SYMBOLIC AND GRAPHICAL REPRESENTATIONS OF FUNCTIONS

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(Please feel free to email me questions and /or comments.)

Key Topic: Pre-Requisites: Functions and Equations

Abstract:

Every teacher of calculus knows that their students need to have a firm foundation in the symbolic and graphical representations of functions before they start learning calculus. This activity is designed to evaluate that understanding. It can be used as a pre-test to determine which topics must be reviewed before the student starts studying calculus, or as a post-test after the student has been given a review of functions.

Prerequisite Skills:
- Ability to read a graph.
- Ability to find the domain of a function, both graphically and algebraically.
- Ability to evaluate functions, both graphically and algebraically.
- Ability to find intercepts, both graphically and algebraically.
- Ability to recognize when a graph is not the graph of a function.
- Ability to deal with functional notation, both graphically and algebraically.
- Basic understanding of lines and their equations.
- Basic understanding of the terms increasing and decreasing.

Degree of Difficulty: Easy

Needed Materials: TI-89

NCTM Principles and Standards:

- Content Standards – Algebra
    - Represent and analyze mathematical situations and structures using algebraic symbols
    - Use mathematical models to represent and understand quantitative relationships
    - Draw a reasonable conclusion about situation being modeled

- Process Standards
    - Representation
   - Connections
    - Problem Solving

SYMBOLIC AND GRAPHICAL
REPRESENTATIONS OF FUNCTIONS

**Can you read a graph? If so, you will have no trouble with the following questions.**

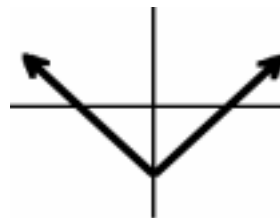1. Given the graph of $y = f(x)$ pictured at the right, find each of the following.
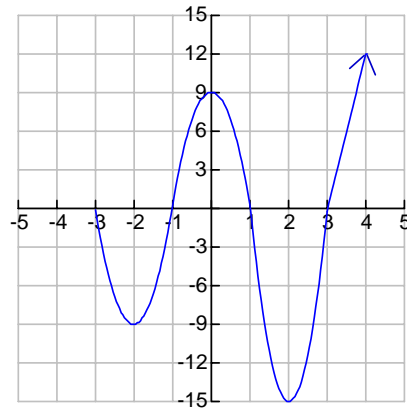
   a. The domain of f.

   b. $f(0)$

   c. The value(s) of x for which $f(x) = -15$.

   d. The x-intercepts of the graph.

   e. The values of x where is f increasing.

   f. The number of time the line $y = -10$ crosses the graph.

2. Answer the following questions about the four graphs shown below. Some questions can have more than one answer.
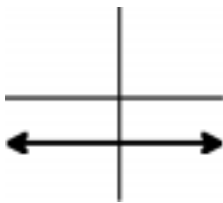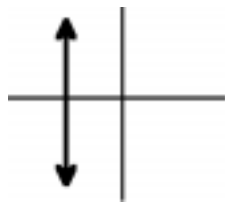
A	B	C	D

   a. Which graphs, if any, are not graphs of a function?

   b. Which graphs are defined for all x?

   c. Which graphs are graphs of a constant function?

   d. Which graphs have asymptotes?

   e. Which graphs look like the graph of $y = -2$?

3. Kenny, who got his driver's license two weeks ago, got permission to borrow his Dad's car for the day. His plan was to drive around for awhile and then meet his friends at 3 pm. The graph you see at the right tells you what Kenny's day with his Dad's car was like. The x-axis shows the time of day with 0 denoting 8 am, and the y-axis shows you how many miles Kenny was from his home.
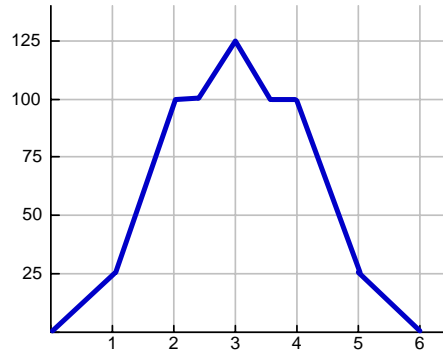
Use this graph to answer the following questions about Kenny's day.

a. How fast, on the average, was Kenny driving between 9 and 10am?

b. What was Kenny doing between 10:00 and 10:15 am?

c. What does the decreasing part of the graph indicate?

d. Did Kenny take his friends for a ride in his Dad's car at 3 pm? Explain.

Do you understand functional notation? If so, you will have no trouble with the following questions.

4. Scientists have found that the number of times a cricket chirps per minute is a function of the outdoor temperature. In particular, they have found that the function $f(x) = \frac{10}{3}(x-40)$ describes this relationship when the temperature is measured in degrees Fahrenheit.

a. What does x denote: chirps per minute or temperature?

b. Without using any "math lingo", describes in words the meaning of each of the following. Your description should fit the context of the problem and should use words like "chirps", "temperature", "hotter", etc. Your description should make sense to a 5 year old child.

  i. $f(2x)$ 　　　　　　　　　　　　ii. $2f(x)$

  iii. $f(x+5)$ 　　　　　　　　　　iv. $f(x)+5$

5. If $f(x)=3x+4$,

    a. sketch the graph of $y=f(x)$.

    b. find the *x*-intercept of this graph.

    c. find the *y*-intercept of this graph.

    d. find, and simplify, $f(-4)$.
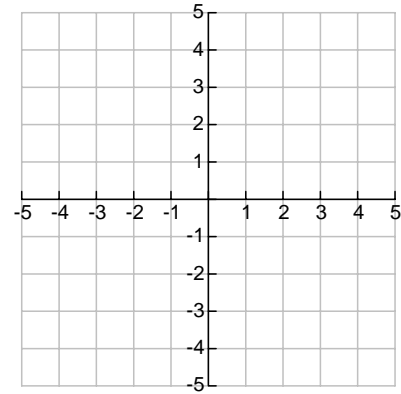
    e. find, and simplify, $3f(x)$ when $x=3$.

    f. find x when $f(x)=12$.

    g. find, and simplify, $f(x+5)$.

    h. find, and simplify, $\dfrac{f(x+h)-f(x)}{h}$ where $h\neq 0$.

6. Find the domain of each of the following functions.

    a. $f(x)=\dfrac{x-2}{x^2+x-6}$ b. $g(y)=\sqrt{4-y}$

ANSWERS:

1. a. $x \geq -3$ b. 9   c. 2
   d. $-3,\ -1,\ 1,\ 3$ e. $-2 < x < 0,\ x > 2$ f. 2

2. a. **B**   b. **A, C**   c. **A**   d. **D**   e. **A**

3. a. 75 mph
    b. He stopped for about 20 – 30 minutes. Maybe he was picked up for speeding.
    c. He was traveling back home.
    d. No. According to the graph, he arrived home around 2 pm and didn't use his
        Dad's car after that.

4. a. Temperature. The first sentence tells you that the number of chirps is a *function
        of temperature*. So # chirps $= f(x)$, where $x =$ temp.
    b. i.   the number of chirps when it's twice as hot
        ii.   double the number of chirps
        iii. the number of chirps when it's $5°$ hotter
        iv. 5 more chirps

5. a. see the graph at the right
    b. $-\dfrac{4}{3}$ c. 4   d. -8 e. 13
    f. 8   g. $3x + 19$ h. 3
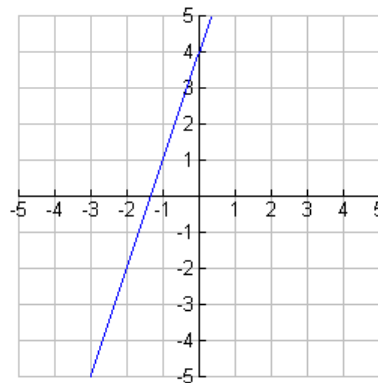
6. a. $x \neq -3,\ 2$. That is, all real numbers except -3 and 2.
    b. $y \leq 4$. That is, all real numbers less than or equal to 4.