

Square Triangles

Teacher Notes

Introduction

This is an activity that uses all of the applications available within TI-Nspire and is very rich in mathematics across geometry, algebra and statistics. It will take a minimum of three one-hour lessons to do and is worth revisiting in subsequent years.

The problem

A square is divided into three triangles and a quadrilateral by a diagonal and a line joining one of the other vertices to the midpoint of a side.

What fraction of the square is each part?

You can draw the diagram on page 1.3 or see it on page 1.4

By looking at a simple geometric construction, learners can gather data and put it into lists that can then be analysed. Learners who wish to develop their skills using dynamic geometry can construct the diagram themselves, though it is provided for use if that skill is not to be developed at the time.

Screen 1: Shows a square divided into four regions: a quadrilateral (areaq) of 33.75 cm², two triangles (areabt and areamt) of 27 cm² each, and a small triangle (arealt) of 6.75 cm². The total area of the square (areas) is 81 cm².

Screen 2: A scatter plot showing a positive linear relationship between variables s and t. The data points form a straight line starting at approximately (72, 24) and ending at (96, 31).

Screen 3: A question asking learners to look at the equation of the line of best fit on page 1.10 and determine what fraction of the square is the large triangle. The answer is provided as 3/4.

Screen 4: A graph of a coordinate plane showing a square with vertices at (-2, -2), (2, -2), (2, 2), and (-2, 2). A line with the equation $y = -x$ passes through the origin and the midpoints of the square's sides. Another line with the equation $y = 0.5x - 1$ passes through the midpoints of the top and bottom sides of the square.

This activity is suitable for middle to high attainers in Year 9 or above.

Instructions are given within the TI-Nspire document, and the objectives are listed below.

Objectives:

- Interpret a written statement using dynamic geometry
- Calculate fractions of a square empirically or theoretically
- Collect data in lists
- Use a scattergraph and line of best fit to compare two data sets
- Prove two triangles are similar
- Use similarity to find the linear ratio of two triangles
- Extend a problem
- Explore the links between geometry and algebra
- Solve simultaneous equations graphically

This activity shows how powerful the links between geometry and algebra can be. It also links geometry, algebra and statistics with some number work (learners need to be able to recognise fraction equivalents of decimals).

Teaching notes

Learners may have to be shown how to use the dynamic geometry to construct the shape, but you may wish to skip that. However the time investment is well worth the effort since it helps them in the future. It was a steep learning curve for the author, but one that helped consolidate his knowledge of TI-Nspire!

Complete screen shots

The image shows two side-by-side screenshots of the TI-Nspire CX CAS software interface. Both screens have a top menu bar with buttons for 1.1, 1.2, 1.3, 1.4, RAD, AUTO, and REAL, along with a battery icon.

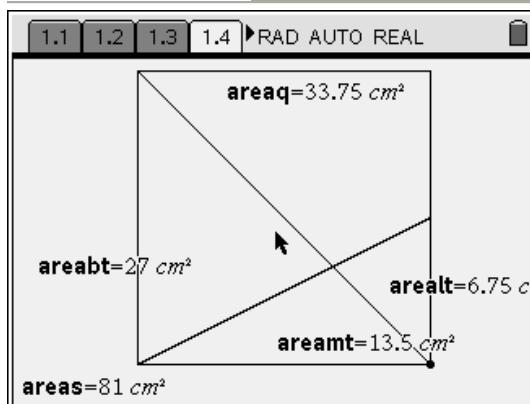
Left Screen (Page 1.3):

- Title:** Square triangles
- Text:** This activity looks at the fractions of a square formed when it is divided by two lines.
- Text:** You can draw the diagram, tables and charts yourself, or use the ones already drawn.
- Text:** You may wish to use paper and pencil first.

Right Screen (Page 1.4):

- Title:** The problem
- Text:** A square is divided into three triangles and a quadrilateral by a diagonal and a line joining one of the other vertices to the midpoint of a side.
- Text:** What fraction of the square is each part?
- Text:** You can draw the diagram on page 1.3 or see it on page 1.4.

Page 1.3 is a blank geometry page for learners' own construction.



1.2 1.3 1.4 1.5 ► RAD AUTO REAL

You can collect the data about the area of each shape yourself, filling in the lists on page 1.6.

Or else use the data from page 1.4 in the lists on page 1.7. Moving point will gather more data.

The variables are defined below.

Page 1.6 is a blank lists page for learners' own data capture.

◀ 1.4 1.5 1.6 1.7 ► RAD AUTO REAL

	A lt	B mt	C bt
◆	=capture(area _{lt} ,1)	=capture(area _{mt} ,1)	=capture(area _{bt} ,1)
1	7.84083	15.6817	
2	7.52083	15.0417	
3	7.36417	14.7283	
4	7.20833	14.4167	
5	6.97875	13.9575	

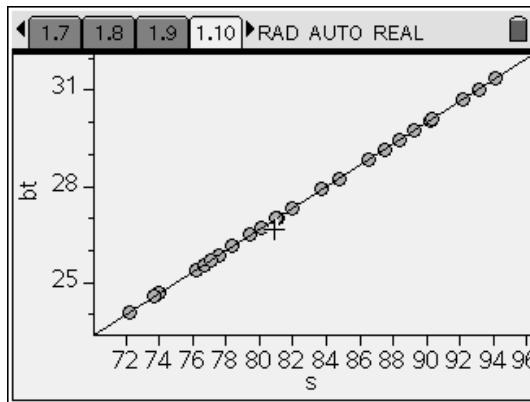
A1 |

1.5 1.6 1.7 1.8 ► RAD AUTO REAL

You can draw a scattergraph of your own data on page 1.9, or use the one on page 1.10 to explore the data.

The scattergraph on page 1.10 is set up to look at the relationship between the areas of the big triangle and the square.

Page 1.9 is a blank scattergraph page for learners' own list data.



1.8 1.9 1.10 1.11 ► RAD AUTO REAL

Question

Look at the equation of the line of best fit on page 1.10

What fraction of the square is the large triangle?

Answer

◀ 1.9 1.10 1.11 1.12 ► RAD AUTO REAL

Now change the variables on page 1.10, or 1.9 (if you set them up yourself) to find out what fraction of the square the other shapes are.

Then record what you find below.

1.10 1.11 1.12 1.13 ► RAD AUTO REAL

Question

Geometrically the large and small triangles are similar. Why?

Answer

◀ 1.11 1.12 1.13 1.14 ▶ RAD AUTO REAL

Question

The ratio of the sides in the large and small triangles is 2:1. Why?

Answer ▾

[Empty box for answer]

◀ 1.12 1.13 1.14 1.15 ▶ RAD AUTO REAL

Question

How does the previous ratio help you work out what fraction of the square the triangles are?

Answer ▾

[Empty box for answer]

◀ 1.13 1.14 1.15 1.16 ▶ RAD AUTO REAL

Now find out what happens when the line is joined to a point that divides the side in the ratio 1:2, rather than in the ratio 1:1 (the midpoint).

You will need to set up a new diagram.

◀ 1.14 1.15 1.16 1.17 ▶ RAD AUTO REAL

On the next page we will explore the same problem by looking at the situation algebraically.

The square has been put on a grid and the equations of the lines are shown.

Can you work out the areas of each shape

