] key.)

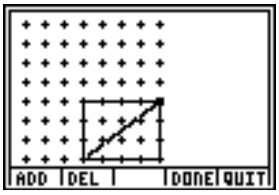


7. All of the examples above are rational numbers. Suppose you have a square with an area of 20 square units. Use the TI-73 to find the length of one side. This number is an irrational number.

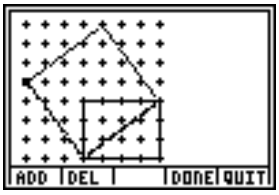


$\sqrt{16} < \sqrt{20} < \sqrt{25}$
or
$4 < \sqrt{20} < 5$

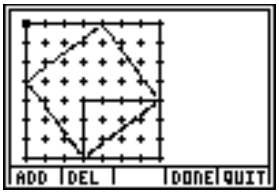
8. Go back to the main Geoboard menu and select **3:8x8**. Draw the rectangular pasture described in the introduction. Your geoboard should look like the screen shown at the right. The diagonal of the rectangle divides the rectangle into two congruent right triangles. The diagonal is called the hypotenuse of the right triangles. It is the path that Jay and Joyce take together.



9. Build a square in which the triangle's hypotenuse is the length of the square's side. Remember that squares are rectangles in which all four sides are equal.



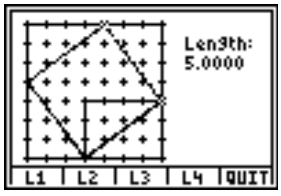
10. Find the area of the square by surrounding it with a 7×7 square and subtracting the four triangular corners.



49 – (4 x 6) = 25 sq. units

11. If the area of the square built off of the hypotenuse is 25 square units, the length of the hypotenuse is _____. Therefore, Jay and Joyce would travel _____ miles back to the starting point together.

12. Use the measurement tool **1:Length** to check your answer in Step 11. To use this tool, press [ENTER] on each endpoint of the hypotenuse.



Student Activity

Name _____

Date _____

Activity 7: Follow the Fence, Then the Hypotenuse

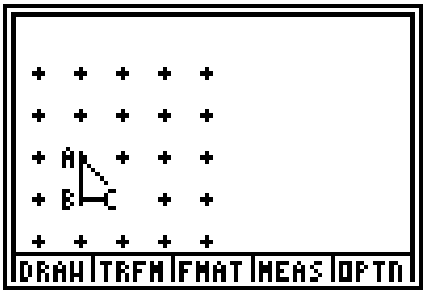
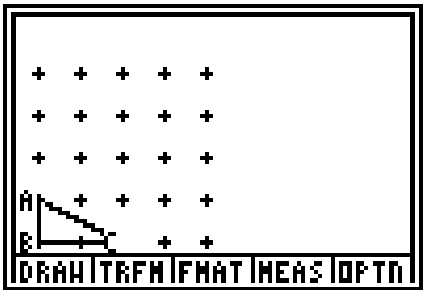
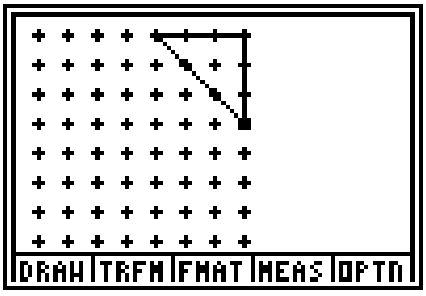
Assume that the vertical or horizontal distance between adjacent pegs is 1 unit.

a. Draw each triangle on your geoboard.

b. Build a square on the hypotenuse.

c. Find the area of the square you built.

d. Use the area of the square to find the length of the triangle's hypotenuse.

1. Area of the square: _____ Length of the hypotenuse: _____	
2. Area of the square: _____ Length of the hypotenuse: _____	
3. Area of the square: _____ Length of the hypotenuse: _____	

⑦ Draw a right triangle on a 6×6 geoboard and find the length of the hypotenuse 	⑦ The right angle's vertex is in the upper right corner. Find the length of the hypotenuse 
⑦ The vertical side is 1 unit long Find the length of the hypotenuse 	⑦ The horizontal side is 3 units long Find the length of the hypotenuse 
❼ The area of the right triangle is 1.5 square units 	❼ The length of the hypotenuse is not a rational number 

Teacher Notes

Activity 7

Follow the Fence, Then the Hypotenuse

Objective

- To find the length of the hypotenuse of right triangles

NCTM Standards

- Select and apply techniques and tools to accurately find length…to appropriate levels of precision
- Recognize and use connections among mathematical ideas

Investigation

This purpose of this investigation is twofold. It is designed to reinforce finding areas using the surround method and to introduce the idea of finding the lengths of diagonal segments, which will lead into *Activity 8: Right Triangles are Cool Because of Pythag's Rule.*

Students may need some guidance in drawing squares from diagonals. You may want to walk them through this process to get to adjacent vertices. For example, go from Vertex 1 (up 3 right 4) to Vertex 2, then from Vertex 2 (up 4, left 3) to Vertex 3, from Vertex 3 (down 3, left 4) to Vertex 4 and from Vertex 4 (down 4, right 3) back to Vertex 1.

Answers to Student Activity pages

1. c. 2
   d. $\sqrt{2} \approx 1.41$
2. c. 5
   d. $\sqrt{5} \approx 2.24$
3. c. 18
   d. $\sqrt{18} \approx 4.24$

Group Problem Solving: The length of the hypotenuse

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

With a little experience, students should be able to design their own group problems. They could then switch problems with other groups for additional problem solving practice.

Solution for this problem solving exercise:

The length of the hypotenuse is $\sqrt{10} \approx 3.1623$