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| **Thursday Night Precalculus Series**  **February 8, 2024**  In this *AP Precalculus Live* session, we will explore several examples on solving trigonometric equations and inequalities using both a restricted domain and finding all solutions. We will rewrite trigonometric expressions using trigonometric identities. |  |
| **About the Lesson**   * This Teacher Notes guide is designed to be used in conjuction with the AP Precalculus Live session and Student Problems document that can be found on-demand:   [**https://www.youtube.com/watch?v=D61KxiGWlwg**](https://www.youtube.com/watch?v=D61KxiGWlwg)   * *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the “Live Session” but can be enriched by that resource.* * This session involves solving trigonometric equations and inequalities. It also involves rewriting trigonometric expressions in equivalent forms. * The trigonometric identities used include:   + The Pythagorean identities,   + The sum and difference identities for sine and cosine,   + The double angle identities for sine and cosine. * Students should be able to use the TI-Nspire to check solutions to equations and inequalities as well as confirm the equivalence of representations of trigonometric functions. * **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.   **AP Precalculus Learning Objectives**   * 3.10.A: Solve equations and inequalities involving trigonometric functions. * 3.12.A: Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity. * 3.12.B: Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities. * 3.12.C: Solve equations using equivalent analytic representations of trigonometric functions.   Source: AP Precalculus Course and Exam Description, The College Board | **Materials:**  *TI-Nspire document*   * Trig\_Eq,\_Ineq,\_Identities.tns   *Student document*   * Problems\_02\_08   *Solutions*   * Precal\_problems\_solutions\_02\_08   *YouTube*   * <https://www.youtube.com/watch?v=D61KxiGWlwg> |

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| **Problem 1.** | |
| (a)Find all the values of that satisfy the equation  (b) Find all the values of in the interval that satisfy the inequality  **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem. | |
| **Teacher Tip:** Students need practice with solving equations or inequalities with  as the argument of the trigonometric function. Students should then progress to equations or inequalities with  as the argument, such as the equation in 1. (a).  **Class Discussion:**  *We have two intervals as solutions to the inequality when the closed interval is*  *If the closed interval is changed to how many intervals would we have as solutions to the inequality?*  **Possible Answers:** There would be four intervals as solutions to the inequality if the closed interval is now  **Class Discussion:**  *What is the period of the function*  **Possible Answers:** The period is  **Teacher Tip:** Revisit these discussions as we work through the graphing calculator solutions. | |
| Graph the function  Use the Window Settings shown to the right. If we set the xScale to we can see that the two zeros of the function in the interval are  and  **Technology Tip:** It is possible to set the attributes on the x-axis to show the values at the tick marks as shown to the right. |  |
| To verify the solutions to the inequality in 1. (b), graph the trigonometric function on the given closed interval Use piecewise functions as shown to verify that the solution to the inequality consists of two intervals.  Review the Class Discussion since the graph confirms the four intervals on  **Class Discussion:**  *We have two intervals as solutions to the inequality when the closed interval is*  *If the closed interval is changed to how many intervals would we have as solutions to the inequality?*  **Possible Answers:** There would be four intervals as solutions to the inequality if the closed interval is now |  |
| **Problem 2.** | |
| (a) Find all the values of  that satisfy the equation  (b) Find all the values of in the interval that satisfy the inequality  **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem.  **Class Discussion:**  *In 2. (b) analytical solution of*  *why can’t we divide both sides by*  **Possible Answers:** One issue is that there are values of for which  We also want to use the zero product property with sign charts.  **Teacher Tip:** Sign charts are helpful with solving inequalities. | |
| Use the graphing application to confirm the solution. Graph the two functions on windows shown to the right. Use an XScale of  Trace will also be useful to check the intervals.  **Technology Tip:** Set the Trace to |  |
| **Problem 3.** | |
| What are all the values of  for which and  **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem. **Class Discussion:**  *In previous problems, we added* *to the endpoints for an additional interval. Why didn’t we do that here?*  **Possible Answers:** We have a restricted domain. We can check the interval where was added to the endpoints to obtain  This interval is not in the given domain. | |
| **Problem 4.** | |
| 1. Rewrite as an expression in which appears once and no other trigonometric functions are involved.      1. Rewrite as an expression in which appears once and no other trigonometric functions are involved. | |
| **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem. | |
| For 4. (a), graph the initial expression and the rewritten expression to verify the solution. Use a Zoom Trig window. The two graphs should match. |  |
| For 4. (b), use Xmin = 0 and Xmax = 2π.  **Class Discussion:**  *In 4. (b) we only graphed the first function. How many cycles do we see in the interval from x = 0 to x = 2π? How could this information be used to determine the sin function that is represented in the rewrite?*  **Possible Answers:** There are three (3) cycles shown in the intervalfrom x = 0 to x = 2π. The sine function would be  Graphing would confirm. |  |
| ***Note:*** *The following* p*roblems, 5 and 6, are not discussed in the video.* | |
| **Problem 5.** | |
| Suppose  and  where and are in the interval  Evaluate the expression  **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem. | |
| **Problem 6.** | |
| The function is given by  The function is given by  What are the zeros of  on the interval  **Sample Solution:**  Refer to the Teacher Solutions Document for the full solution to this problem. | |

**Wrap Up**

Upon completion of the discussion, the teacher should ensure that students understand:

* The graphing application can be used to verify solutions to both equations and inequalities.
* The calculator application can be used to solve equations.
* The graphing application is useful in verifying equivalence of trigonometric expressions.

For more videos from the AP Precalculus Live series, visit our playlist <https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI>

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