

Slope Fields Forever

ID: 12321

Time Required
20 minutes

Activity Overview

In this activity, students will dynamically explore a particular solution to a differential equation for different initial conditions. Slope fields are investigated. Self-check multiple-choice questions help students build the foundation for the DE matching activity. Students will check their solutions by using the Differential equation Graph type. An extension involves the use of the **deSolve** command.

Topic: Slope fields

- Graphical solutions to differential equations.
- Identify the differential equation for a certain slope field.

Teacher Preparation and Notes

- Students will write their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students can then press **menu** and select **Check Answer** (or **ctrl** + **▲**). If desired, by using the TI-Nspire Teacher Edition software, teachers can change these self-check questions to exam mode so students cannot check their answer. On any question, click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.
- Students should know how to navigate between pages with **ctrl** + **▶** or **◀**, and navigate between split screen with **ctrl** + **tab**.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “12321” in the keyword search box.**

Associated Materials

- SlopeFieldsForever_Student.doc
- SlopeFieldsForever.tns
- SlopeFieldsForever_Soln.tns

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Slope Fields Introduction (TI-Nspire technology) — 16097
- Logistic Growth, Differential Equations, Slope Fields (TI-89 Titanium) — 5514
- Differential Equations and Slope Fields – Activity 7 (TI-89 Titanium) — 4279

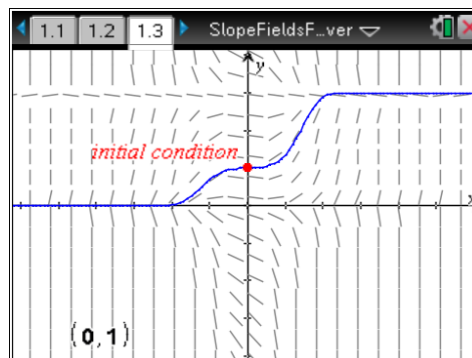
Part 1 – Introducing Slope Fields

On page 1.2 and 1.3, students are introduced to a slope field. Students will learn that slope fields graphically give a particular solution to a differential equation when given an initial condition. Students will grab the initial condition point and move it to dynamically explore the solutions to the DE $y' = \frac{1}{2}x^2(3y - y^2)$.

Discussion Questions

- Looking at the differential equation, what values give a slope of zero? Can you see that this is true on the slope field? (Answer: $x = 0$, $y = 0$, $y = 3$)
- What value might there be in using a slope field to graphically find a solution? (Answer: If students have already been introduced to separable differential equations, they may think that all DEs can be solved to find the general and particular solution. However, many differential equations cannot be solved algebraically. Numerically, the slope at each point on the graph can be determined and a graphical solution can be found.)

Question 2 asks students to see why the slopes are what they are for several representative points. Question 3 has students consider what makes the differential equation zero. Question 4 leads students in beginning the thought process for matching a differential equation to a slope field. See Part 2 for more details.



You saw that the slope field for the DE $\frac{dy}{dx} = \frac{1}{2}x^2(3y - y^2)$ has many short segments that show the slope at that point. Find the value of the slope at $(-1, -1)$, $(0, -3)$, $(1, 1)$, $(1, -1)$ using the equation. Record your answers below. Confirm graphically by comparing your slope with what it appears to be on page 1.3.

Slope for $(-1, -1) =$ for $(0, -3) =$
 for $(1, 1) =$ for $(1, -1) =$

Which DE produces this slope field?

The slopes depend on

x

y

both x & y

$\frac{dy}{dx} = y^2$

$\frac{dy}{dx} = x + 1$

$\frac{dy}{dx} = \frac{1}{2}y$

TI-Nspire Navigator Opportunity: Quick Poll
See Note 1 at the end of this lesson.

Student Solutions

1. Students may observe that the slope is different at different points. It is not the same all along a horizontal or vertical line. This shows that the DE depends on both x and y . As the initial condition is moved about, different curves are formed. The solution always approaches $y = 0$ and/or 3 and levels out when $x = 0$.
2. $m = -2$ for $(-1, -1)$, $m = 0$ for $(0, -3)$, $m = 1$ for $(1, 1)$, and $m = -2$ for $(1, -1)$
3. The slope is zero when $x = -1$.
4. The slope only depends on y , since it has the same slope along horizontal lines.
 $\frac{dy}{dx} = \frac{1}{2}y$. (It cannot be the first choice because the slopes are not always positive.)

Part 2 – DE matching

The following are the strategies mentioned on the student worksheet. This is what students can think about in order to identify the DE that corresponds to the slope field. The last of these 5 will often be the most important in determining the match. It is recommended that the teacher pause and allow students to give the answer to the problems listed below.

- i) If the DE only depends on x , then the slopes are the same along _____ (Answer: vertical lines).
- ii) If the DE only depends on y , then the slopes are the same along _____ (Answer: horizontal lines).
- iii) If the DE is something like $\frac{dy}{dx} = x + y$, then the slopes are the same along _____ (Answer: oblique lines).
- iv) Does the slope field look sinusoidal? There will be a sine or cosine function in the DE.
- v) Consider a specific point. What values make the slope zero? What values of x and y make the slope 1?

TI-Nspire Navigator Opportunity: Quick Poll

See Note 2 at the end of this lesson.

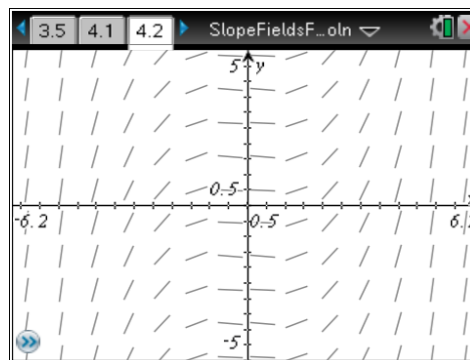
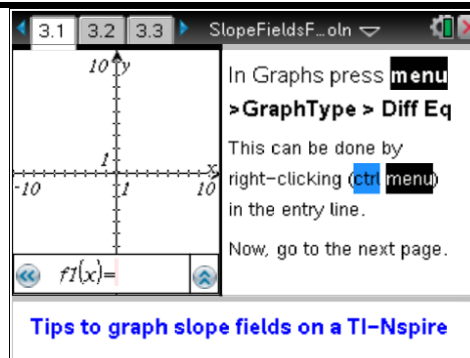
Page 3.1-3.5 provide a tutorial for using TI-Nspire’s built-in capability of graphing slope fields.

Important TIPS: On the TI-Nspire, slope fields can be created on a Scratchpad Graph or on a Graph application of a current document. Press **menu** > **Graph Type** > **Differential Equation**, or right-click (**ctrl menu**) in the entry line.

You can change the y to a different letter, like P for population. However, the independent variable will need to be x . For example, $dP/dt = P+t$ will be entered as $y1' = y1+x$.

You can only have one DE graphed at a time. Use the check boxes to the left of $y1'$, $y2'$, etc., to select one and deselect the others.

Students will check their answers to the DE matching activity using the slope field on page 4.2. The window size is set up to correspond to the worksheet.



TI-Nspire Navigator Opportunity: Live Presenter

See Note 3 at the end of this lesson.

Student Solutions

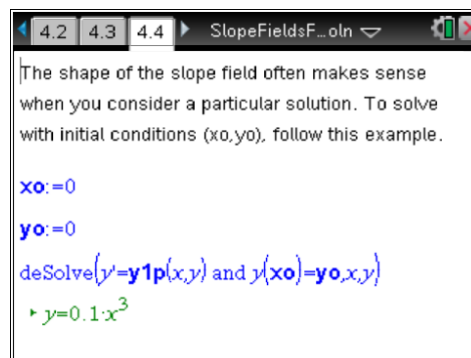
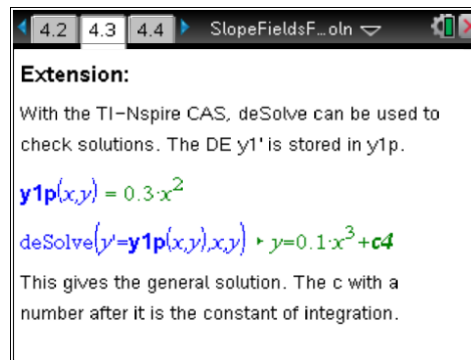
- | | | | |
|------|------|------|-------|
| 1. E | 4. G | 7. I | 10. K |
| 2. C | 5. B | 8. H | 11. J |
| 3. A | 6. D | 9. F | |

Extension

The TI-Nspire CAS can use the command `deSolve` to find the general or particular solution of a differential equation. Examples are provided on page 4.3 and 4.4. The examples take advantage that a differential equation is graphed on 4.2 by using `y1p` as the $y1'$ that is already entered. The differential equation dy/dx must be entered as y' . The syntax after the comma provides the independent variable, then the dependent variable. The “and” followed by the initial condition is optional as shown on 4.3.

`deSolve(y'=f(x,y) and y(x0)=y0,x,y)`.

The initial conditions in the Math Boxes on page 4.4 can be changed by deleting what it is already stored as.



TI-Nspire Navigator Opportunity: Screen Capture

See Note 4 at the end of this lesson.

TI-Nspire Navigator Opportunities**Note 1****Part 1, Quick Poll**

Send a Quick Poll for the questions on page 2.2.

Note 2**Part 2, Quick Poll**

Send a Quick Poll for the three extra questions that are provided in the “teacher” document SlopeFieldsForever_Soln.tns on page 2.3-2.5. These questions lead students to the DE matching strategy. The student .tns file has the strategy listed out on page 4.1 and are visible if they scroll down.

Note 3**Part 3, Live Presenter**

After giving students a few moments to look through the tutorial from page 3.1-3.5 have a volunteer become the *Live Presenter* to demonstrate the process of graphing slope fields on a *Scratchpad*.

Note 4**Extension, Screen Capture**

After student have tried the matching activity, have students check their work. Have pairs of student graph the same number. Then use *Screen Capture* to compare page 4.2 and 4.3 of these student. Discuss the constant of integration and how that corresponds to the family of values.

Discuss “what was the most helpful part of this TI-Nspire file?” and “What questions do you have? What is not as clear as they would like regarding slope fields or differential equations?” Consider making this last question an open response *Quick Poll* so students can anonymously respond.