NUMB3RS Activity: Narrowing the Search
Episode: "Undercurrents"

Topic: Length of an *n*-dimensional vector **Grade Level:** 10 - 12
Objective: Extend the Pythagorean Theorem to find the length of a vector in *n* dimensions.
Time: 15 minutes

Introduction

In "Undercurrents," Charlie examines the algorithm used to flag dangerous containers entering the country through the Port of Los Angeles. The flagged containers are then subject to more intense inspection. Because the actual search criteria are not made public, this activity looks at a fictional algorithm similar to the type that could be used. It expands the concept behind the Pythagorean Theorem to *n*-dimensional space so that the length of the vector (x_1, x_2, x_3, …, x_n) is given by $\sqrt{\sum_{i=1}^{n} x_i^2}$. Because some high school texts refer to this as the length of a vector and others as the distance from the origin, both references are used in the activity.

Discuss with Students

Students should review the Pythagorean Theorem. If they are not familiar with its relationship to the distance formula, show them how they are essentially the same. Also, review how a vector is represented as an ordered *n*-tuple and how this *n*-tuple can be represented as a point in *n*-dimensional space.

There is no discussion on the student page about concepts such as orthogonality and dot products, but if students are advanced enough for these concepts, they can easily be connected.

Student Page Answers:

1a. $\sqrt{x^2+y^2}$ **1b.** $\sqrt{x^2+y^2+z^2}$ **1c.** 13 **2.** $\sqrt{8} \approx 2.828$ **3a.** $\sqrt{48} \approx 6.93$; it would not be flagged because 6.93 < 7. **3b.** $\sqrt{52} \approx 7.21$; it would be flagged, because 7.21 > 7 **4.** 7, because if one component is 7 and the others are the minimum (1), the length is $\sqrt{56}$, which is greater than 7. On the other hand, a 6 with the other seven components being 1 would be of length $\sqrt{43}$, which is less than 7. (Note that to prove 7 is the minimum for this, one must show that it would set off the trigger ***and*** that no lower number would. A common mistake is to only do half of this proof.) **5.** Three, since three 4s and five 1s gives a length of $\sqrt{53}$, which is greater than 7. However, two 4s and six 1s yields $\sqrt{38}$ which is less than 7. **6.** If rating *m* is the same for each category, the length of the vector is $\sqrt{8m^2} < 7$. So, the rating for each category would have to be less than $\frac{7}{\sqrt{8}} \approx 2.475$.

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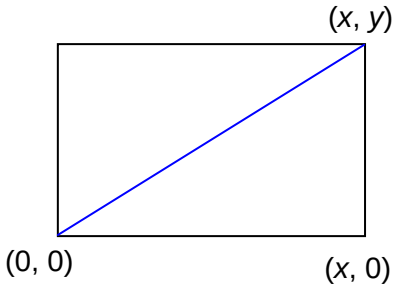
Name: _____ Date: _____

NUMB3RS Activity: Narrowing the Search

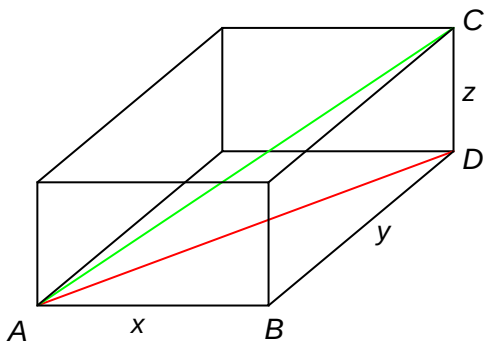
In "Undercurrents," Charlie examines the algorithm used to flag dangerous containers entering the country through the Port of Los Angeles. One way that these algorithms work is to view each potential characteristic of the container (e.g., port of origin, ports of call of the ship enroute, reputation of the shipping company, volume and/or density of the cargo, type of cargo, destination, etc.). Each of these characteristics is assigned a number from 1 to 10 to indicate its relative danger, and then each becomes a component of a vector. If there are only two categories, expressed as (x, y), the relative threat for that container can be defined as the magnitude of the vector (which is the distance to that point from the origin). Note that this algorithm is reasonable to use because each characteristic x and y is considered to be equally important.

For a right triangle whose hypotenuse has endpoints (0, 0) and (x, y), by the Pythagorean Theorem, the distance from the origin to (x, y) is $\sqrt{x^2 + y^2}$.



We can extend this concept to three dimensions (x, y, z) by considering x, y, and z as the lengths of the sides of a box.

In the "box" (rectangular prism) at the right, the base is a rectangle with diagonal AD. Suppose $AB = x$, $BD = y$, and $CD = z$.



1. **a.** Use the Pythagorean Theorem to find an expression for the distance from A to D.
b. Now, using your expression for AD and the given fact that $CD = z$, consider triangle ADC. Note that angle ADC is a right angle. Again, use the Pythagorean Theorem to find an expression for AC.
c. Suppose that $x = 4$, $y = 12$, and $z = 3$. Find AC.

When there are more than 3 components, they can be notated as x_1, x_2, x_3, …, x_n (assuming that these are mutually perpendicular). Even though we cannot draw illustrations, we can extend the ideas from 2- and 3-dimensional vectors and see that the distance from the origin (magnitude of the vector) is still found by taking the square root of the sum of the squares: $\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

For the purposes of the remainder of this activity, assume there are 8 components and that each category is assigned a value from 1 (minimal threat) to 10 (maximum threat). A container is subject to further inspection if the length of the vector is greater than or equal to 7.

2. The lowest possible security threat is when all 8 components have a value of 1. What is the length of that vector?

3. **a.** Consider a shipment that comes in with values (1, 3, 2, 2, 3, 1, 4, 2). What is the length? Would it be flagged for further inspection? Why?
b. Consider a shipment that comes in with values (1, 1, 4, 2, 3, 1, 4, 2). What is its length? Would it be flagged? Why?

4. If one component is sufficiently large (dangerous) by itself, the whole shipment receives closer examination. What is the smallest number such that if it appears in ANY position, the container would be chosen for further scrutiny no matter what the other components were? Justify your answer.

5. How many components with a value of 4 would it take to warrant further inspection regardless of what the rest of the scores were?

6. If the ratings were the same for each of the 8 categories, what is the highest rating possible before the shipment would receive extra attention?

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

A similar kind of strategy can be used in scouting opponents of a sports team. For example, suppose that the value of a component depends on the probability of a football team using a passing play. This probability would be high for such situations like third down with long "yards to go," the other team is winning by a large margin, there is little time remaining in the game, etc. But what if only some of these conditions apply on a particular play? The lower values of the components that do not apply would lessen the probability of a pass, and lower its likelihood, causing the defense to react accordingly. Professional teams have very sophisticated computer methods of studying the tendencies of opponents and make decisions accordingly.

Ask coaches at your school if there is a similar program in place for your school's teams. If not, offer your mathematical expertise to work with them. Work with the coaching staff's knowledge of the sport to develop such a system for your teams. Using scouting data, check how well you can predict an opponent's reaction.

Additional Resources

- For more information about the Pythagorean Theorem visit the Web site below.
  http://www5.geometry.net/theorems_and_conjectures/theorem_of_pythagoras.html
- Duane Nycamp at the University of Minnesota shows how a 9-dimensional vector can be used to measure the motion of a runner, including an animation you can manipulate to see how the values of each component are measured.
  http://www.math.umn.edu/~nykamp/m2374/readings/ndimex/index.html
- For more information about higher dimensions, see the *NUMB3RS* Activity **Tesseract** which accompanies the episode "Rampage." This activity can be downloaded for free from the Web site below.
  http://www.cbs.com/primetime/numb3rs/ti/activities/Act2_Tesseract_Rampage_final.pdf

Related Topics

- In some areas, trauma centers have a program similar to what is described in this activity. They often need to decide who should be treated first in the event of a disaster, where there are more patients than the staff on hand can handle simultaneously. The components might be vital signs that could be transmitted from the ambulances while en route to the hospital, saving valuable time.

- In all of these examples, the mathematics is only as good as the professional judgment that sets the relative value for each part of each component. This is why mathematicians work hand-in-hand with other experts. In today's world, there is more and more collaboration between math and other disciplines. Research fields like actuarial science are highly mathematical and use insurance data to evaluate risks and set insurance rates. In this case the components might be the customer's age, health, occupation, smoking habits, etc.

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